BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

EDITED

IN CONSULTATION WITH FRANCIS GALTON

BY

W. F. R. WELDON   KARL PEARSON

AND

C. B. DAVENPORT

VOLUME IV

JUNE 1905 TO MARCH 1906

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Memoirs.

Subject and Name Indices to the first five volumes will appear with the next volume.

* By an unfortunate slip of the engraver, this plate is erroneously entitled *Round* Barrow Crania. N. facialis.

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BIOMETRIKA.

RESULTS OF CROSSING GREY (HOUSE) MICE WITH ALBINOS.

BY E. H. J. SCHUSTER, M.A., New College, Oxford.

THE experiments here recorded were commenced in the department of comparative anatomy in Oxford in the autumn of 1902, but various personal circumstances have delayed the publication of any account of them for more than a year after their completion.

They were made at the suggestion and with the advice and help of Professor Weldon, and I have therefore numerous reasons for thanking him.

Source of Material.

The original stock of white mice was obtained from various dealers, among whom Jeunet, of Paris, figures most largely. Some also were given to me by Mr Darbishire.

The grey mice were caught in numerous houses in Oxford, and were brought to me by their captors in answer to an advertisement in *The Oxford Times.* In order to avoid, as much as possible, any possibility of using grey mice with "fancy" blood in their veins, those caught in the precincts of the Museum, where Mr Darbishire's experiments had been going on for some months, were in all cases rejected.

Previous Experiments of a Similar Nature.

Breeding experiments similar to mine have been made by Cuénot ('02 and '03) and Allen ('04), and the fact that my results do not seem to be in exact accordance with theirs must serve for an apology for the appearance of an otherwise superfluous publication. That such differences should have arisen at first seemed surprising; but an ingenious theory of Cuénot's explains the possibility of their occurrence, and I should like, before going on to the consideration of results, to give a short account of this theory.

Cuénot ('04) suggests that all mice, including albinos, have some definite colour in them, but that it needs a special faculty to make this latent colour apparent on their coats, and that the difference between all coloured mice and all albino mice lies in the possession or non-possession of this faculty.

Coat-colour, then, is a compound character which must be split up primarily into two component parts : (1) nature of the colour; (2) power, or lack of power of producing colour. He represents the constitution of the gametes of a mouse with regard to these two things by two letters, the first indicating their condition with regard to (2). When they are colour-producing the letter C (chromogène) is used, when non-colour-producing A (albino). The second letter denotes the nature of the colour. Thus G grey, B black, and Y yellow.

For example :

CG represents an ordinary house mouse.

AG represents an albino mouse whose gametes would produce the grey colour if possessed of the colour-producing faculty.

$CB =$ a pure black mouse.

AB a mouse similar to AG, only with Black for the latent colour.

CY a pure yellow mouse.

AY albino with yellow latent.

The six symbols described above denote six pure races of mice. When any two of these are mated together hybrids will of course result whose gamete formula is a combination of the gamete formula of the two parents. The colour of the hybrids is determined by the relative dominance : (1) of colour productiveness and albinism; (2) of the nature of the two colours latent in the two parental gametes. As in all cases colour dominates albinism, it only remains to be considered which of the three colours employed is dominant over which of the others. Here we find G dominant over B, Y dominant over B, Y dominant over G. Thus a grey CG crossed with a white AY will produce yellow mice whose formula is CG-AY. A black CB crossed with either a grey CG or a white AG will produce grey mice, though in the first case the formula will be CB-CG, and in the second it will be CB-AG.

All this is not a modification nor an extension of Mendel's laws, but merely an application of them to a special case ; Cuénot's theory as to the two factors which determine coat colour renders such an application possible. The further behaviour of the hybrids when crossed among themselves or with pure mice can therefore be predicted by any one conversant with these laws and possessed of a knowledge of the gametic formula of the hybrids used.

A third element is introduced by the fact that the colour of a mouse's coat is not always uniform, but may be marked with white. This marking may occur with mice of any colour; it is transmitted independently of the colour and may be latent in albinos. A uniform coat dominates a piebald coat.

Thus if one mates a piebald yellow mouse with an albino with uniform greyness

latent in it one will obtain a uniform yellow hybrid, for yellow dominates grey, uniformity piebaldness, and colour-productiveness albinism. If such yellow hybrids are paired together and a sufficient number of young obtained they should be in the following proportion: 16 albinos, 9 piebald yellow, 27 uniform yellow, 3 piebald grey, and 9 uniform grey: see Appendix II, p. 12. Eight of these 64 mice would be pure, all of different constitutions, and there would be two of each of 12 different kinds of hybrids, four of 6 different kinds, and eight of one kind. Thus a reasonable degree of complication is obtained when only two colours are employed; when more than two colours are introduced the complication is largely increased. I have brought forward this example to explain my reasons for not examining in detail the occurrence of each of the various types of mice produced in my experiments. For in addition to the inherent difficulties of such an examination, I am in total ignorance as to colours latent in the original stock of white mice.

Types of mice produced.

Bateson ('03) gives a list of a number of types of Fancy mice with a statement of the colour of the actual pigments contained in each, and almost all the mice produced in these experiments can be referred to one or other of the groups in this list; I cannot, however, guarantee that I have referred them rightly, as no microscopic examination of them has been made, so that the colour as it appears to the naked eye is the only basis of the classification.

After the name of each colour is given the abbreviation employed as a symbol for it. In giving instances of Cuénot's theory, for the sake of uniformity I have used my own abbreviations and not those employed in his original paper.

(1)   *Grey (G)* (Cinnamon or Agouti) figure 4.

(2)   *Golden agouti (GA).*

(3)   *Yellow (Y).*   In this are included both yellow and cream, as these shade insensibly into one another and form indeed a highly variable group of colours.

Figures 5, 7, 8 represent three conditions of yellowness, but the lightest yellow of all obtained was almost completely white, with but the faintest suggestion of pigment. It had, however, perfectly black eyes like all the coloured mice produced.

(4)   *Sooty Yellow (SY).*

(5)   *Chocolate (Ch.)* figure 6.

(6)   *Chinchilla (Cc.)* figure 2.   I cannot find this anywhere on Bateson's list. In the best specimens it is a beautiful silvery grey in colour but it shades insensibly into the ordinary grey coat, figure 3 being one of the intermediate forms. Allen ('04) records having obtained some light grey mice in crossing house mice with albinos, these may possibly have been of the same nature as my Chinchillas. His apparently died in childhood so that he had no opportunity of breeding from them.

Intermediate forms between pairs of these colours did occur and are represented by the symbols of the two colours joined by a hyphen. Thus figure 1 represents

a mouse intermediate in colour between a black and a grey, and would therefore be denoted by *B-G*.

Each of these colours occurred both uniform and marked with white. The white marking, as Cuénot observes, is subject to continuous variation. The parts first affected are the tail and the paws; then a small white mark may be found on the belly and on the top of the head. The white mark on the belly may grow forwards and backwards, so as to cover the whole ventral surface; laterally it sends out arms of whiteness immediately in front of the hind legs, these grow dorsalwards and may meet and form a complete white belt. The mark in the head spreads forwards till it forms a good white blaze on the forehead, or further still till the front of the muzzle is completely white, and eventually it becomes continuous with the ventral white mark.

The letter *P* in conjunction with the symbol for a colour means that the mouse is piebald and of this colour; no attempt is made to indicate the degree to which the white marking has spread, though of course a record of this has been kept.

Results.

106 white mice were mated with grey mice and of these crosses 74 proved fertile.

70 families contained only grey mice. Of the remaining four 2 contained mice of varying shades of yellow in addition to grey, and the original chinchillas appeared in the other two. Thus the first generation of hybrids consisted of 342 mice, of which 329 were grey, seven yellow, and six chinchilla.

The appearance of yellow in this generation is easily understood, if we assume, in the absence of any definite information on the subject, that the albino parents in each case were hybrids containing yellow and some other colour latent. Such mice would according to Cuénot's hypothesis produce some yellow and some grey when mated with pure grey mice, as yellow is dominant over grey and grey over any other colour which they might contain.

Having no sort of evidence either as to the nature of the chinchilla coat, or as to its behaviour when pure and mated with other colours, it would be futile to indulge in speculation on the possible reasons for its occurrence here.

The yellow mice when mated together produced yellow and grey and white. Thirteen young from four families were obtained, and of these five were grey, six yellow and two white. These numbers are of course too small for the proportions to have any significance, but the theoretical expectation for sixteen mice would be four albino, nine yellow and three grey.

From the original seven yellow mice all the yellows and sooty yellows occurring in the experiments were produced.

The original pairs are labelled *A* and *B* in the Appendix, the *A*s having the ♂ white and the *B*s the ♀.

The offspring of the *A* and *B* crosses were mated with white mice and with one another.

In the former case the crosses produced are labelled C and D, C having the ♂ a product of an A or B cross and the ♀ an albino, and D being the reciprocal of this.

The Cs produced 282 young of which 130 were albino and 152 coloured, the Ds produced 255 young of which 131 were albino and 124 coloured. Thus together we have 537 young of which 261 are albinos and 276 coloured. Considering merely the characters, colour productiveness and albinism, the Mendelian expectation would be 268·5 of each, so that the actual result obtained is in fairly close agreement with it.

If we go on to an enumeration of the actual colours obtained, we find among the Cs 25 black, 102 grey, 2 intermediate between grey and chinchilla, 2 golden agouti, 16 grey marked with white, and 7 black marked with white.

Among the Ds we have 26 black, 77 grey, 1 golden agouti, 1 yellow, 1 sooty yellow, 1 chocolate, 12 grey marked with white and 7 black marked with white.

Speculation as to the proportion of the various colours seems to be useless, but if one assumes that some of the albinos employed both in the A and B and in the C and D crosses had any other characters but uniformity and greyness latent in them, then one would expect that these characters would become apparent in the coats of some of the mice produced.

This remark applies with equal force to the consideration of the results of the various other crosses described below, but it will not be repeated again.

The offspring of the A and B crosses were paired together and these pairs are labelled E. The Es produced 119 albinos, 27 blacks, 17 intermediate between black and grey, 201 greys, 9 intermediate between chinchilla and grey, 9 chinchilla, 8 golden agouti, 9 yellow, 3 sooty yellow, 1 chocolate, 19 grey marked with white, 3 black marked with white, 1 chocolate marked with white, and 1 chinchilla marked with white. That is to say 119 albinos and 308 coloured. 119 is at any rate a possible approximation to the Mendelian quarter. The 308 coloured mice should be composed of $\frac{1}{3}$ pure colour producers and $\frac{2}{3}$ hybrids between albinos and colour producers.

The F and G crosses were made to test the truth of this. The Fs have coloured offspring of the Es for ♀s and albinos for ♂s. The Gs are reciprocal. Out of 87 of these crosses 32 produced no albino young and 55 produced some albino young. 32 is a fair approximation to $\frac{1}{3}$ of 87.

The colours of the young contained in these 32 families is as follows: 18 black, 1 intermediate between black and grey, 95 grey, 2 intermediate between chinchilla and grey, 1 yellow, 8 sooty yellow, 1 golden agouti, 6 chocolate, 20 grey marked with white, 13 black marked with white.

The 55 families produced 141 albinos and 137 coloured mice. If these are to be regarded as hybrids equal numbers of each should be expected; and a fair degree of equality is obtained. If we add these numbers to the results of the C and D crosses with which they are to be associated, we get out of 815 mice 402 albino and 413 coloured.

The actual colours obtained in the 55 *F* and *G* families are as follows:—
23 black, 74 grey, 2 golden agouti, 1 chinchilla, 1 yellow, 6 sooty yellow,
2 chocolate, 1 yellow marked with white, 24 grey marked with white and 3 black
marked with white.

Cross *L* was made by breeding together those coloured mice which had been
used in the *F*s and *G*s and which had produced only coloured young. Their
offspring were all coloured, consisting of 57 grey, 3 grey marked with white,
3 black and 5 sooty yellow.

In cross *M* the remaining coloured mice which had been used for the *F*s and
*G*s were mated together and produced 30 albinos and 71 coloured mice, the colours
being as follows:—52 grey, 5 grey marked with white, 7 black, 1 golden agouti,
6 sooty yellow.

The white offspring of the *E*s were also mated together, and in all cases bred
true.

In conclusion, it is hardly necessary to point out that with regard to characters,
colour-productiveness and albinism the mice under consideration here behave in
complete accordance with Mendel's laws, both with regard to dominance and to
segregation. If one considers the nature of the colour and the question as to
whether it is distributed uniformly or is marked with white there is nothing in
my results incompatible with similar behaviour with regard to these characters
also.

Certain minor difficulties do indeed suggest themselves, for instance the
occurrence of intermediate forms and the tendency of yellow mice when crossed
either with white or grey hybrids to produce sooty yellow young; this colour
figuring somewhat conspicuously in some of the latter generations.

It is only to be regretted that the true nature of the albinos was not known
and it is to be hoped that in the future the experiments may be repeated without
this very serious drawback.

LIST OF LITERATURE REFERRED TO.

'02. L. Cuénot. "La Loi de Mendel et l'Hérédité de la Pigmentation chez les Souris."
Arch. Zool. Exp. et Gén. Notes et Revue, 1902.

'03. ——. Ibid. 1903.

'04. ——. Ibid. 1904.

'03. W. Bateson. "Colour Heredity in Fancy Mice and Rats." P.Z.S. 1903, Vol. II.

'04. Glover M. Allen. "The Heredity of Coat-Colour in Mice." Proc. Amer. Acad.
Art. and Sci. Vol. XL.

DESCRIPTION OF FIGURES.

These are taken from water-colour drawings ad nat. by Miss Adeline Ritchie.

(1) Intermediate between grey and black.
(2) Chinchilla.
(3) Intermediate between chinchilla and grey.
(4) Grey.
(5), (7), (8) Three shades of yellow.
(6) Chocolate.

II.

IV.

I.

III.

V.   VI.   VII.   VIII.
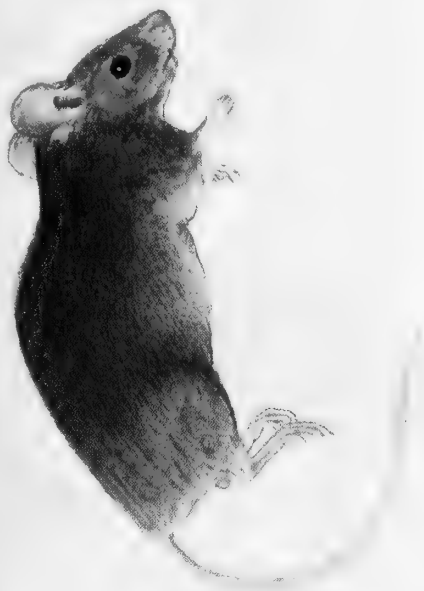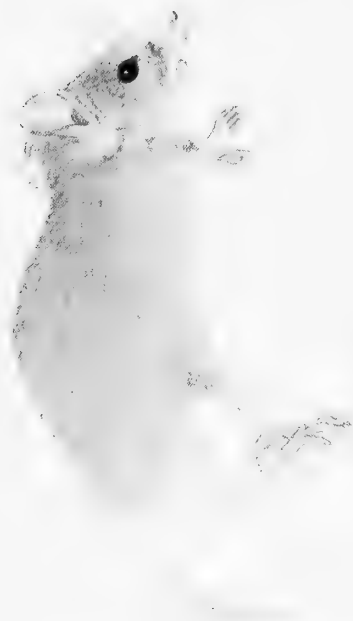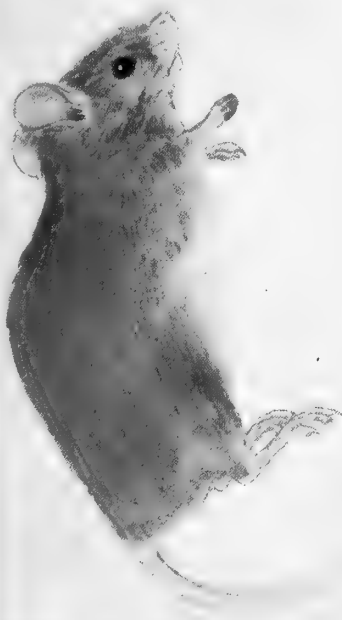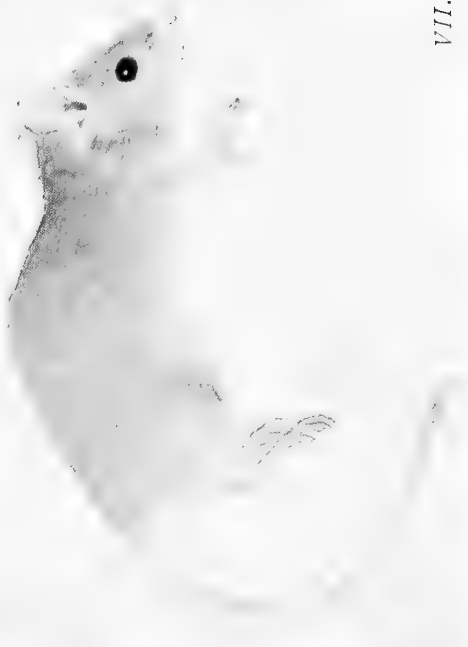
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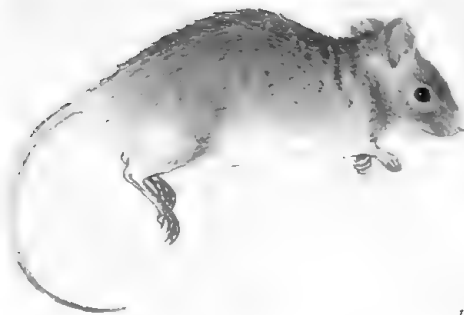
II.

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VII.

VIII.

COAT-COLOURS IN MICE.
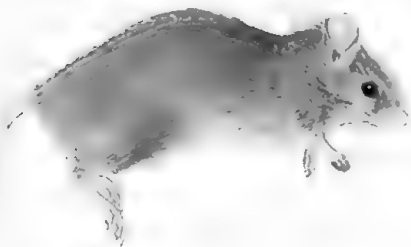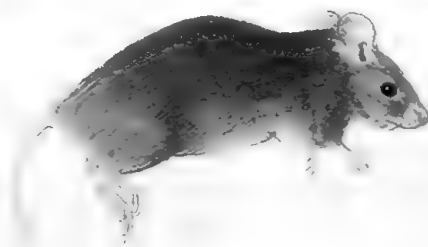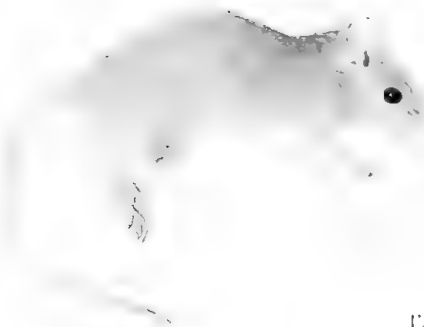
APPENDIX I.

A List of all the Mice Produced.

The following abbreviations are used in describing the mice:—

A, Albino.   Cc. Chinchilla.   SY, Sooty Yellow.

G, Grey.   Ch. Chocolate.

Y, Yellow (including Cream).   GA, Golden Agouti.

Intermediate colours are represented by the symbols of the colours between which they lie placed together.

P joined to the symbol for a colour means that the mouse was marked with white.

Crosses A and B.   A, Father Albino, Mother Grey.   B, Mother Albino, Father Grey.   70 A and B pairs produced only G mice.

Family.   A 32, 3Y and 2G.

          B 17, 4Cc.

          B 26, 4Y, 2G.

          B 43, 2Cc., 2G.

Cross C.   Products of A and B Crosses mated to Albino Does.

	Results		Results
C 1	2A, 1B, 1G	C 41	3A, 1G
C 2	3A, 2B	C 42	2A, 2G
C 3	1A, 1GA, 1BP	C 43	2A
C 4	3A, 2B, 2GP	C 45	2A, 3GP
C 5	2A, 2G	C 46	3A, 1BP
C 6	4A, 1G	C 48	3A, 1B, 1G
C 7	3A, 3B	C 49	2A, 1G
C 8	1G, 3G	C 47	1A, 1G
C 9	3A, 2B, 1G	C 50	2A, 2G, 1GP
C 10	6G	C 51	3A, 1B
C 13	2A, 2B, 1G, 1BP	C 52	2A, 2B
C 12	1A, 5G	C 53	3G
C 14	1A, 5G	C 54	4A, 4G
C 15	2A, 1B, 1G, 1GP	C 57	2A, 1B
C 16	2A, 1G	C 59	3A, 3B
C 19	5G	C 60	1A
C 20	5G	C 63	4A, 2G, 2GP
C 21	3A, 4G	C 62	3A, 2G, 2GP, 1BP
C 22	2A, 2G, 2Ch. G	C 61	3A, 4G, 2GP
C 23	5A	C 64	1A, 2G, 1GP
C 25	2A, 2B, 2G	C 65	2A, 2B
C 26	1A, 5G	C 66	2A, 2G
C 27	3A, 2G	C 67	1A, 3G
C 28	1A, 2BP	C 68	4A, 1G
C 29	4A, 1G	C 69	3A
C 30	3A, 1B, 3G, 1GP	C 71	2A, 2G
C 32	3A	C 72	1G
C 37	2A, 1B, 3G	C 73	4A, 1B, 1G
C 40	3A, 3G, 1GP	C 74	1A, 1G, 1GA
C 39	3A, 1G		
Total of Cs	130A, 25B, 102G, 2Ch. G, 2GA, 16GP, 7BP		

*Cross D.   Products of **A** and B Crosses mated with Albino Bucks.*

	Results		Results
D 2	3*G*, 2*G*	D 33	2*A*, 2*G*, 1*GA*
D 3	1*A*, 1*GP*	D 35	2*A*, 1*B*
D 4	1*A*, 1*G*, 1*GP*, 1*BP*	D 37	2*A*, 2*G*
D 6	2*A*, 1*G*	D 38	2*A*, 1*B*, 2*G*
D 7	4*A*, 1*B*	D 40	3*A*, 1*G*
D 8	1*A*, 4*G*	D 41	4*A*, 2*G*
D 9	2*A*, 4*G*, 1*GP*, 1*BP*	D 42	3*A*, 1*B*, 2*G*
D 12	2*A*, 1*B*	D 44	2*A*, 2*G*, 1*GP*
D 13	2*A*, 3*G*, 1*GP*	D 45	2*A*, 1*GP*
D 14	2*A*, 1*B*, 2*G*, 1*BP*	D 46	3*G*
D 15	2*A*, 2*G*, 1*BP*	D 47	2*A*, 4*B*, 1*G*
D 16	1*A*, 2*B*, 2*GP*	D 48	2*A*
D 17	3*A*, 2*B*	D 52	3*A*, 2*BP*
D 18	2*A*, 1*B*, 2*G*	D 50	2*A*, 3*G*
D 19	2*A*, 2*G*	D 53	2*A*, 1*G*, 1*GP*
D 20	5*A*, 1*G*	D 54	1*A*, 1*G*, 1*GP*
D 21	3*A*, 1*G*	D 55	5*A*, 1*G*
D 22	2*A*, 3*G*	D 57	4*A*, 1*B*, 1*BP*
D 23	3*A*	D 58	2*A*, 1*B*, 1*G*
D 24	2*A*, 3*G*	D 51	2*A*, 1*B*
D 26	1*A*, 3*G*	D 59	4*A*
D 27	3*A*, 1*B*, 1*G*	D 60	3*A*, 1*Y-G*
D 28	3*A*, 1*B*, 1*G*	D 61	2*A*, 1*G*, 1*Y*, 1*GP*
D 29	1*B*, 3*G*	D 62	2*A*, 1*G*
D 30	3*A*, 3*G*	D 64	3*A*, 1*G*
D 32	4*A*, 3*B*	D 65	1*A*, 1*B*
D 66	3*A*, 1*Ch.*	D 67	1*A*, 1*B*, 1*GP*, 1*G*
		D 68	4*A*
Total of *D*s	131*A*, 26*B*, 7*BP*, 77*G*, 12*GP*, 1*GA*, 1*Y*, 1*SY*, 1*Ch.*		
Total of *C*s and *D*s	261*A*, 51*B*, 14*BP*, 179*G*, 28*GP*, 2*Cc-G*, 3*GA*, 1*Y*, 1*SY*, 1*Ch.*		

Cross E. Products of A and B Crosses mated with One another.

	Results			Results
E 2	1A, 1B, 2G		E 61	1A, 2G, 1GA
E 3	2A, 2G		E 62	1B, 2G, 2GA, 1GP
E 5	2B, 2G		E 63	3A, 1B, 2G
E 8	2G		E 64	1A, 2B, 2G
E 9	2A, 1G		E 66	1B, 3G
E 10	1A, 4G		E 67	1A, 3G
E 12	1A, 2B, 2G		E 68	2A, 2G
E 11	4G		E 69	1A, 3G
E 14	1A, 2B, 1G		E 70	1A, 4G
E 15	4G		E 71	1A, 2B, 4G
E 16	4G		E 72	1A, 1B, 2G
E 17	1A, 4G		E 73	1A, 1CcG
E 18	3A, 3G		E 74	2G, 1GP, 2CcP
E 19	1A, 1B, 1G		E 75	1A, 1CcG, 1Cc
E 20	1A, 3G, 2GP		E 76	1A, 3G
E 21	2A, 2G, 1GP		E 77	2A, 3G
E 22	3A		E 78	1A, 4G
E 23	3G		E 79	3A, 2G
E 24	2A, 2G, 1GP		E 80	1A, 3G, 1GP
E 25	3A, 2G		E 82	1A, 3G
E 26	1A, 2G		E 83	2A, 3G
E 27	1A, 1G		E 84	1A, 3B, 1GP
E 28	2A, 2G		E 85	2A, 1G, 1Ch., 1GP
E 29	1B-G, 3G		E 86	2A, 2B-G, 1G
E 30	1A, 3GA, 1Ch. P		E 87	2A, 3G
E 31	1A, 3G, 1GA		E 88	1A, 2G
E 33	3A, 1G, 2Y		E 89	2A, 3G
E 34	1A, 4G		E 90	2A, 3G
E 35	4A, 1G		E 94	3B-G
E 40	1B, 3G		E 95	1A, 1B-G, 1GP
E 41	3G		E 96	1A, 1GP
E 42	2G, 1SG		E 97	1A, 1B, 1G
E 44	2A, 2G		E 98	2A, 3G
E 45	1A, 3G		E 99	1A, 3G
E 46	2A, 2G		E 100	1A, 1B, 2G
E 47	1A, 1G, 1SG		E 101	1A, 1B, 3G
E 48	2A, 1G, 1SG		E 102	1A, 2G
E 49	1A, 2G		E 103	1A, 2G, 1GP
E 50	2G, 1GA, 1Y		E 104	3BG
E 51	4G		E 108	5G
E 52	1A, 1B, 1GP		E 105	5BG, 1G
E 54	2A, 2GP		E 109	1BG, 1G, 1GP, 2BP
E 54*	1A, 1GP		E 110	2G
E 55	3A, 1B, 1G, 1GP		E 111	1A, 2Y
E 56	1A, 2G		E 112	1A, 3Cc
E 57	3A, 1G		E 113	2G
E 58	2A, 2GP		E 113*	1A, 2CcG, 1Cc
E 59	3A, 1G		E 114	1A, 1G, 2Cc
E 60	5G		E 114*	1A, 2CcG, 2Cc
E 36	3G		E 115	2G, 2Y
E 37	2A		E 116	1A, 1CcG, 1Y
E 38	1A, 1B, 2G		E 117	1G, 1Y
E 39	1B, 2G			

Total of Es	119A, 27B, 17B-G, 201G, 9CcG, 9Cc, 8GA, 9Y, 3SY, 1Ch., 19GP, 3BP, 1Ch.P, 1 Cc.P=119 Albino, 308 coloured.

*Cross F. Being coloured ♂ Products of E Crosses mated with **Albino** ♀s. Those which produced Albinos among their young are taken first.*

	Results		Results
F 1	3A, 2G	F 23	3A, 1GP
F 2	4A, 2G	F 24	1A, 4G
F 3	2A, 2G	F 31	3A, 2G
F 4	3A	F 32	1A, 1B, 1GA
F 5	2A, 2GP	F 33	3A
F 6	2A, 2BP	F 34	1A, 2G
F 10	3A, 3G	F 35	2A, 3G
F 13	2A, 1GP, 1BP	F 36	1A, 1G
F 14	2A	F 39	1A, 1Cc, 2GP
F 14*	4A, 1B	F 43	1A, 1B, 1G, 1GP
F 16	4A, 1G	F 44	2A, 1G
F 22	3A		

Total of *F*s containing albinos. 53A, 3B, 24G, 1Cc, 1GA, 7GP, 3BP.

*Cross G. Being coloured ♀ Products of E Crosses mated with **Albino** ♂s. Those which produced Albinos among their Young.*

	Results		Results
G 1	4A, 1G	G 31	2A, 1B, 1G
G 2	4A, 1G, 1GA	G 32	1A, 4G
G 3	3A, 2G	G 35	2A, 8GP
G 6	3A, 1G, 1Ch.	G 36	2A, 2B, 3G
G 8	4A, 4B	G 37	4A, 2G, 1GP
G 9	5A, 2G	G 38	2A, 2B, 2G
G 11	4A, 1B	G 39	3A, 2Cc, 3GP
G 12	2A, 3B	G 41	5A, 2B, 1G
G 13	3A, 3G	G 42	5A
G 15	1A, 4G	G 45	2A, 1G
G 17	1A, 1B, 2G	G 46	2A, 1B
G 18	2A, 4G	G 50	4A, 1G, 1Ch.
G 19	2A, 5G	G 51	4A, 2G, 1GP
G 22	1A, 2G, 5SY	G 53	1A, 1B, 3G, 1GP
G 25	4A, 1Y, 1GP	G 52	3A, 1G, 1GP
G 28	2A, 1G, 1SY, 1GP, 1YP	G 47	1A, 2B, 1G

Total   88A, 20B, 50G, 1GA, 1Y, 6SY, 2Ch., 17GP, 1YP

Fs and Gs which have produced no Albino Young.

	Results			Results
F 9	7G		G 4	2G
F 7	4G		G 5	5B, 3G
F 11	2G, 1GA		G 7	10G
F 15	1B, 3G		G 10	2B, 3G
F 19	2G, 2GP		G 14	3B, 1G, 3GP
F 20	2G		G 20	8G, 1GP
F 26	1BP		G 23	1B, 1G, 3BP, 2GP
F 27	3G, 2GP		G 24	6BP
F 29	2B, 2G		G 26	5G
F 30	2B, 2G		G 27	3G, 4GP
F 37	1G, 1Cc.G, 1Ch.		G 29	2G, 4SY
F 38	4G, 1BP		G 30	3G, 1GP
F 41	2G, 1Ch., 1BP		G 33	3G, 1GP, 1BP
F 42	1G		G 34	5G, 2GP
F 45	2G, 3Ch., 1GP		G 48	3G, 1Y, 4SY
F 46	1B-G, 1Ch., 1GP		G 49	2B, 4G
Total	5B, 1B-G, 37G, 2Cc-G, 1GA, 6Ch., 6GP, 3BP		Total	13B, 56G, 1Y, 8SY, 14GP, 10BP

Cross K.   Pairs of Albino Young from Cross E.

These gave all Albino Young.

Cross L.   Pairs made of the Coloured Offspring of E pairs which have given all Coloured Young when Crossed with Whites.

	Results			Results
L 1	4G		L 9	4G
L 2	9G		L 10	1B, 4G
L 3	6G		L 11	3G
L 4	5G		L 13	1B, 4G
L 5	5G, 1GP		L 14	3G, 2GP
L 7	2G, 5SY		L 15	3G
L 8	1B, 5G			
Total	57G, 3GP, 3B, 5SY.			

*Cross M. Pairs made of Coloured Offspring of E pairs which have
given some White Young when Crossed with White.*

	Results		Results
M 1	3A, 3SY	M 11	1A, 1G
M 2	1GA, 2G, 1GP	M 12	2A, 2G
M 3	3A, 1GP, 2G	M 13	5G
M 4	1A, 6G	M 14	1A, 4B, 2G
M 5	3A, 3G	M 15	1A, 1B, 3G
M 6	2A, 2G, 1B	M 16	5G
M 7	2A, 3G	M 17	1A, 2G
M 8	2A, 2GP, 1G	M 18	3G
M 9	1A, 5G, 1GP	M 21	1A, 3G, 1SY
M 10	3A, 2SY, 1G	M 20*	3A, 1B, 1G
Total	30A, 52G, 5GP, 7B, 1GA, 6SY.		

APPENDIX II. (See p. 3.)

The eight pure mice would have the following gametic formulae *CGU*, *CGP*, *CYU*, *CYP*, *AGU*, *AGP*, *AYU*, *AYP*. The last four being white, and the first four coloured respectively, uniform grey, piebald grey, uniform yellow and piebald yellow. Of the 56 hybrids, there are 24 which are hybrids with regard to only one pair of characters;—two of each of the following 12 different kinds: *CGU-CGP*, *CGU-AGU*, uniform grey in colour; *CGU-CYU*, *CYU-CYP*, *CYU-AYU*, uniform yellow; *CGP-AGP*, piebald grey; *CGP-CYP*, *CYP-AYP*, piebald yellow; *AGU-AGP*, *AGU-AYU*, *AGP-AYP*, *AYU-AYP*, albino. There are 24 which are hybrids with regard to two pairs of characters, 4 of each of the following six different kinds, *CGU-CYP*, *AYU-CGU*, *AYP-CYU*, uniform yellow; *AGP-CGP*, uniform grey; *AYP-CGP*, piebald yellow; *AGU-AYP*, albino. Also 8 hybrids with regard to all three pairs of characters; of these there can be only one kind, which is uniform yellow in colour.

BIOMETRICAL STUDIES ON MAN.

I. VARIATION AND CORRELATION IN BRAIN-WEIGHT*.

By RAYMOND PEARL, Ph.D.

CONTENTS.

1. *Introduction.*

THE subject of brain-weight in man has for a long time been given considerable attention by anatomists and anthropologists. The reason for this is obvious. Since the brain is the organ of the mind it appeared to earlier workers that size of brain ought to be an index of intellectual capacity. The substance of the brain is of such physical homogeneity that its weight is a fairly accurate, as well as simple, "size" measure of the organ. Consequently, by earlier workers, brain-weights were collected, studied and discussed to a considerable extent as measures

or indices of brain power. It became evident after a time, however, that there was not the close and definite relationship between brain-weight and intellectual capacity which had been supposed to exist. Individuals of marked intellectual power were found in not a few instances to have brain-weights below the average, while on the other hand it was not at all difficult to find individuals of very mediocre intellectual attainment who possessed brains of unusually large size and weight. These results still held even after rough corrections were made for bodily size, age, etc. While it is thus evident that brain-weight cannot be taken as a close index of intellectual power, holding for individual instances, yet it is clear that, considered from the phylogenetic standpoint, increase in brain-weight and in psychic capacity have in general gone hand in hand, and the weight of the brain increases quite regularly as we go up the taxonomic scale*.

The present trend of investigation in this subject, is rather from the point of view of anthropology than of psychology. Evidently brain-weight is an important and interesting anthropological character, and in this field we may expect significant results.

The method of investigation which has been almost universally followed in brain-weight work has been to tabulate large masses of statistics of weighings, compute means for various groupings, and draw the conclusions which appeared to follow from the tabulations and averages. In other words the only statistical methods which in most cases have been applied to the data have been those of the sociological statistician. Such methods serve fairly well, of course, when only the "types" are wanted, but they are quite inadequate for some of the work which many neurologists have wished to do in this field. Practically all students of the subject have attempted to determine in one way or another the degree of correlation which exists between brain-weight and other physical characters and also age. Knowledge of these correlations is of course much to be desired. The human brain is justly to be regarded as the highest product of organic evolution. Any contribution to a knowledge of the laws governing its variation and correlation cannot fail to be of the greatest interest. Again, only through a knowledge of the degree of the correlation of brain-weight with other characters of the body is it possible to make scientifically such suitable corrections for bodily differences as will make fair any comparison of the brain-weights of different races, or of different groups of the same race. Now, as everyone knows who has even an elementary knowledge of statistics, it is possible to make the same statistical material lead to quite different conclusions, by grouping it in different ways, when the tabulations and averages are the only sources from which conclusions may be drawn. As a matter of fact this has happened in work on brain-weights. Different investigators, working in different ways, have arrived at

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* Cf. Ziehen, Th.: "Das Gehirn, Massverhältnisse," Bardeleben's *Handbuch der Anatomie des Menschen*, Bd. IV. 1—3 Abtheilung, pp. 353—386, 1899, p. 362 et seq.; and more recently Spitzka, E. A.: "Brain-weight of Animals with special reference to the Weight of the Brain in the Macaque Monkey," *Jour. Comp. Neurol.* Vol. XIII. pp. 9—17, 1903.

quite different numerical appreciations of the relation existing between brain-weight and other characters.   The very interesting question of variation in brain-weight has been scientifically investigated only by Pearson*, so far as is known to the writer.

The reason which led me to undertake the present piece of work was primarily a desire to apply adequate statistical methods to a biometrical problem of peculiar intrinsic interest, and for which fairly large collections of reliable data were available.   It was desired to determine as a part of a general plan of work outlined as exactly as possible from the data at hand, what were the actual conditions of variation and correlation in brain-weight.   It was hoped that by such work some light might be thrown on the question of the evolution of man's brain.   The work was originally planned to include the analysis of only one large collection of brain-weight data (see below p. 16) but when the constants for this material had been determined certain of the results were seen to be rather peculiar. Not being satisfied that these indicated the true state of affairs, but thinking that they might be caused by some hidden anomalies of the raw material, it was decided to extend the work to other data in order to test the first results.

The specific problems to which attention has been directed in this work are:

1.   The amount of variation in the weight of adult male and female brains.

2.   The relation of brain-weight to age in the adult.

3.   The relation of brain-weight to stature and body-weight in the adult.

4.   The relation of brain-weight to skull characters.

5.   The nature of the regression of brain-weight on these other characters.

6.   The effect of natural selection on brain-weight.

7.   The value of brain-weight statistics from the anthropological standpoint.

* Pearson, K.: '' Variation in Man and Woman,'' *The Chances of Death*, Vol. i. pp. 319—323.

2. *Data.*

The statistical data for this study were obtained from several sources, as has been mentioned above. The work was begun on Marchand's statistics, then extended to include the old, but still valuable collection of brain-weights by Bischoff. With the results from these two series in hand, I was still not satisfied that I had analyzed sufficient material to be certain of the conclusions. So I determined to include the following material in the discussion : (1) the admirable series of brain-weights of Swedes collected by Retzius. (2) Matiegka's Bohemian series. (3) The Boyd-Marshall data observed at the St Marylebone Infirmary. As will appear later the last mentioned series was used only for a special purpose. The general results of this paper are based on the analysis of the Marchand, Bischoff, Retzius and Matiegka series.

Marchand's[*] data were obtained from a series of brain-weighings made between the years 1885–1900 in the Pathological Institute at Marburg. The total number of brains weighed was 1234. Of this number 389 were from individuals under 15 years of age, and hence were unavailable for adult brain-weight studies. The data recorded from each individual in Marchand's work included, in addition to brain-weight, sex, age, and stature. It was not possible, however, to determine all of these points in every case, so that omissions of one or more records for an individual occur not infrequently in these statistics. The same is of course true of all extensive series of brain-weight data ever collected. After all such incomplete records have been thrown out there remain in Marchand's tables 475 male and 281 female complete records, or a total of 756 of both sexes. Marchand did not include body-weight in his determination, because he considered that: "das Körpergewicht bei dem Leichenmaterial der Krankenhäuser ein so wechselnder Faktor ist, dass bestimmte Beziehungen zum Gehirngewichte sich daraus kaum ableiten lassen würden."

The brains were weighed in the fresh condition, usually immediately after removal from the cranial cavity. The membranes were not removed before weighing. The weighings were made in the majority of cases to the nearest 5 grams, greater accuracy not being possible on account of such unavoidable sources of error as varying amounts of blood in the vessels of the brain, amount of fluid in the ventricles and membranes, etc. These sources of error cause variations in the total weight outside the limits of 5 grams. Marchand's material comes from the "hessische Bevölkerung."

Bischoff's[†] data comprise the results of the weighing of something over 800 brains of individuals between the ages of 17 and 85. The attempt was to determine for each individual the following characters in addition to brain-weight : sex, age, cause of death, stature and body-weight. Omissions in the records were

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    * Marchand, F.: "Ueber das Hirngewicht des Menschen," *Abhandl. d. math.-phys. Cl. d. Königl. Sachs. Gesellsch. d. Wiss.* Bd. xxvii. No. iv. pp. 393—482, 1902.

    † Bischoff, T. L. W. v.: *Das Hirngewicht des Menschen*, Bonn, 1880, Pp. vi. and 171, plus Tables.

fairly numerous, so it was only possible to extract full data on sex, stature, body-weight and brain-weight for 365 males and 241 females. This material I have referred to in this paper as the "short" Bavarian series. For determining the correlation between brain-weight and age I was able to extract 529 male and 323 female records from Bischoff's tables. This larger group I have called the "long" Bavarian series. The brains were weighed in the fresh condition, soon after removal and with the membranes. Nearly all the individuals had died in hospitals and a considerable number of them were convicts. Practically all were members of the middle and lower classes of society. The causes of death for this and Marchand's series were very varied, including a wide range of diseases and accidents, so that the material is not vitiated by the great preponderance of some one cause of death which might have a specific effect on brain-weight. The great majority of the individuals included in Bischoff's series were Bavarians.

The Swedish data used were collected by Retzius*. The source of the material was in the main the autopsies at the great Sabbatsberg Krankenhaus in Stockholm, although some of the returns were from autopsies at the Maria Krankenhaus. This series of brain-weights is undoubtedly one of the best which has ever been made with reference to accuracy in the individual weighings and uniformity in the conditions and method of weighing. It is only to be regretted that the series is not more extensive. Regarding the racial homogeneity of the material Retzius says (*loc. cit.* p. 55): "ich überzeugte mich aber in den allermeisten Fällen davon, dass sie eine echt schwedishe Herkunft anzeigten." The method of weighing was as follows (*loc. cit.* p. 55): "Die Gehirne wurden ohne Abnahme der weichen Hirnhaute, in der Regel kurz nach der Herausnahme, d. h. nur nach dem Verlust einer dabei stets ausfliessenden geringen Menge Blut und Cerebrospinalflüssigkeit, direct auf die Waageschale gelegt." The series as published by Retzius gives the weights of 450 male brains, and 250 female. In addition to brain-weight, age, stature, and cause of death are recorded. After throwing out individuals incompletely recorded, and those falling outside the age limits 20—80, there was left available for this study 416 male records and 233 female.

Matiegka† has furnished some very valuable data for the student of brain-weight. His material came from the autopsies at two institutions in Prague, the Institut für gerichtliche Medizin, and the Pathologisch-Anatomische Institut. The series from the former source is the larger and more complete, and is the only one which is used in the present discussion. This includes the brain-weights of 372 males and 197 females, together with records of age, stature, skull-length and skull-breadth. The individuals were all adults between the ages of 20 and 80. The great majority of them were Czechs. The method of weighing the brain

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* Retzius, A.: "Ueber das Hirngewicht der Schweden," *Biol. Untersuchungen*, N. F. Bd. ix., Cap. iv. pp. 51—68, 1900.

† Matiegka, H.: "Über das Hirngewicht, die Schädelkapacität und die Kopfform, sowie deren Beziehungen zur psychischen Thätigkeit des Menschen," *Sitzber. des kön. böhmischen Gesellsch. d. Wissensch. Mathem.-Naturwiss. Classe*, Jahrg. 1902, No. xx. pp. 1—75.

was that usually followed: the pia and arachnoid were not removed before weighing. The special value of this material of Matiegka's lies in the fact that it is the most extensive series of brain-weights in which skull characters are so recorded as to make it possible to determine their correlation with the weight of the brain. In addition, data are given for the determination of the correlation between brain-weight and stature, and brain-weight and age. Unfortunately Matiegka's material is open to criticism in certain important respects. In the first place detailed tables showing the exact records for each individual are not given, but instead the material is thrown into the form of correlation tables. In making up these tables the units of grouping were not altogether wisely chosen. This results in introducing a certain error into the absolute values of the constants deduced from these tables. The exact cause, nature and amount of this error will be pointed out later in the paper. Again, the skull-length and skull-breadth were measured in a very unreliable manner. Regarding the method of making these measurements Matiegka says (*loc. cit.* p. 46): "Bei dem von mir verarbeiteten Materiale und zwar in beiden Instituten wurde gewöhnlich auch die *Länge* und *Breite* des *Schädeldaches* d. i. an dem behufs der Hirnentnahme vorgenommenen *Horizontalschnitte* gemessen. Das so erlangte Längenmass ist daher bedeutend *kürzer* als die *grösste Schädellänge,* die Breite wohl häufig etwas *kleiner* als die grösste Schädelbreite*." The effect of this procedure on the biometric constants is discussed later in this paper. My chief reason for including this somewhat questionable material was for the sake of comparison, and because reasonably long series of brain weighings are not so plentiful as to allow one who would study the subject a great range of choice in material.

The data on the brain-weight of the English were very kindly placed at my disposal by Prof. H. H. Donaldson. He was able to obtain, some years ago, a copy of the original detailed manuscript tables which the English anatomist Marshall had compiled from Boyd's original data collected at the St Marylebone Infirmary and the Somerset County Lunatic Asylum. These detailed tables were never published, though it was on them that Marshall's well-known paper, " On the Relations between the Weight of the Brain and its Parts and Stature and Mass of the Body in Man †," was based. For the privilege of examining and using this unique and valuable material I am very grateful to Prof. Donaldson.

From the biometrical standpoint the raw material available for a statistical study of the weight of the brain is peculiarly complex. The great bulk of the statistics consists of material gathered at the autopsies performed in large general hospitals or other public institutions of similar character. The result of this is that the only large collections of brain-weight data available are not representative "random" samples of the general population. Instead they represent a group of the population which has been subjected to a rather stringent selective process of a peculiar kind. It would be very difficult, if not impossible, to formulate all

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* Italics Matiegka's.

† *Jour. Anat. and Physiol.* Vol. xxvi. pp. 445—500, 1892.

the factors which contribute to the differentiation of the "general hospital population" from the "general population" of the same locality. Of some of them, however, we may be fairly certain. In the first place, there are many classes (social) of the general population which will never be represented in any significant proportion in the general hospital population. Furthermore, the nature of the injury or disease from which an individual is suffering in many cases determines whether the individual shall be in a hospital. On this point, Greenwood*, in an able discussion of the matter, says : "Evidently the population of a general hospital will chiefly consist of: (i) persons acutely ill, (ii) those suffering from surgical injuries or diseases, (iii) sufferers from medical affections requiring special treatment. Chronic maladies of old age, such as bronchitis, indeed, any highly chronic disease, will be under-represented in comparison with the general death-rate. Similarly, the number of cases of valvular heart disease and rarer disorders, such as *Diabetes Mellitus* or *Insular Sclerosis* and other nervous lesions, will be above the general average."

In addition to the selection which occurs in the formation of the general hospital population there is a still farther weeding out when we come to deal with autopsy records, for the reason that not every individual dying in a hospital is subjected to a post-mortem examination. Whether there shall be an autopsy or not in a given case depends on several factors, one of which is the cause of death. The individuals whose last illnesses have given doubtful or rare clinical features are more likely to be subjected to a post-mortem examination, other things being equal, than the individuals in whose case the fatal illness has run a perfectly typical clearly cut course of some common disease. Consequently there is a tendency for the statistics to be unduly weighted with deaths from some of the rarer maladies. There can be no doubt that the statistical constants relating to any character of the body which is in any way affected by the disease causing death will be different, in material coming from hospital and pathological laboratory autopsy returns, from those which would be obtained could we get a perfectly random sample of the general population dying outside a hospital, and subjected to post-mortem examination regardless of the cause of death.

These peculiarities attendant upon hospital autopsy records are of particular importance when the character studied is the weight of the brain, for the reason that this character is definitely affected both by the age of the individual, and, in certain cases, by the nature of the disease causing death. It is evident that there will be a more or less definite relation between the nature of the cause which brings an individual into a general hospital and the age of the individual. This combined relation of age and disease is a factor of importance in analysing brain-weight data. In what manner will be apparent from a moment's consideration. Suppose we take the first of Greenwood's classes of the general

hospital population mentioned above, namely, "persons acutely ill." It is at once clear that this will include two sub-groups. First in number and importance will be the group of individuals suffering from diseases medically classified as "acute" in the strict sense of the term, for example, typhoid fever. Second will be the group of individuals acutely ill at the termination of a long-standing chronic disease. These individuals in most instances have been cared for at home as long as the disease remained in the chronic or sub-acute form, and only go into the hospital when it becomes acute. A good example of such cases is afforded by Bright's disease. There can be little doubt on *à priori* grounds that the mean age of the individuals will be lower in the first as compared with the second of these groups. This is shown to be the case by the actual statistics in the brain-weight series where "cause of death" is tabulated. In the lower age groups the causes of death returned are preponderantly "acute" in the strict sense. In the higher age groups we get a preponderance of the chronic affections. Now it is altogether likely that the acute illness which leads to death in a comparatively short time has much less effect on the weight of the brain than the chronic "wasting" sickness. So, then, the general result is that in the higher age classes where the brain-weight has decreased as the natural result of senescence there is a further artificial lowering on account of the preponderance of individuals who have been afflicted with "wasting" diseases. On the other hand the returns during early adult life give mean values for the brain-weight which are probably nearer the true normal value for the general population.

In the present paper it was desired to consider only adult brain-weights and consequently only material which fell in age between 20 and 80 years was usually used. It is the general opinion of anatomists (cf. for example, Ziehen, *loc. cit.* p. 359, and Marchand, *loc. cit.* p. 404), that after the age period 15—20 there is very little increase of weight in the brain with advancing age. In other words the brain is considered to attain practically its complete growth in about the first 20 years of life. According to Marchand (*loc. cit.* p. 402 *et seq.*) the maximum brain-weight remains practically constant till the individual is about 50 years old. After that age he believes that senile degeneration begins. Accordingly, in the discussion of the Marchand data, I have considered the period from 15—80 years to constitute the "adult period," so far as brain-weight is concerned. As a matter of fact, the result would have been but little different had the period 20—80 years been used in this case (as it was with all the other data), because there are only 36 males and 17 females falling in the age class 15—19, and these individuals are fairly evenly distributed among the brain-weight classes as will be seen by consulting Tables 17 and 18 of the Appendix. When the work was begun I had, of course, no means of knowing how it would turn out, so it was necessary to decide how to treat the material on the basis simply of a careful general inspection of the statistics and of the opinion of other workers on the subject. As one point which I wished especially to investigate was the change of brain-weight with age, it became a nice problem as to how to handle the

material so as to best get at this. I finally decided after a good deal of consideration to adopt provisionally Marchand's view stated above, viz., that the weight of the brain reaches a maximum between the ages 15 and 20, remains constant till about age 50, then declines through the old age period. Acting on this plan I separated the material in each case into what I have called a "young" and the "total" group. The "young" series included the individuals falling between the ages 20 to 50 (in the Hessian material, 15—50, and in the Bohemian 20—59 were the limits, in the former case from choice, in the latter from necessity). The "total" series included all the individuals between ages 20 and 80 (in the Hessian series 15 was the lower limit). The results show that on the whole this method of handling the material was adequate, considering the ends to be gained and the amount of material available.

3. *Fundamental Constants.*

As a preliminary to the discussion of the correlation of brain-weight with other characters it is necessary to exhibit and discuss the constants, measuring type and variability of the characters considered. Certain problems which are of considerable interest on their own account present themselves here. Of first importance perhaps is the question as to the amount of differentiation in respect of brain-weight which exists between different sub-races of men, both in type and variability. Further, the rather extensive material worked over makes it possible to settle the problem of the relative variability of the sexes with respect to the weight of the brain. On this point Pearson (*loc. cit.*) has made a brief communication based on an analysis of four short series of English data, Bischoff's Bavarian series, and a French series. His general conclusion is that in respect of brain-weight the sexes are sensibly equally variable. Another interesting problem which merits discussion here is as to how the relative variability in brain-weight compares with the variability of other organs and characters of the body.

In Table I are exhibited the values of the Means, Standard Deviations and Coefficients of Variation together with their probable errors for each of the characters studied in this work. While, of course, the primary object of study is the weight of the brain, yet it is necessary for the correlation determinations that we have the fundamental constants for the other characters, age, stature, etc. To effect economy of space the constants for these other characters have been included in this first table. The frequency distributions from which the constants have been deduced will be found in the fundamental tables, numbered in Arabic numerals from 1 to 44, collected at the end of this paper. Regarding the calculation of the constants the following explanations should be made. In preparing the correlation tables from the raw statistical material the following values for the units of grouping were chosen, and used uniformly throughout the work, except in certain cases where it was impossible to keep to them.

Character	Standard Unit of Grouping
Brain-weight ...	50 grams
Age	5 years
Stature	3 centimetres
Skull length ...	5 millimetres
Skull breadth ...	5 millimetres
Body-weight ...	5 kilograms

These standard units were chosen after considerable study and experimenting and are believed to be the best values for the data discussed. However, it has been shown recently by other workers * that "the system of grouping adopted is within wide limits immaterial." An examination of those cases recorded in the present paper where it was necessary to use different units of grouping—notably in the Matiegka data where the brain-weights were grouped into 100 gram classes and the ages into 10 year classes—leads to the same conclusion.

The standard deviation was evaluated according to the equation $\sigma = \sqrt{\mu_2}$. In obtaining μ_2 Sheppard's correction was used throughout for the characters brain-weight, stature, skull length, skull breadth and body-weight. The uncorrected "rough" moment was used in getting the standard deviation in age, as in this case there is no approach to high contact at either end of the range.

The decimals have been retained to three places in the tabulated values in order that the significant value of the first place may be seen. It will be understood, of course, that in the computations the decimals were retained to a larger number of places for arithmetical reasons.

4. *Brain-weight Types.*

In brain-weight work hitherto it has not been possible to make accurate comparisons of the mean weight of the brain in different races. There were two reasons for this: one that the probable errors of the means were not determined, and the other that no method was available by which allowance could be made for the differences in the mean stature and age of the samples of material to be compared. Obviously if brain-weight is correlated with stature and age, we should expect to get a considerable difference between the mean brain-weight of two groups which differed widely with respect to these other characters. Inasmuch as a knowledge of the degree of correlation between these characters affords a method whereby the material can be reduced to a "standard stature-age base," it will be worth while to examine the actual differences in brain-weight types in the four racial groups here discussed.

* "Assortative Mating in Man." A Cooperative Study. *Biometrika*, Vol. II. pp. 481—498, 1903.

TABLE I. *Fundamental Constants.*

Race	Character	Mean						Standard Deviation			Coefficient of Variation	
		No.	♂	Table	No.	♀	Table	Table	♂	♀	♂	♀
Swedes ...	Brain-weight (Total) ...	416	1400·481 ± 3·516	1	233	1252·682 ± 4·452	2	2	106·329 ± 2·486	100·757 ± 3·148	7·592 ± ·179	8·043 ± ·253
„	„ (Young) ...	262	1415·267 ± 4·550	3	127	1269·488 ± 6·317	4	4	109·180 ± 3·217	105·543 ± 4·467	7·714 ± ·229	8·314 ± ·354
Hessians	„ (Total) ...	475	1391·737 ± 3·487	13	281	1259·875 ± 4·119	14	14	112·675 ± 2·466	102·368 ± 2·642	8·096 ± ·178	8·125 ± ·233
„	„ (Young) ...	291	1405·756 ± 4·402	15	173	1280·202 ± 5·223	16	16	111·328 ± 3·113	101·841 ± 3·693	7·919 ± ·223	7·955 ± ·290
Bohemians	„ (Total) ...	372	1454·839 ± 3·973	27	197	1310·914 ± 4·650	28	28	113·608 ± 2·813	96·772 ± 3·288	7·809 ± ·197	7·382 ± ·252
„	„ (Young) ...	266	1460·150 ± 4·731	25	133	1313·910 ± 5·453	26	26	114·395 ± 3·345	93·227 ± 3·855	7·835 ± ·231	7·095 ± ·295
„	„ (Skull) ...	299	1463·712 ± 4·503	29	159	1321·698 ± 5·073	30	30	115·440 ± 3·159	94·840 ± 3·587	7·887 ± ·219	7·176 ± ·273
Bavarians	„ (Total, Long*) ...	529	1363·185 ± 3·245	37	323	1220·356 ± 3·820	38	38	110·644 ± 2·295	101·776 ± 2·701	8·118 ± ·122	8·340 ± ·223
„	„ (Young, Long*) ...	365	1369·110 ± 3·841	39	238	1235·504 ± 3·997	40	40	108·776 ± 2·716	91·421 ± 2·826	7·945 ± ·200	7·399 ± ·230
„	„ (Total, Short*) ...	365	1357·466 ± 3·888	35	241	1218·776 ± 4·616	36	36	120·128 ± 2·749	106·246 ± 3·264	8·849 ± ·223	8·717 ± ·270
Swedes ...	Stature (Total) ...	416	169·789 ± ·225	1	233	158·710 ± ·297	2	2	6·807 ± ·159	6·719 ± ·210	4·009 ± ·094	4·233 ± ·133
„	„ (Young) ...	262	170·176 ± ·280	3	127	159·673 ± ·418	4	4	6·728 ± ·198	6·987 ± ·296	3·954 ± ·117	4·375 ± ·186
Hessians	„ (Total) ...	475	167·359 ± ·223	13	281	156·178 ± ·277	14	14	7·189 ± ·157	6·895 ± ·196	4·296 ± ·094	4·400 ± ·125
„	„ (Young) ...	291	167·294 ± ·282	15	173	156·980 ± ·384	16	16	7·118 ± ·199	7·490 ± ·272	4·255 ± ·119	4·771 ± ·173
Bohemians	„ (Young) ...	266	169·417 ± ·303	25	133	157·650 ± ·418	26	26	7·324 ± ·214	7·152 ± ·296	4·323 ± ·127	4·537 ± ·188
Bavarians	„ (Total, Short*) ...	365	166·549 ± ·226	35	241	154·712 ± ·270	36	36	6·393 ± ·160	6·213 ± ·191	3·838 ± ·096	4·016 ± ·124
Swedes ...	Age (Total) ...	416	45·024 ± ·452	5	233	47·629 ± ·671	6	6	13·671 ± ·320	15·195 ± ·475		
„	„ (Young) ...	262	36·317 ± ·327	7	127	35·689 ± ·480	8	8	7·847 ± ·231	8·027 ± ·340		
Hessians	„ (Total) ...	475	42·889 ± ·521	17	281	44·795 ± ·656	18	18	16·843 ± ·369	16·304 ± ·464		
„	„ (Young) ...	291	31·572 ± ·390	19	173	34·061 ± ·497	20	20	9·861 ± ·276	9·685 ± ·351		
Bohemians	„ (Total) ...	372	45·699 ± ·523	27	197	41·447 ± ·792	28	28	14·957 ± ·370	16·482 ± ·560		
Bavarians	„ (Total, Long*) ...	529	43·171 ± ·396	37	323	40·333 ± ·561	38	38	13·495 ± ·280	14·954 ± ·397		
„	„ (Young, Long*) ...	365	35·582 ± ·255	39	238	32·731 ± ·340	40	40	7·216 ± ·180	7·768 ± ·240		
Bohemians	Skull length (Young) ...	299	176·547 ± ·287	29	159	170·142 ± ·294	30	30	7·368 ± ·203	5·500 ± ·208	4·173 ± ·115	3·233 ± ·122
„	„ breadth (Young) ...	299	149·841 ± ·260	31	159	144·953 ± ·296	32	32	6·654 ± ·184	5·530 ± ·209	4·441 ± ·123	3·815 ± ·145
Bavarians	Body-weight (Total, Short*)	365	49·925 ± ·376	41	241	42·894 ± ·461	42	42	10·646 ± ·266	10·601 ± ·326	21·324 ± ·608	24·715 ± ·804

* Cf. p. 17.

Taking the gross values given in Table I the following Table II has been prepared to facilitate comparison.

TABLE II.

Brain-weight Types.

	Hessians	Bavarians	Swedes	Bohemians
Total Males				
Mean brain-weight	1391·737	1363·185	1400·481	1454·839
Mean age	42·889	43·171	45·024	45·699
Mean stature ...	167·359	166·549	169·789	—
Young Males				
Mean brain-weight	1405·756	1369·110	1415·267	1460·150
Mean age... ...	31·572	35·582	36·317	40·545*
Mean stature ...	167·294	—	170·176	169·417
Total Females				
Mean brain-weight	1259·875	1220·356	1252·682	1310·914
Mean age... ...	44·795	40·333	47·639	41·447
Mean stature ...	156·688	154·712	158·710	—
Young Females				
Mean brain-weight	1280·202	1235·504	1269·488	1313·910
Mean age... ...	34·061	32·731	35·689	35·613†
Mean stature ...	156·980	—	159·673	157·650

It is apparent at once that, considering the magnitude of the probable errors, there are significant differences between the Bavarians, Swedes, and Bohemians, with respect to brain-weight, in both sexes and age classes. Between the Swedish and Hessian means the differences are smaller, and as the following table shows, are not significant when their probable errors are taken into account:

Total Males :   Swedish mean—Hessian mean = 8·744 ± 4·952.

Total Females : Hessian mean— Swedish mean = 7·193 ± 6·065.

Young Males :   Swedish mean—Hessian mean = 9·511 ± 6·331.

Young Females: Hessian mean— Swedish mean = 10·714 ± 8·197.

In no case is the difference as great as even twice its probable error, hence we must conclude that the differences exhibited might, so far as the extent of material allows us to judge, be due to random sampling.

* This value was calculated by taking the first four age classes of the "total" series. It includes more individuals than the "young" series for the other characters. This was the only way in which the mean age of the "young" series in this case could be even approximated to, as the raw material given by Matiegka is incomplete in this respect. The age as given is probably a little too high, but the error cannot be great.

Is this agreement between the Hessian and Swedish peoples in the matter of brain-weight real, or only apparent and the result of compensating differences in the other correlated characters? An answer to this question can be reached in two ways; either by reducing one set to the same "stature-age base" as the other and then comparing results, or by analyzing the effects on the brain-weight of the observed stature and age differences in the two sets of material. Both of these methods depend on the use of characteristic equations, the derivation of which will be described further on in the paper. For the present I shall make practical use of such equations without further discussion of their derivation or validity. Table II shows that in the samples with which we are dealing the Hessians, both male and female, average *younger* and *shorter* than the Swedes. Reducing the Hessians to the same "stature-age base" as the Swedes, by means of the appropriate equations given on p. 63 *infra,* the following results are obtained:

TABLE III.

The probable brain-weight of a group of Hessian ♂ ♂ having the
same age and stature as the Swedish ♂ ♂ (Total) ... ... $= 1396\cdot480$ (23)
Swedish brain-weight observed ... ... $= 1400\cdot481$

Swedish *heavier.*   Difference ... $= \quad 4\cdot001$

The probable brain-weight of a group of Hessian ♀ ♀ having the
same age and stature as the Swedish ♀ ♀ (Total) ... ... $= 1258\cdot232$ (25)
Swedish brain-weight observed ... ... $= 1252\cdot682$

Hessian *heavier.*   Difference ... $= \quad 5\cdot550$

The probable brain-weight of a group of Hessian ♂ ♂ having the
same age and stature as the Young Swedish ♂ ♂ ... ... $= 1407\cdot457$ (24)
Swedish brain-weight observed ... ... $= 1415\cdot267$

Swedish *heavier.*   Difference ... $= \quad 7\cdot810$

The probable brain-weight of a group of Hessian ♀ ♀ having the
same age and stature as the Young Swedish ♀ ♀ ... ... $= 1283\cdot540$ (26)
Swedish brain-weight observed ... ... $= 1269\cdot488$

Hessian *heavier.*   Difference ... $= \quad 14\cdot052$

These differences after the stature-age corrections are made are extremely small. This is particularly well shown if they are reduced to a relative basis, by expressing the differences as percentages of the observed Swedish brain-weight:

The difference between Total Swedish ♂ ♂ and Hessians (calculated) $= 0\cdot29\,°/_\circ$ of the observed Swedish brain-weight.
The difference between Total Swedish ♀ ♀ and Hessians (calculated) $= 0\cdot44\,°/_\circ$ of the observed Swedish brain-weight.
The difference between Young Swedish ♂ ♂ and Hessians (calculated) $= 0\cdot55\,°/_\circ$ of the observed Swedish brain-weight.
The difference between Young Swedish ♀ ♀ and Hessians (calculated) $= 1\cdot11\,°/_\circ$ of the observed Swedish brain-weight.

It appears then that, so far as the available material may be considered valid as representing the whole population, the conclusion is justified that the Hessian and Swedish peoples are sensibly alike in respect to their brain-weight. The agreement would be closer than that actually observed if the assumption made in the course of reasoning here followed, that the regression of brain-weight on age and stature is strictly linear, were exactly true. As will be shown later these regressions are not strictly linear but they approach linearity with sufficient closeness to serve for all practical purposes. This, together with the difference in age distribution of the " total " and " young " series, also accounts for the fact that while in the " total " series and male " young " series the gross differences between Swedes and Hessians are lowered when we reduce to a common "stature-age base," these differences are slightly increased, on the other hand, when we deal in the same way with the shorter female " young " series.

This is in agreement with the general fact that these peoples are probably the most closely related ethnically of any with which we are dealing in the present paper. The Swedes may be considered to be among the purest representatives of the original blonde, dolichocephalic Teutonic race (Ripley[*] and Deniker[†]). In the case of the Hessians some intermixture of this Teutonic with the brachy-cephalic, characteristically brunette Alpine type has occurred. The differentiation from the Swedes in such important characters as skull form and stature is not great however[‡].

Turning now to the other racial groups discussed, viz., the Bavarians and Bohemians (Czechs), we find as would be expected, that the differences in brain-weight are greater. Using the Hessian data as a basis for comparison, the gross differences with their probable errors are exhibited in the following table :

Total Males :   Hessian mean—Bavarian mean $= 28 \cdot 552 \pm 4 \cdot 763$.
Total Females :   Hessian mean—Bavarian mean $= 39 \cdot 519 \pm 5 \cdot 618$.
Young Males :   Hessian mean—Bavarian mean $= 36 \cdot 646 \pm 5 \cdot 842$.
Young Females :   Hessian mean—Bavarian mean $= 44 \cdot 698 \pm 6 \cdot 577$.

Total Males :   Bohemian mean—Hessian mean $= 53 \cdot 102 \pm 5 \cdot 286$.
Total Females :   Bohemian mean—Hessian mean $= 51 \cdot 039 \pm 6 \cdot 212$.
Young Males :   Bohemian mean—Hessian mean $= 54 \cdot 394 \pm 6 \cdot 462$.
Young Females :   Bohemian mean—Hessian mean $= 33 \cdot 708 \pm 7 \cdot 551$.

In all cases the differences are seen to be well above what might arise from errors in statistical sampling. Reducing to a common " stature-age base " by the method followed above the following results are obtained :

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[*] Ripley, W. Z.: *The Races of Europe*, New York, 1899, pp. xxxii, 624.

[†] Deniker: *The Races of Man*, London and New York, 1900, pp. xxiii, 328 and 611.

[‡] This is well shown graphically in the maps indicating the distribution of stature and cephalic index in Europe, given by Ripley (*loc. cit.* pp. 96 and 53). Cf. also Deniker, *loc. cit.* pp. 328 and 329.

TABLE IV.

	Probable Brain-weight of a group of Hessians of the same *sex*, *age* and *stature* as the	Mean Brain-weight *observed* (Bavarian or Bohemian)	Difference expressed as in excess or defect of Hessian values	Difference as per cent. of observed mean (Bavarian or Bohemian)	Equation on which Hessian estimate is based
Bavarian ♂ ♂, Total	1388·984	1363·185	− 25·799	− 1·9 °/₀	No. 23, p. 63
,, ♀ ♀, Total	1265·137	1220·356	− 44·781	− 3·7 °/₀	,, 25, ,,
,, ♂ ♂, Young	1397·157*	1369·110	− 28·047	− 2·0 °/₀	,, 24, ,,
,, ♀ ♀, Young	1277·317*	1235·504	− 41·813	− 3·4 °/₀	,, 26, ,,
Bohemian ♂ ♂, Total	1394·570†	1454·839	+ 60·269	+ 4·1 °/₀	,, 23, ,,
,, ♀ ♀, Total	1259·310†	1310·914	+ 51·604	+ 3·9 °/₀	,, 25, ,,
,, ♂ ♂, Young	1399·527‡	1460·150	+ 60·623	+ 4·2 °/₀	,, 24, ,,
,, ♀ ♀, Young	1278·935‡	1313·910	+ 34·975	+ 2·7 °/₀	,, 26, ,,

This table brings out several points of considerable interest. In the first instance is to be noted the general effect of reducing the Hessians to the same "stature-age base" as the other races, upon the interracial differences in mean brain-weight. In the case of the Bavarians the deviations from the Hessian means are reduced slightly in both male series, and the "young" female series, when stature and age differences are eliminated. In the "total" female series the difference is increased over the original gross difference. The explanation for this discrepancy in the female "total" series is to be found in a peculiar abnormality which this series shows in its elemental frequency distribution, and which will be discussed later (p. 40). In the case of the Bohemians we get the somewhat remarkable result that a reduction to a common "stature-age base" actually increases the differences of this racial group in brain-weight as compared with the Hessians. Or in other words, in the samples with which we are dealing the stature and age differences act in a compensatory way and bring the mean brain-weights closer together than they would be if we dealt with selected samples of the populations, each sample having the same mean age and stature. The fact already noted (p. 18) that the mean brain-weights deduced from Matiegka's tables for the Bohemians are somewhat too large, may now be examined in detail. This Bohemian material was published by Matiegka in the form of correlation tables with unduly large units of grouping (cf. Tables 25 to 28, Appendix). The base unit for brain-weight was 100 gr. Now in calculating the general population mean from these tables one assumes that the individuals in each elemental frequency group centre in brain-weight at the mid-point of that group. Thus, the individuals recorded as having a brain-weight of between 1300 and 1400 grams are assumed to centre at 1350 gr. But evidently this assumption will not be true except at the middle of the whole range. For example, the brain-weights recorded

* This assumes that the mean stature of the Bavarians would be the same in the "young" group as it is in the "total" group. The error introduced by this procedure is practically negligible.

† Here again, on account of lack of data, the mean stature is assumed to be the same in "total" and "young" groups.

‡ See footnote on p. 24.

as between 1500 and 1600 will centre somewhat below 1550. Now, of course, if the distribution is exactly or approximately normal and the number of individuals is sufficiently large, the errors from this cause on one side of the general population mean will balance those on the other, and we shall still be able to get a very close value for this mean from the frequency distribution. Unfortunately, in Matiegka's series, however, the total numbers are not sufficiently large to overcome entirely this error. So we have the following differences: by direct calculation from the individual observations Matiegka finds for the mean brain-weight of the " young " (20—59) series in the male 1450·4 gr., in the female 1305·5 gr. Our corresponding values are 1460·150 gr. and 1313·910 gr. or there is an excess of 9·7 gr. and 8·4 gr. respectively. For the " total " series (20—80) I have calculated from regression values which Matiegka gives (p. 7), based on the individual observations, general population means which gives values as follows: males 1441·4 gr., females 1294·8 gr. The corresponding values from the tables are: males 1454·8 gr., females 1310·9 gr., or the excess of the table means is 13·4 gr. for the males and 16·1 for the females. These differences are small and for practical purposes negligible.

The general conclusion may be drawn that, *apart from all differences in stature, and in the mean age of the samples studied*, the Bavarian mean brain-weight is lower than the Hessian (roughly about 2 °/₀), and the Bohemian mean brain-weight is higher than the Hessian (roughly about 3·5 °/₀).

These results seem to be of some importance as indicating a method whereby brain-weight may have some scientific validity as an anthropological character. This can hardly be said to be the case when nothing more is done than to tabulate means without probable errors, and no attempt is made to get rid of disturbing stature and age effects. Interracial differences in brain-weight may mean something or nothing. With modern biometrical methods it is possible to measure exactly these differences, and when such methods are used it is the belief of the writer that brain-weight can become a really significant anthropological character. In the cases discussed here we see ethnic affinities and differences clearly reflected in the brain-weight. Of the four racial groups studied the two most closely related in origin and general anthropological characters, the Swedes and the Hessians, have mean brain-weights sensibly identical. The Bavarians and Bohemians, though close neighbours geographically, arise from totally distinct ethnic stocks (" Alpine " and Slav respectively) and associated with this we find a wide difference in the weight of the brain.

On the question of the homogeneity of brain-weight statistics, some evidence is furnished in the third and fourth columns of Table IV. If the inclusion of aged persons (50 years and over) introduced any considerable element of heterogeneity so far as brain-weight constants are concerned it would be expected that the interracial differences in the means would not be equal in the " total " and in the " young " series. As a matter of fact in the material here discussed they are sensibly equal, or perhaps better, they are not significantly different. The whole

question of the homogeneity of this material will be treated fully in connection with the discussion of the variability in brain-weight.

The interesting questions brought up by this table of means, regarding sexual differences, association of brain-weight with other characters, etc., will be discussed in later sections of the paper.

5. *Variation in Brain-weight. Homogeneity of Material.*

Variability in the weight of the brain has been subjected to exact investigation by only one worker, Pearson (*loc. cit.*), so far as is known to the writer. He deduced from his material (cf. p. 15 *supra*) coefficients of variation ranging in value from 7·93 °/₀ to 10·64 °/₀, the higher values being from admittedly heterogeneous series.

The numerical values for the variation constants and their probable errors found in the present work are given in Table I.

The question of homogeneity of material should be first discussed. For reasons which have been set forth above it would be hopeless to look for any high degree of homogeneity in any collection of human brain-weighings at present available. The best we can hope for is a fair degree of homogeneity, and reasonably the same degree in different series which are to be compared. Unfortunately there is not available here, as in the case of craniometrical investigations, series in which fair homogeneity can be inferred with high probability, so that it is not possible to make a direct estimate by comparing variabilities with such a "known base." Instead resort must be had to indirect methods. The best of such indirect methods is based on the fact that if a random sample be taken from a *homogeneous* collection of material the variation constants for the sample and the whole collection will not significantly differ. On the contrary if the material is non-homogeneous such a sampling will give different values for the constants. If the sample be *selected*, i.e. not random, the variation constants for the character selected will of course be lowered. Now in the material as treated here we have in the "young" series for each racial group a *selection* from the "total" series, but a selection based on age, not on brain-weight directly. If there were no correlation between brain-weight and age such a selection on an age basis would be, of course, a random sample so far as brain-weight is concerned. Unfortunately for the argument, there is, as will be shown later, a sensible though in general low correlation between age and brain-weight. This being the case it would be expected that such a selection as has been made in separating out the "young" from the "total" series would result in a lowering of the standard deviation and coefficient of variation in brain-weight. Now, as a matter of fact, as the following table shows, the correlation between brain-weight and age is so low that in the relatively small series under discussion, the "young" series forms practically a random sample of the "total" series, within the limits of error, with respect to brain-weight.

TABLE V.

Difference in Variation Constants between " Young " and " Total" Series.

	♂		♀	
	S. D.	C. of V.	S. D.	C. of V.
Swedes	+2·851 ± 4·066	+·122 ± ·291	+ 4·786 ± 5·465	+·271 ± ·435
Hessians	−1·347 ± 3·971	−·177 ± ·285	− ·527 ± 4·541	−·170 ± ·372
Bavarians ...	−1·888 ± 3·556	−·173 ± ·234	−10·357 ± 3·909	−·941 ± ·320
Bohemians ...	+ ·787 ± 4·371	+·026 ± ·304	− 3·545 ± 5·067	−·287 ± ·388
,, (Skull)*	+1·832 ± 4·230	+·078 ± ·295	− 1·932 ± 4·866	−·206 ± ·372

Difference + when "Young" is greater.
,,   −   ,,   ,,   less.

In only one case—the Bavarian females—is the difference as large as its probable error, and in this case the difference is not three times the probable error. The conclusion seems justified that the series are reasonably homogeneous in other respects than age. The age distribution is such as to exclude growth effects, and, as the differences indicate, the effects of senescence on brain-weight are so insignificant that for practical purposes the material may be considered sufficiently homogeneous to warrant further biometrical study. From the character of the differences it would appear that the most homogeneous series are, on the whole, the Swedish and the Bohemian.

A direct examination of the standard deviations and coefficients of variation confirms the conclusion. In the following table I have arranged in order according to ascending value, the variation constants for the "total" series, using as the basis for the grouping the coefficient of variation:

TABLE VI.

*Comparison of Different Races in Respect to Variation in Brain-weight
and Skull Capacity.*

	Males			Females	
	S. D.	C. of V.		S. D.	C. of V.
Swedes	106·329	7·592	Bohemians	96·772	7·382
Bohemians	113·608	7·809	Swedes	100·757	8·043
Hessians	112·675	8·096	Hessians	102·368	8·125
Bavarians	110·664	8·118	Bavarians	101·776	8·340
English †	124·48	9·20	English †	118·7	9·72
Skull Capacity, English ‡	—	8·28	Skull Capacity, English ‡	—	8·68
,, ,, Germans §	—	7·74	,, ,, Germans §	—	8·19

* See p. 17 and Table I. for the series here referred to.
† Pearson: *The Chances of Death*, Vol. i. p. 321.
‡ W. R. Macdonell: *Biometrika*, Vol. iii. p. 221.
§ Pearson: *The Chances of Death*, Vol. i. p. 333.

All of the four series used in this paper are in good agreement, both among themselves and with the skull capacity values. They are distinctly lower than the figures from the admittedly heterogeneous, Reid, Peacock, Sims and Clendinning English series, worked over by Pearson. Taking all the evidence together we are driven to the conclusion that the series are fairly homogeneous. If any one of our series is markedly heterogeneous all the others must be equally so, which would be a very improbable result. That the series cannot be very heterogeneous in their make-up is also shown by the values of the coefficients of variation for skull capacity from two admittedly homogeneous series, the Whitechapel skulls studied by Macdonell, and Ranke's Bavarian series.

A comparison of the variability in brain-weight with that shown by other organs and characters of the human body may next be undertaken. I have arranged in the following table (VII) the coefficients of variation for a variety of characters which have been studied by biometrical workers. The arrangement is in general that of descending order of values in the male series. The attempt has been made in the table to include representatives of all the different classes of organs and characters for which we have biometric data available.

The most noticeable and remarkable fact brought out by the foregoing table is that with the exception of capacity, all skull characters are roughly only about half as variable as brain-weight. Some such a relation as this might have been predicted, on the general ground that brain-weight measurements and statistics are on many accounts rather "loose," and would *indicate* a higher variability than would exact measurements on skulls, even though it had no real existence. Such reasoning, however, takes no account of the agreement, which is really remarkably close when we recall the numerous sources of error in brain-weight returns, between skull capacity and brain-weight in their variability. All will admit that capacity is the most difficult skull character to measure accurately, yet no one would maintain that the difference in variability between cephalic index, for example, and skull capacity was entirely, or even in any considerable part, due to the element of error in the measurement of the latter. The agreement in variability between skull capacity and brain-weight is, of course, to be expected on theoretical grounds. That it should turn out in practice to be so close is a first-rate guarantee of the general trustworthiness of brain-weight statistics.

As to the explanation of the great variability in brain-weight and skull capacity as compared with the other skull characters we may tentatively reason about the matter in the following way. If the list of organs and characters given in the foregoing table be examined a natural division into three groups almost immediately suggests itself. First, we have at the bottom of the list the "bone" measurements, in general including all those characters which depend primarily for their values on the dimensions of various parts of the skeleton. These characters give values for the coefficient of variation up to from 5 to 7, certain of the mandibular variabilities exceeding this limit. Next comes the group giving values for the coefficients of from 7 to 10, with the limits fairly sharply marked off.

TABLE VII. *Coefficients of Variation for Man.*

	♂	♀
Weight of Spleen (General Hospital Population)*	50·58	—
„ „ „ (Healthy)†	38·21	—
Dermal Sensitivity‡	35·70	45·70
Weight of Heart (General Hospital Population)*	32·39	—
Keenness of Sight‡	28·68	32·21
Weight of Kidneys (General Hospital Population)*	24·63	—
Weight of Body (Bavarians)	21·32	24·715
Weight of Liver (General Hospital Population)*	21·12	—
Swiftness of Blow‡	19·4	17·1
Weight of Heart (Healthy)†	17·71	—
„ Kidneys (Healthy)†	16·80	—
Breathing Capacity‡	16·6	20·4
Strength of Pull‡	15·0	19·3
Weight of Liver (Healthy)†	14·80	—
Height of Mandible (English, both sexes)§	11·73	11·73
Weight of Body (English)‡	10·37	13·37
Skull Capacity (Etruscan)‖	9·58	8·54
Brain-weight (French)‡	**9·16**	**9·14**
Skull Capacity (Modern Italian)‖	8·34	8·99
„ „ (English)¶	8·28	8·68
„ „ (Egyptian Mummies)‖	8·13	8·29
Brain-weight (Bavarian)	**8·118**	**8·340**
„ „ *(Hessian)*	**8·096**	**8·125**
„ „ *(Bohemian)*	**7·809**	**7·382**
Skull Capacity (Modern German)‖	7·74	8·19
„ „ (Naqada)‖	7·72	6·92
Brain-weight (Swedish)	**7·592**	**8·043**
Skull Capacity (Parisian French)‖	7·36	7·10
„ „ (Aino)‖	7·07	6·90
Mandible, Distance between Foramina mentalia (English, both sexes)§	6·23	6·23
Length of Forearm**	5·24	5·21
„ „ Femur (French)‡	5·05	5·04
„ „ Tibia „ ‡	4·975	5·365
„ „ Humerus (French)‡	4·89	5·61
„ „ Radius „ ‡	4·87	5·23
Skull, Height to Breadth Index (English)¶	4·86	4·16
„ Breadth to Height „ ¶	4·83	4·17
Length of Finger (English Criminals)††	4·74	—
Skull, Ratio of Height to Horizontal Length (English)¶	4·61	4·10
Length of Foot (English)††	4·59	—
Skull, Cephalic Index for Horizontal Length (English)¶	4·38	3·99
Length of Cubit (English Criminals)††	4·36	—
Skull, Least Breadth of Forehead (English)¶	4·29	4·55
„ Height (English)¶	4·21	3·96
„ Length of Base (English)¶	4·07	4·11
„ Cephalic Index for Greatest Length (English)¶	3·95	4·03
Stature (English)**	3·99	3·83
Skull, Ratio of Height to Greatest Length (English)¶	3·80	4·21
„ Greatest Breadth (English)¶	3·75	3·54
„ Auricular Height (English)¶	3·73	4·12
„ Face Breadth (English Criminals)††	3·707	—
Skull, Cross Circumference (English)¶	3·70	3·97
„ Sagittal „ ¶	3·63	3·90
Head Breadth (English Criminals)††	3·333	—
Skull, Length (English)¶	3·31	3·45
Head Length (English Criminals)††	3·154	—
Skull, Horizontal Circumference (English)¶	2·87	2·92

* Greenwood, M. : *Biometrika*, Vol. iii. p. 66.   † *Ibid.* p. 67.

‡ Pearson, K. : *The Chances of Death*, Vol. i. pp. 293—377.

§ Macdonell, W. R. : *Biometrika*, Vol. iii. p. 225.   ‖ *Ibid.* p. 221 (after Pearson).

¶ *Ibid.* p. 222.   ** Pearson and Lee : *Biometrika*, Vol. ii. p. 370.

†† Macdonell, W. R. : *Biometrika*, Vol. i. p. 202.

Finally we have all the characters giving values above 10. Now as one passes from one end of the series to the other a definite biological relationship is plain. At the upper end of the series (coefficient of variation 10 and over) the organs and characters tabulated are such as depend in a very considerable degree for their values as determined by measurement, on the general *metabolic condition of the organism* as a whole at the time the measurements are made or immediately before. This will be freely admitted, I think, for such characters as "Dermal sensitivity," "Keenness of sight," "Weight of body," "Swiftness of blow," "Strength of pull," "Breathing capacity" and the like. The same thing, though less apparent perhaps, is, I believe, true when the characters are the weights of viscera. Indeed Greenwood's own results show this to be the case, when we get such different results for means, variabilities and correlation accordingly as we deal with the "general hospital population," "healthy" organs, or the same viscera in diseased conditions of different characters. Wynn* has only recently shown that in rabbits there is a considerable degree of probability that continued administration of digitalis will in a short time raise the mean weight of the heart appreciably, while at the same time the body weight is lowered. It is to be regretted that his series of experiments included so (statistically) few individuals.

Furthermore, it seems reasonable to assume that in this upper group the thing measured in the majority, if not in all cases, is not the thing natural selection has acted upon directly, allowing that it has acted at all. I think it may be fairly assumed that so far as natural selection has acted at all on these organs and characters, the selection has been in the direction of *ability to function properly* so as best to conserve the physiological economy of the organism as a whole, rather than in the direction of absolute size of organ or character. The physiologically balanced functioning with reference to the needs of the organism as a whole is the important thing in such organs as the spleen, pancreas, liver, etc. Absolute size of organ can hardly be a very close direct measure of ability to function well. So then, I am inclined to attribute the high variability observed in those organs and characters falling together in the uppermost part of the table to these two factors: (*a*) the value obtained by measurement depends to a considerable degree on the general metabolic condition of the individual at the time, and (*b*) the thing measured is not the thing with which natural selection, so far as it has acted at all, has had directly to do.

On the other hand, if we turn to the lowest group in the above table (coefficients of variation under 5—7 for the males) we find the two factors just mentioned almost exactly reversed. Here the characters are either of the skeleton directly, or in the case of the anthropometrical data are characters which closely depend for their measured values on the size of portions of the skeleton. Obviously the general metabolic condition of the organism has, within wide limits, little effect on the length of the femur or the skull, for example, and if there is an effect it manifests itself very slowly. Further it is to be presumed that the absolute

* *Journal of the American Medical Association*, Vol. XLIII. pp. 164, 165, 1904.

size is of greater direct selective value in the case of parts of the skeleton than in the case of organs like the liver, etc. In other words, size is much more directly related to proper functioning in the former than in the latter case. The skull may at first sight appear to form an exception here. It must be kept in mind, however, that the skull serves two very important—perhaps equally important— functions; viz., (1) the enclosing and protecting of the brain, and (2) the serving as a basis for the attachment of the complexes of muscles which actuate the masticatory apparatus and support and move the head. With reference to each of these factors natural selection may act on the skull. So far as the second factor at least is concerned, size of skull (in a broad sense) will be of selective value. So that here again, since size and function are interrelated, we should expect to find the results of selective action reflected in the size measurements of characters. In general, I think it can safely be maintained that the low variability shown in the group of characters under discussion is what might be expected to result from the operation of the factors just mentioned.

In the middle group (coefficients of variation ranging from 7 to 10 in the male) including brain-weight and skull capacity we seem to have also an intermediate condition, with respect to the two factors which have been mentioned as among the causes which contribute to the observed variability. In the first place, there can be no doubt in face of the evidence both from the biometrical and anatomical standpoints, that the cranial capacity is quite highly correlated with volume of the brain *. Admitting this, the discussion may be restricted to the brain. Now it has been known from the time of the earliest collection of brain-weight statistics that the weight of the brain is influenced to a certain extent by the general metabolic condition of the individual preceding death. I am inclined to think however that the extent of this influence has been over-estimated. My primary reason for this view comes from the analysis of the statistics themselves. If the metabolic condition of the individual preceding death influenced brain-weight to a marked degree, one would hardly expect to find the agreement shown in the constants of variation and correlation tabulated in this paper, when different groups of individuals with different distribution of ante-mortem affections were compared. We might expect by chance to get two groups in agreement, but on this supposition the odds would be great against getting four groups to agree so closely as they actually do. Furthermore, there is a certain amount of evidence from other sources indicating that the influence of ante-mortem general metabolic conditions on brain-weight, while always present as one factor, does not produce so marked an effect on the weight of the brain as on the size measurements of some of the organs in our most variable group. For example, Mühlmann† studied microscopically the brains of 24 individuals of different ages from birth to 90 years to determine the amount of pigment degeneration in the ganglion cells at the different ages. In the adults the causes of death included such divergent con-

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* On the anatomical side, cf. Symington, *Nature*, Vol. 68, pp. 539—544.

† *Verhandl. d. deutsch. Pathol. Gesellsch.* Bd. III. pp. 148—157.

ditions as endocarditis, pneumonia cruposa, and nephritis chronica. Yet the differences found in the appearance of the brain in different cases were clearly associated with differences in age, and not with causes of death. On this whole question more data are needed. Exact measurements of the correlation between skull capacity and brain-weight are much to be desired. We also lack definite biometrical evidence regarding the effect of morbid conditions on the weight of the brain. The evidence at present available seems to lead to the conclusion here indicated.

On the other hand, it can hardly be maintained that natural selection has acted on the size of the brain to the degree that it has on skeletal structures. The brain, in this respect, comes nearer to the organs included in our most variable group. So that, on the one hand with a less marked dependence of the measurement on the previous metabolic condition of the organism than is found in the most variable group of organs and characters, and on the other hand, with a less stringent selection with reference to size as compared with the least variable group, an intermediate condition in variability for the brain is to be expected.

Of course, too much stress cannot be laid on this argument as to the relative variability of the characters of the human body in view of the meagreness of the data at present available. The data at hand, however, plainly suggest some such an explanation, and it is only as a tentative suggestion worthy of being kept in mind as further data are available that the matter has been developed here. To summarise; when a series of the various organs and characters of the body whose variation has been measured are arranged in the order of their relative variability, those characters falling at the most variable end of the series are those whose measured value is affected markedly by the previous general metabolic condition of the organism, and in which ability to function properly is not closely related to size of organ, and in which natural selection, if it has acted at all on the character, has not acted directly upon the thing measured. The opposite end of the series includes the least variable organs and characters, in which both the above mentioned conditions are reversed. Brain-weight and skull capacity occupy an intermediate condition both with reference to variability and the conditioning factors discussed.

Attention may be turned next to the question of the relative variability of the different races in respect to brain-weight. In the following scheme the races are arranged in descending order of mean brain-weight in the left-hand columns, and of variability (measured by the coefficient of variation) in the right-hand column. The lists are based on "total" series.

From these lists it would appear that as a general rule the higher the mean brain-weight is, the lower will be the variability. But it must be kept in mind that with our present series none of the differences in the variability columns is significant. Taking the extremes, Bavarians and Swedes for the males, and Bavarians and Bohemians for the females, the differences between the coefficients of variability with their probable errors are respectively ·526 ± ·217 and ·958 ± ·337.

In these cases the difference is less than three times its probable error and cannot be considered as certainly significant.   The lists are introduced merely to show the

♂		♀	
Mean	C. of V.	Mean	C. of V.
Bohemians	Bavarians	Bohemians	Bavarians
Swedes	Hessians	Hessians	Hessians
Hessians	Bohemians	Swedes	Swedes
Bavarians	Swedes	Bavarians	Bohemians

general trend of the results.   It is possible that with much larger series of brain-weight statistics and consequently reduced probable errors some such relation as that just stated might be definitely proven.

For the sake of comparison and reference I include here tabular arrangements of the means and variabilities for the characters stature and age, made in the same way as that for brain-weight.

Stature.

♂		♀	
Mean	C. of V.	Mean	C. of V.
Swedes	Bohemians	Swedes	Bohemians
Bohemians	Hessians	Bohemians	Hessians
Hessians	Swedes	Hessians	Swedes
Bavarians	Bavarians	Bavarians	Bavarians

Age.

♂		♀	
Mean	S. D.	Mean	S. D.
Bohemians	Hessians	Swedes	Bohemians
Swedes	Bohemians	Hessians	Hessians
Bavarians	Swedes	Bavarians	Swedes
Hessians	Bavarians	Bohemians	Bavarians

Comparing the different characters the means seem to be entirely chaotic, but there is more regularity as regards the variabilities.   The relative variabilities in stature and age for the different samples run parallel, with the exception of the Hessians and Bohemians in the male series.   The samples which show the greatest variability in age of individuals included, also show the greatest variability in stature, and *vice versâ*.   This relation is, of course, to be expected, but the relation between

the brain-weight series and the other two is rather curious. There is no parallelism between the variability columns of the brain-weight series and the other two, but the variability columns of the stature series together with the females of the age series exactly agree (with the exception of the transposition of the Hessians and Swedes) with the columns of *means* of brain-weight. In other words in those series showing the greatest variability in stature (and in the females of age also) we have the highest mean brain-weight, and *vice versâ.* No reason for this curious parallelism is apparent, and it may be purely accidental, but it seems worth noting.

The questions of the relative variability of the sexes in brain-weight and the racial differences as affected by sex will be discussed in a later section of the paper.

6. *On the Nature of the Frequency Distribution.*

In this section the variability in brain-weight will be discussed analytically, according to the methods and nomenclature of Pearson's fundamental memoir on Skew Variation[*] and its supplement[†]. My purpose in considering these curves analytically is not primarily that data may be furnished so that the material may be fitted with appropriate curves, but rather in order that definite knowledge may be had as to whether the variation in this character obeys the "normal" law of the deviation of errors. Both Miss Fawcett[‡] and Macdonell[§] have reached the conclusion that, for practical purposes at least, the majority of skull characters may be considered to conform to this law in their variation. It is of prime importance to determine in how far the same is true of the weight of the brain. Since this was my chief object in analyzing the data, I shall not at this time deal graphically with the curves, but instead shall merely present the chief analytical constants arranged in tabular form. Furthermore, from considerations of the time involved in computation, I have not determined the analytical constants for all the brain-weight frequency distributions given in this paper. Instead, after examining all the data, I decided to confine myself to the "total" series for both sexes of the four races.

The analytical constants for the selected series are exhibited in Table VIII. In the second column is given the number of brains on which the calculation in each case is based. The third column gives the unit in terms of which the second, third and fourth moments about the mean (μ_2, μ_3, and μ_4) are calculated: the next two columns give β_1 and $\sqrt{\beta_1}$, and the two following, β_2 and $3 - \beta_2$. Following this are given in order the "criterion" ($\kappa_1 = 2\beta_2 - 3\beta_1 - 6$)[||], the mean, mode and skewness. The skewness was calculated from the moments directly by the formula

$$Sk = \tfrac{1}{2} \frac{\sqrt{\beta_1}\,(\beta_2 + 3)}{5\beta_2 - 6\beta_1 - 9}, [¶]$$

and from this the distance from mean to mode was obtained by multiplying by σ.

* *Phil. Trans.* Vol. 186, A, pp. 343—414. † *Ibid.* Vol. 197, A, pp. 443—459.

‡ *Biometrika*, Vol. i. p. 443. § *Ibid.* Vol. iii. p. 227.

|| *Phil. Trans.* Vol. 197, A, p. 444.

¶ Pearson, K.: "On the Mathematical Theory of Errors of Judgment and on the Personal Equation," *Phil. Trans.* Vol. 198, A, pp. 235—299. Page 277.

TABLE VIII.

Analytical Constants of Curves.

Brain-weight.

Race and Series		No.	Unit	μ_2	μ_3	μ_4	β_1	$\sqrt{\beta_1}$	β_2	$3 - \beta_2$	Criterion	Mean	Mode	Skewness
Swede	"Total" ♂	416	50 g.	4·5223	1·6292	57·18995	·0287	·1694	2·7964	·2036	−·4934	1400·481	1389·627	·1021
,,	,, ♀	233	50 g.	4·0608	1·8441	51·1703	·0508	·2254	3·1031	−·1031	·0539	1252·682	1241·526	·1107
Hessian	,, ♂	475	50 g.	5·0783	−4·1458	86·8638	·1312	·3623	3·3682	−·3682	·3428	1391·737	1373·311	·1635
,,	,, ♀	281	50 g.	4·1917	−·2601	49·8514	·0092	·0958	2·8372	·1628	−·3531	1259·875	1254·294	·0545
Bohemian	,, ♂	372	100 g.	1·2907	·5908	5·2300	·1623	·4029	3·1396	−·1396	−·2078	1454·839	1430·292	·2161
,,	,, ♀	197	100 g.	·9365	·1033	2·5144	·01298	·1139	2·8671	·1329	−·3048	1310·914	1304·762	·0636
Bavarian	,, ♂	529	50 g.	4·8986	2·4919	79·2516	·0528	·2298	3·3026	−·3026	·4467	1363·185	1352·047	·1006
,,	,, ♀	323	50 g.	4·1433	−·9391	77·7815	·0124	·1114	4·5408	−1·5408	3·0444	1220·356	1217·221	·0308
,,	"Young" ♀	238	50 g.	3·3431	3·0329	45·6227	·2462	·4962	4·0820	−1·0820	2·4255	1235·504	1219·334	·1769

Considering first the *skewness*, it is seen to be in all the series positive, or the mean is greater than the mode. Further we note that in all cases the value is low. Whether the values can be considered significant however can only be determined by an examination of the probable errors. The formula for the probable error of the skewness is $\cdot 67449 \sqrt{\dfrac{3}{2n}}$ *. The limiting values for this probable error for values of n ranging from 197 to 529 as in the present case are respectively $\cdot 0589$ and $\cdot 0359$. Having regard to the number of cases on which the calculations are based it appears that in six out of the nine cases tabulated the skewness can be regarded as certainly or probably insignificant. In all of these six cases the skewness is less than thrice its probable error, in two cases it about equals its probable error, and in one is less. The remaining three cases out of the total (skewness, $\cdot 1635$, $\cdot 2161$ and $\cdot 1769$) are very probably or certainly significant. In general we may safely conclude, I think, that, *in the case of the weight of the brain, the distance from the mean to the mode will be very small. If the mean and mode do not coincide the mean will be greater than the mode.* This agrees with Miss Fawcett's[†] conclusion for the most important skull characters in the Naqada race. Macdonell[‡] finds, however, that in the case of the English, considering the same skull characters, "if we were to draw the curves, the mean would be found in half the number of the curves to be less, and in the other half to be greater than the mode." In neither brain-weight nor skull series does there appear to be any definite preponderance in the value of the skewness of one sex over the other.

We may turn now to the other constants, which are of most significance in determining whether the distribution may be considered normal within the limits of error; viz., $\sqrt{\beta_1}$, β_2, and the criterion. The probable error of $\sqrt{\beta_1}$ $\left(= \cdot 67449 \sqrt{\dfrac{6}{n}} \right)$ ranges in value for our series between $\cdot 0718$ $(n = 529)$, and $\cdot 1177$ $(n = 197)$; that of β_2 $\left(= \cdot 67449 \sqrt{\dfrac{24}{n}} \right)$ between $\cdot 1437$ $(n = 529)$ and $\cdot 2354$ $(n = 197)$; and that of the criterion $\left(= \cdot 67449 \sqrt{\dfrac{96}{n}} \right)$ between $\cdot 2873$ $(n = 529)$ and $\cdot 4708$ $(n = 197)$. Considering the probable errors of $\sqrt{\beta_1}$, it is seen at once that of the eight "total" series three give certainly insignificant values ($\cdot 0958$, $\cdot 1139$, $\cdot 1113$) for $\sqrt{\beta_1}$; two others give values which are probably insignificant ($\cdot 1694$, $\cdot 2254$). One ($\cdot 2298$) is probably significant; and the two remaining values ($\cdot 3623$ and $\cdot 4029$) are certainly significant. The Bavarian "young" ♀ series gives a certainly significant value for $\sqrt{\beta_1}$. Taking next the deviation of β_2 from 3 in comparison with the probable error of β_2 we see that in four cases $(3 - \beta_2 = - \cdot 1031$, $\cdot 1628$, $- \cdot 1396$, $\cdot 1329)$ β_2 differs from 3 by an insignificant amount. In one case ($\cdot 2036$) the difference is less than twice the probable error and hence may be considered

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* The formulae for the probable errors of the analytical constants are given on p. 278 of Pearson's memoir on "The Mathematical Theory of Errors of Judgment," *loc. cit. supra*.

† *Loc. cit.* p. 443.

‡ *Biometrika*, Vol. III. p. 227.

as very probably insignificant. In two cases ($\cdot$3682 and $\cdot$3026) the difference is less than thrice the probable error and hence may be considered possibly, or even perhaps, probably insignificant. The Bavarian ♀ series, both " total " and " young," give certainly significant values for $3 - \beta_2$. Considering finally the criterion it is seen that in all cases except the two Bavarian ♀ the criterion differs from zero by a certainly or very probably insignificant amount. These two Bavarian ♀ series differed so greatly from the normal curve in most of the analytical constants that it was thought desirable to determine their position precisely by means of another constant κ_2*. For the " total " series I found $\kappa_2 = \cdot$0032, and for the " young " series $\kappa_2 = \cdot$0816. By the scheme given by Pearson (*loc. cit.* p. 445) we see that the " total " series, when the probable errors of the constants are considered, comes very close to the condition demanding a curve of Type II ($\kappa_2 = 0$, $\beta_1 = 0$, β_2 not $= 3$). The " young " series clearly demands a curve of Type IV ($\kappa_2 > 0$ and < 1). The deviation of these Bavarian female curves from the normal type I believe to be due to an undue accumulation of individuals in one brain-weight class; viz., that from 1250—1300 gr. It seems altogether probable that some of the individuals which should have gone into the next higher class (represented in the " total " series by a frequency of only 26 as against 69 in the class next lower) have by some error been entered in Bischoff's lists with too low brain-weights. What the source of error was it is, of course, impossible now to determine. The abnormality of the Bavarian females has already been noted in the discussion of the means and variabilities. Leaving these two series out of account I think that on the whole we may safely conclude, as Miss Fawcett and Macdonell (*loc. cit.* p. 443, and p. 227 resp.) have for skull characters, that :

With series of brain weighings such as are considered in this paper we shall reach for most practical purposes adequate graphical representations of the frequency by using the normal curve of deviation: $y = y_0\,e^{-x^2/2\sigma^2}$.

It should always be kept in mind, however, that our series, both on the brain-weight and skull sides, are too small to fix absolutely the normality or non-normality of the variation in these characters. Some of the distributions certainly differ from normality. The conclusion stated above is to be considered simply as a *practical* result, rather than as a theoretical generalization.

This result seems to be of considerable importance as indicating the worth of brain-weight statistics. It shows that such statistics justify careful study and analysis, and that, contrary to the statements of certain recent writers on the subject, there is no general fallacy inherent in the data themselves which renders abortive any attempt to reach through them the truth regarding the mass relations of the brain.

In order to test exactly how well the normal curve represents the data in a single case I have fitted the Swedish male " total " series with a normal curve. The frequency histogram and its fitted curve are shown graphically in the

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* $\kappa_2 = \dfrac{\beta_1\,(\beta_2 + 3)^2}{4\,(\beta_2 - 3\beta_1)\,(2\beta_2 - 3\beta_1 - 6)}$, Pearson, *Phil. Trans.* Vol. 197, A, p. 444.

accompanying diagram. The unit of x is 50 grams. The scale of frequency at the left of the diagram gives y.

DIAGRAM I. Showing the distribution of variation in the brain-weight of adult Swedish Males.
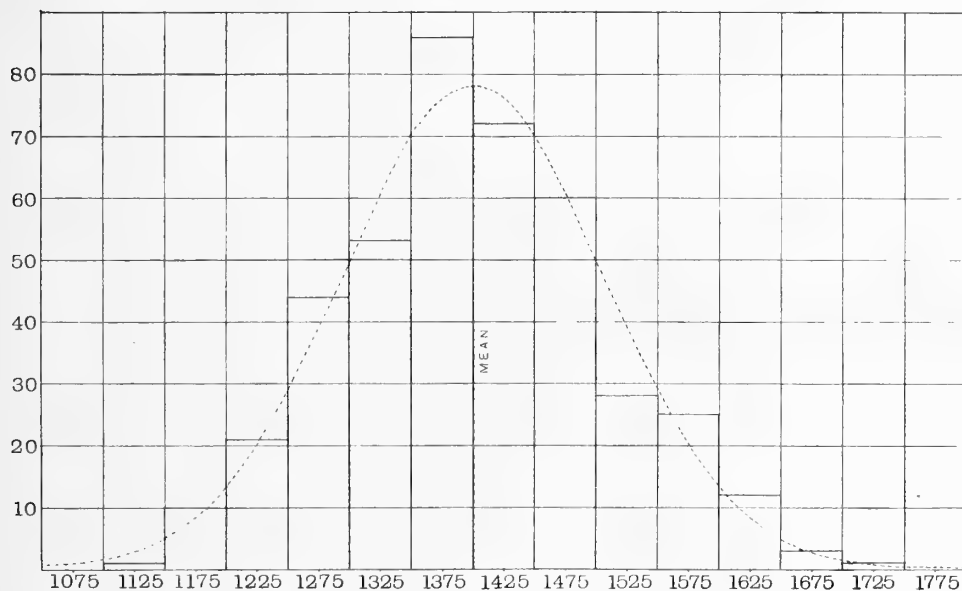
The general equation $y = y_0 e^{-x^2/2\sigma^2}$, where in this case $y_0 = \dfrac{416}{\sigma \sqrt{2\pi}}$ and $\sigma = 2 \cdot 1266$, becomes $y = 78 \cdot 0401 \, e^{-\cdot 01106 \, x^2}$, with the origin at $1400 \cdot 481$. Applying Pearson's test of the goodness of fit*, and comparing *areas* instead of mid-ordinates, as is necessary where the number of frequency groups is so small, we get the results shown in the accompanying table. The table gives the observed frequencies ($m_r{}'$),

Grams of brain-weight	Observed	Calculated	$\dfrac{(m_r - m_r{}')^2}{m_r}$
Under 1100	0	·981	·981
1100—1150	1	2·9	1·24
1150—1200	10	8·5	·26
1200—1250	21	20·3	·02
1250—1300	44	39·0	·64
1300—1350	53	60·4	·91
1350—1400	86	75·2	1·55
1400—1450	72	75·3	·14
1450—1500	60	60·8	·01
1500—1550	28	39·4	3·29
1550—1600	25	20·6	·94
1600—1650	12	8·7	1·25
1650—1700	3	2·9	·003
1700—1750	1	·8	·05
1750 and over	0	·036	·036
Totals	416	415·817	11·320

* *Phil. Mag.* Vol. L. pp. 157—175.

the calculated frequencies (m_r), and the ratio of the squared difference between the two to the calculated $\left(\dfrac{(m_r - m_r')^2}{m_r} \right)$.

Here n' the number of frequency groups is 15 and $\chi^2 = 11 \cdot 32$. From Elderton's Tables* the value of P may be obtained. With $n' = 15$ and $\chi^2 = 11$ the value of P as given in the table is $\cdot 686036$. Or, expressed in words, if the brain-weight of Swedish males obeyed the "normal" distribution of frequencies we should expect to get a greater divergence between theory and observation in roughly 65 out of every 100 trials when the trials were based on random samples of 416 individuals each. In other words, the fit may be considered very fair, and certainly entirely satisfactory for all practical purposes.

7. *The Correlation of Brain-weight with other Characters.*

In the introductory portion of this paper it was stated that one of the chief objects for which the work was undertaken was to measure exactly the degree and nature of the association between brain-weight and other characters in man. All workers on the subject have realized the importance of such determinations and numerous attempts have been made to arrive at them. The methods used, however, have not been such as to lead to definite and sound conclusions. These methods have in the main consisted in tabulating the mean brain-weights for various groupings of the other characters under consideration. In this way it can of course be determined whether there is any shift of the mean as the other character changes. Further than this one cannot go. Besides, in this method there lurk numerous pitfalls unless one uses it with a clear understanding of some of the fundamentals of statistical science. Different results can be obtained as the material is differently grouped. Hence, it is not strange that we find conservative anatomists making only very general statements as to the correlation of brain-weight with other characters, and, in those cases where an attempt at greater precision of statement is made, considerable difference of opinion as to what conclusions shall be drawn from the data. What are wanted in the case are not simply inspections and general "appreciations" of tabular lists of brain-weighings, or of ratios and indices, or even of correlation tables or regression lines, but instead definitely determined coefficients of correlation with their probable errors. With these we can get in given cases precise statements of the degree of correlation between different characters.

It would of course be highly desirable to measure the correlation between the weight of the brain and a great variety of other organs and characters of the body. Such characters will at once suggest themselves to anyone interested in the problems of correlation. For instance, aside from age, stature and body-weight for which material is available, there are the whole range of skull characters; the

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* *Biometrika*, Vol. I. p. 161.

weights of the various viscera; the weight and volume of the spinal cord; characters of the muscular system, etc. A study of the correlation between any or all of these characters and the weight of the brain would be most interesting. But, unfortunately, material for such study cannot be had. Brain-weight statistics usually provide data for but few other characters; in the majority of cases only sex, age and stature. In two of the series used in this work, viz., the Swedish and Hessian, this was the case. For these two groups I have determined all possible correlations for both sexes, and for two age groupings ("young" and "total"). This gives as the pairs of characters, (a) brain-weight and stature, (b) brain-weight and age, (c) stature and age. Bischoff's Bavarian material furnished in addition to the above, data on the body-weight. The following correlations have been determined for this material: (a) brain-weight and age ("total" and "young"), (b) brain-weight and stature ("total"), (c) brain-weight and body-weight ("total"), (d) body-weight and stature ("total"). In the case of the Bohemian material data were furnished from which the correlation between skull length and skull breadth could be determined so that for this group I have been able to calculate coefficients of correlation for the following pairs of characters: (a) brain-weight and stature ("young"), (b) brain-weight and age ("total"), (c) brain-weight and skull length ("young"), (d) brain-weight and skull breadth ("young"), (e) skull length and skull breadth ("young"). In Table IX are exhibited the coefficients of correlation together with their probable errors. The correlation tables from which the coefficients were calculated are given in the appendix to this paper, Tables 1 to 44. References are given in Table IX telling for each coefficient the original table from which it was calculated. The coefficients of correlation were determined from the usual Bravais formula according to which the coefficient of correlation

$$r = \frac{S\,(xy)}{N\sigma_1\sigma_2},$$

where x and y are deviations from the means of the two correlated characters and σ_1 and σ_2 are the respective standard deviations; the usual[*] method of evaluating r from the above equation was used.

The most striking fact of a general nature shown by this table is the generally low degree of correlation which exists between the weight of the brain and other characters. The coefficients run noticeably lower even than those of skull characters (cf. Macdonell's Table V[†]) and very markedly below those between different characters of the long bones[‡]. On the other hand our brain-weight correlations give values of the same general order of magnitude as those found by Greenwood[§] for various abdominal and thoracic viscera. Attention may be called

[*] Yule: *Jour. Roy. Stat. Soc.* Vol. LX. Part IV. pp. 1—44 (Reprint).
[†] *Biometrika*, Vol. III. p. 232.
[‡] Lee and Pearson: *Phil. Trans.* Vol. 196, A, pp. 228, 229.
[§] *Loc. cit.* p. 19 above.

TABLE IX.

Brain-weight. Correlations.

Character and Series	No.	♂ (Swedes)	Table	No.	♀ (Swedes)	Table	No.	♂ (Hessians)	Table	No.	♀ (Hessians)	Table
Brain-weight and Stature (Total)	416	·1830 ± ·0320	1	233	·3490 ± ·0388	2	475	·1823 ± ·0299	13	281	·1828 ± ·0389	14
,, ,, ,, (Young)	262	·1796 ± ·0403	3	127	·3390 ± ·0530	4	291	·1741 ± ·0383	15	173	·1809 ± ·0496	16
Brain-weight and Age (Total)	416	−·2493 ± ·0310	5	233	−·2336 ± ·0418	6	475	−·1673 ± ·0300	17	281	−·3598 ± ·0350	18
,, ,, ,, (Young)	262	−·1705 ± ·0405	7	127	−·1512 ± ·0585	8	291	−·0750 ± ·0393	19	173	−·1650 ± ·0499	20
Stature and Age (Total)	416	−·1620 ± ·0322	9	233	−·2279 ± ·0419	10	475	−·0481 ± ·0308	21	281	−·0915 ± ·0399	22
,, ,, (Young)	262	−·2549 ± ·0390	11	127	−·2120 ± ·0572	12	291	·2102 ± ·0378	23	173	−·0474 ± ·0512	24

Character and Series	No.	♂ (Bohemians)	Table	No.	♀ (Bohemians)	Table	No.	♂ (Bavarians)	Table	No.	♀ (Bavarians)	Table
Brain-weight and Stature (Total)	266	·2034 ± ·0397	25	133	·2168 ± ·0557	26	365	·1664 ± ·0343	35	241	·2236 ± ·0413	36
,, ,, ,, (Young)												
Brain-weight and Age (Total)	372	−·2045 ± ·0335	27	197	−·2558 ± ·0449	28	529	−·1225 ± ·0290	37	323	−·2405 ± ·0354	38
,, ,, ,, (Young)												
Brain-weight and Skull Length (Young)	299	·5482 ± ·0273	29	159	·3604 ± ·0465	30	365	−·0100 ± ·0353	39	238	·0114 ± ·0412	40
,, ,, Skull Breadth (Young)	299	·4655 ± ·0306	31	159	·5041 ± ·0399	32	365	·1671 ± ·0343	41	241	·2260 ± ·0412	42
Brain-weight and Body-weight (Total)												
Skull Length and Skull Breadth (Young)												
Body-weight and Stature (Total)	299	·4006 ± ·0327	33	159	·1954 ± ·0514	34	365	·3962 ± ·0298	43	241	·4334 ± ·0352	44

here to the general uniformity of the correlations for the same characters in different series. All the brain-weight and stature correlations are seen to be positive, while with a single exception (Bavarian female "young" series) all the correlations of brain-weight with age are negative. The coefficients are generally lower for the "young" than for the "total" series, as is to be expected. In the case of the correlation of brain-weight with age, some of the coefficients for the "young" series are evidently insignificant in comparison with their probable errors. The correlations between stature and age are negative with the exception of the Hessian male series. The positive sign in these series arises from the fact that in this material the age class 15 to 20 was included. All growth in stature has not stopped at age 20, and as a consequence there are included in the tables (21 and 23) five individuals of unusually small stature and low age. These serve to change the sign of the coefficients. Their greater effect in the "young" series is apparent. The detailed discussion of the various brain-weight correlations and the regressions based on them I propose to take up in separate sections of the paper now to follow.

8. *Brain-weight and Sex.*

All brain-weight statistics show that the brain of the male is absolutely heavier than that of the female. In the series here discussed the absolute differences in mean brain-weight between males and females are as follows:

	Male mean—Female mean	
	"Total" series	"Young" series
Swedes ...	147·8	145·8
Bohemians ...	143·9	146·2
Bavarians ...	142·8	133·6
Hessians ...	131·9	125·6

From this table the following points are to be noted:

(a) Considering the size of the probable errors involved it is evident at once that the *absolute* difference is sensibly the same for all four races. Taking the extremes of the "total" column the difference between the Swede and Hessian sex differences is 15·9 with a probable error of ± 7·9, or in other words the difference is almost exactly twice its probable error and cannot be considered certainly significant. The "young" series points to the same conclusion. So then, Weisbach's law for stature that the greatest sex differences occur in those races having the highest mean stature, does not appear to hold for brain-weight, so far as absolute differences are concerned.

(b)   The absolute sex differences are sensibly the same for both age groupings. We may conclude then, that whatever changes occur in the weight of the brain with advancing age act in such a way as to leave the absolute sex difference unchanged.

These absolute differences may be converted into relative differences by expressing them as percentages of the ♂ and ♀ means.   When so expressed they take the following form :

Sex Differences in Relation to Means.

Race	"Total" series		"Young" series	
	Percentage of ♂ mean	Percentage of ♀ mean	Percentage of ♂ mean	Percentage of ♀ mean
Swedes ...	10·6 °/₀	11·8 °/₀	10·3 °/₀	11·5 °/₀
Bavarians ...	10·5 °/₀	11·7 °/₀	9·8 °/₀	10·8 °/₀
Bohemians ...	9·9 °/₀	10·97 °/₀	10·0 °/₀	11·1 °/₀
Hessians ...	9·5 °/₀	10·5 °/₀	8·9 °/₀	9·8 °/₀

The relative differences confirm the conclusions reached from the examination of absolute differences.   The result that the changes which take place in the weight of the brain during adult life affect both sexes to an equal degree relatively seems to be of some importance.   It implies that the factors which bring about the gradual lowering in the weight of the brain with advancing age are organic in a strict sense, and not dependent on environmental conditions.

The same thing is shown if the relative differences expressed in another way, namely by taking the ratio of male to female mean, are examined.   For our series these ratios are as follows :

Sex Ratios.   Means.   Brain-weight.

Race	"Total" series	"Young" series
Swedes ...	1·118	1·115
Bavarians ...	1·117	1·108
Bohemians ...	1·110	1·111
Hessians ...	1·105	1·098

The sex ratios for a number of other characters are given in the following table for comparison.   The values in this table are taken from a table in Pearson's *Chances of Death**.

* Vol. i. p. 374.

Sex Ratios.   Means.   Various Physical Characters.

Organ or Character	Sex Ratio
Body weight, Babies	1·034
„ „ Children	1·038
„ „ Adults	1·193
Weight of Vital Organs	1·130
Stature, Children	1·007
„ Adults	1·077
Height, Sitting	1·032
Long Bones	1·086
Chest Girth	1·024
Squeeze of Hands	1·207
Keenness of Sight and Touch ...	1·061
Skull Capacity	1·124
„ Circumference	1·042
Cephalic Index	·997
Head Index	·995
Profile Angle	·994
Alveolar Angle	·994
Nose and Palate	1·013

We may turn next to the question of the relative variability of the sexes in respect to brain-weight. Pearson[*] found that in the case of the English, the female showed "slightly more" variability than males; the French data which he worked over gave sensibly equal variabilities for the sexes, as did also Bischoff's Bavarian data.   In the following table are shown the differences between the male and female coefficients of variation with the probable errors of the differences. The values tabulated are the male minus the female constant in each case.

Relative Variability of the Sexes.   Male − Female.

Race	Coefficient of Variation	
	"Total" series	"Young" series
Bohemians ...	·427 ± ·320	·740 ± ·375
Hessians ...	− ·029 ± ·293	− ·036 ± ·366
Bavarians ...	− ·222 ± ·254	·546 ± ·305
Swedes ...	− ·451 ± ·310	− ·600 ± ·422

In three cases out of the eight the male has a larger coefficient of variation, but in no instance can the differences between male and female coefficients be considered significant when compared with the probable errors.   In only one case (Bohemian "young" series) does the difference approach closely to a value even twice as great as its probable error.   Therefore we must conclude that, so far as the series here considered are concerned, there is no significant difference between

[*] *Chances of Death*, Vol. I. pp. 321, 322.

males and females in respect to variability in the weight of the brain. In view of the general reliability, from a statistical standpoint, which has been previously demonstrated to exist in the material on which this work is based, it seems not unreasonable to predict that it will be found that equal variability of the sexes in brain-weight holds generally.

Another problem which presents itself in this connection is as to how much of the observed difference between males and females in brain-weight is to be accounted for by the fact that the male body is in general larger than the female. In other words, would a group of females of the same stature and age, say, as a given group of males have the same brain-weight? It is possible to reach a general solution of this problem by the use of characteristic equations based on the correlations and regressions. I propose now to discuss this question by this method. Of course, the prediction may be made by characteristic equations in either direction; i.e., we may predict the probable brain-weight of a group of males having other characters the same as in a given group of females, or we can predict the probable brain-weight of a group of females having the characters the same as in a given group of males. In the former case characteristic equations based on the male data would be used, in the latter the equations would be those deduced from female data. We may consider first the characters, age, and stature. In the following tables are given (*a*) the probable brain-weights of a group of individuals of the sex indicated, as calculated from the characteristic equation referred to in the last column of the table; (*b*) the observed brain-weights of a group of the opposite sex having for the other characters the mean values used in the characteristic equation in calculating (*a*); (*c*) the difference between (*a*) and (*b*); (*d*) a reference to the equation from which (*a*) is calculated.

TABLE X.

Probable Female Brain-weight with Stature and Age equal to Observed Male.

Race		Predicted female brain-weight	Observed male brain-weight	Difference	Equation
Hessian	(Total) ...	1287·963	1391·737	103·774	25
„	(Young) ...	1308·941	1405·756	96·815	26
Swede	(Total) ...	1307·340	1400·481	93·141	21
„	(Young) ...	1319·783	1415·267	95·484	22

The mean difference here is 97·304. The mean observed difference between male and female without any allowance for stature and age is 137·8 for the same races. So then the differences in stature and age between the males and females in our Swedish and Hessian samples account for only 40·5 gr. or about 29 per cent. of the observed difference in mean brain-weight between the sexes.

Let us now proceed to the reversed prediction, and find the probable brain-weights of a group of males having the same stature and age as the means for

these characters in the Hessian and Swedish females. The results are shown in Table XI, which is arranged in the same manner as the preceding table.

TABLE XI.

Probable Male Brain-weight with Stature and Age equal to Observed Female.

Race	Predicted male brain-weight	Observed female brain-weight	Difference	Equation
Hessian (Total) ...	1357·591	1259·875	97·716	23
„ (Young) ...	1369·431	1280·202	89·229	24
Swede (Total) ...	1370·543	1252·682	117·861	19
„ (Young) ...	1391·624	1269·488	122·136	20

The mean difference here is 106·736, or in other words we reach essentially the same result as before. The general conclusion up to this point then is that there is a difference in the mean brain-weight between the sexes of roughly 100 gr. after allowance has been made for stature and age differences by taking the mean brain-weight of an array of individuals of one sex having for type of stature and age the respective means of these characters observed in the other sex. Differences in stature and age account for less than one-third of the observed sex difference in brain-weight.

We may next determine what part of the observed sex difference in brain-weight is to be accounted for by differences in skull length and breadth. Here only one race, the Bohemian, can be used on account of lack of material. Let us examine the effect on the mean brain-weight of selecting a group of individuals of one sex having the same mean skull length and skull breadth as the opposite sex. Making the designated selection first from the males and then from the females we have the results shown in Table XII.

TABLE XII.

Probable Male Brain-weight with Mean Skull Length and Skull Breadth equal to the Observed Female.

Race	Predicted male brain-weight	Observed female brain-weight	Difference	Equation
Bohemian ...	1395·631	1310·914	84·717	35

Probable Female Brain-weight with Mean Skull Length and Skull Breadth equal to the Observed Male.

Race	Predicted female brain-weight	Observed male brain-weight	Difference	Equation
Bohemian ...	1389·736	1454·839	65·103	36

The mean of the two differences is 74·910, while the observed difference between male and female mean brain-weights in the Bohemian "skull" series is 142·014, or in other words by selecting from either sex individuals with mean skull length and skull breadth equal to those found in the opposite sex, we are able to reduce the sex difference in mean brain-weight by about 67 grams, or 47 °/$_\circ$. This result is of considerable interest. We can come nearer to the brain-weight of one sex by selecting skull length and breadth to the means of that sex than we can by selecting stature and age, but still not a great deal nearer. We should on general grounds expect that aside from skull capacity, skull length and skull breadth would be the two characters whose selection would indirectly modify brain-weight the most. Admitting this to be the case there still remains a considerable difference to be accounted for. It is interesting to note that the sexes are brought nearer together in brain-weight when males are selected than when the selection is from the females. This is explained by the fact that the brain-weight is somewhat more highly correlated with the skull characters in the males in our series.

Finally, we may examine the effect of a selection of stature and body-weight on the sex difference in brain-weight. The material here comes from the Bavarian series, and the prediction is made in the same way as in the other cases; viz., by selecting males with mean stature and body-weight equal to the female means and *vice versâ*, and determining the mean brain-weight of the selected groups. The results are shown in Table XIII.

TABLE XIII.

Probable Male Brain-weight with Mean Stature and Body-weight equal to the Observed Female.

Race	Predicted male brain-weight	Observed female brain-weight	Difference	Equation
Bavarian ...	1321·496	1218·776	102·720	29

Probable Female Brain-weight with Mean Stature and Body-weight equal to the Observed Male.

Race	Predicted female brain-weight	Observed male brain-weight	Difference	Equation
Bavarian ...	1261·298	1357·466	96·168	30

Here the better result is obtained by selecting from the females on account of the higher correlation between brain-weight and body-weight and brain-weight and stature in this sex. The mean of the two differences is 99·444, and the observed sex difference in the case is 138·690 gr. So then the differences between

the sexes in body-weight and stature account for 39·246 gr. or 28 °/₀ of the observed difference in mean brain-weights. In other words, by selecting individuals on the basis of body-weight and stature alone we shall produce about the same degree of change in the mean brain-weight as by selecting on the basis of stature and age alone.

Taking all the results together we reach the general conclusion that the difference between the sexes in mean brain-weight is only in part to be accounted for by differences in other bodily characters. While in general it is true that a small body has a brain of low weight, yet in order for the observed difference in mean brain-weight in men and women to be due to this factor alone either the women ought to have very much smaller bodies than they actually possess, or the men ought to have larger bodies, or both sexes ought to be different from what they actually are in size of body in the directions indicated. Of course it is very easy to say by way of speculation that the smaller brain of woman is due to the fact that the female human organism is subjected to less strenuous demands along the lines of motor and intellectual activity than the male. But such speculation leads to nothing on account of our lack of definite scientific evidence as to the degree of correlation between the weight of the brain and amount and intensity of psychic activity.

One other question remains to be considered in connection with the relation of brain-weight to sex. Does either sex show any considerably higher degree of correlation between brain-weight and other characters than the other? Table IX furnishes data on this point. With the exception of the Swedish brain-weight and age correlations and the brain-weight and skull-length correlations it is seen that the female coefficients are uniformly larger than the male. The differences are in many of the cases not significant in comparison with their probable errors but the general tendency towards higher correlations in the females is clearly evident. The meaning of this tendency is in this case difficult to conjecture. It falls in line with the result which has been pointed out by Lee and Pearson in several papers*, viz., that for physical characters generally women usually show a higher degree of correlation than men, in civilized races.

9. *Brain-weight, Age and Stature.*

The coefficients of correlation between brain-weight and age and stature respectively have already been given in Table IX, but before proceeding further in the discussion of their significance it is necessary to determine whether the regressions are linear. One might on general grounds expect the brain-weight and age correlations to be skew, since this is generally the case with growth correlations where one of the variables is age and the other the absolute magnitude of some organ or character. If in any case the correlation turns out to be skew, it of course greatly complicates the problem.

The method of analysis which I have followed in determining the degree of

* E.g., "On the Relative Variability and Correlation in Civilized and Uncivilized Races," *Roy. Soc. Proc.* Vol. LXI. pp. 343—357.

approach to linearity of the regressions is that given by Pearson*. The two constants which are of the greatest significance here are, (a) the mean square deviation of the means of the arrays from the regression line, $\Sigma_{M}{}^{2}$, and (b) a constant η, called the correlation ratio, giving the mean reduction in variability of an array as compared with the whole population. Evidently

$$\Sigma_{M}{}^{2} = \sigma_{M}{}^{2} - r^{2}\sigma^{2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots(i),$$

and

$$\eta = \frac{\sigma_{M}}{\sigma} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(ii),$$

whence by simple substitution $\quad \Sigma_{M}{}^{2} = (\eta^{2} - r^{2})\,\sigma^{2} \quad \dots\dots\dots\dots\dots\dots\dots\dots(iii),$

where in these equations σ is, as usual, the standard deviation of the variates about the mean for the whole population, r is the coefficient of correlation between the two variables concerned, and σ_{M} is the standard deviation of the means of the arrays about the mean of these means. The deviation of Σ_{M} from zero, and of η from r, measure the deviation of the system from linearity†.

I propose to discuss the relation of brain-weight first to age alone, then to stature alone, and finally to both age and stature together.

In Table XIV are exhibited the values of Σ_{M} and η for the correlations between brain-weight and age discussed in this paper. In calculating σ_{M} from which to obtain η according to the relation given above, in this and all other cases the means of the arrays were weighted with the number of cases on which they were based. This procedure of course gave the mean of the means of the arrays the same value as the general population mean calculated from the elemental frequency distribution.

TABLE XIV.

Analytical Constants for Linearity of Regression. Brain-weight and Age.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Swedish (Total) ...	− ·2493 ± ·0310	− ·2336 ± ·0418	·2876	·2770	·1434 σ	·1489 σ
,, (Young)...	− ·1705 ± ·0405	− ·1512 ± ·0585	·2251	·2143	·1469 σ	·1519 σ
Hessian (Total) ...	− ·1673 ± ·0300	− ·3598 ± ·0350	·2002	·3864	·10996 σ	·1409 σ
,, (Young)...	− ·0750 ± ·0393	− ·1650 ± ·0499	·1411	·1961	·1195 σ	·1060 σ
Bavarian (Total) ...	− ·1225 ± ·0290	− ·2405 ± ·0354	·1962	·3481	·1533 σ	·2517 σ
,, (Young)...	− ·0100 ± ·0353	·0114 ± ·0412	·0676	·1958	·0669 σ	·1955 σ
Bohemian (Total) ...	− ·2045 ± ·0335	− ·2558 ± ·0449	·2441	·3033	·1333 σ	·1628 σ

* *Roy. Soc. Proc.* Vol. LXXI. pp. 303—313, especially the footnote, pp. 303, 304. Since the above was written a very full treatment of the whole subject of skew correlation and non-linear regression by Prof. Pearson has appeared as "Mathematical Contributions to the Theory of Evolution, XIV." *Drapers' Company Research Memoirs*, Biometric Series II. 1905.

† Of course the deviation of η from r must be considered numerically simply, because η is necessarily a positive quantity from (ii) above, since neither σ_M nor σ can be negative. If the difference between η and r were taken with reference to sign in cases where r is minus, an altogether false notion of the degree of departure of the regression from strict linearity would be obtained. The degree of this departure will always, of course, be immediately given by Σ_M whatever the sign of r.

It is seen at once that η differs from r and Σ_M from zero in all cases. This, of course, implies departure of the system from linearity. But evidently deviations of a system from linearity may be due to either one or both of two causes. Either, on the one hand, the system may be truly non-linear, in which event the means of the arrays will be fitted better by some curve than by a straight line, or, on the other hand, the points fixed by the means of the arrays may not lie exactly on a straight line and still no curve will represent the relationship between the two variables concerned better than a straight line so drawn that the mean square of the deviations of the points from the line is a minimum. In the first case we have true non-linearity of the regression, while in the second the deviation from linearity is due to the errors of random sampling, and it might reasonably be expected that if the whole population could be studied the regression would become strictly linear. Now evidently in both these cases η will differ from r and Σ_M from zero, so that recourse must be had to some further method in order to determine into which class a given case falls. Two such methods immediately suggest themselves: one, to examine the probable errors involved, the other, to inspect the fitted regression line.

An examination of Table XIV makes it immediately evident that the differences between η and r (without regard to sign) give values of the same order of magnitude as the probable errors of r. In only one case is this difference as great as three times the probable error of r, and in the great majority of cases it does not approach such a value*. So then it seems probable that in our series the regression of brain-weight on age is linear within the errors arising from random sampling. The approach to linearity is sensibly the same both for the whole period of adult life and for the younger half of this period.

In order to bring out the facts graphically I have had prepared a series of diagrams showing the regression lines for brain-weight on age. For the sake of economizing space, and since there is essential agreement between the different series with respect to η, it was decided not to publish all the regression diagrams. I have chosen for representation here the regression lines for the Swedish and the Hessian data. These are shown in Diagrams II to IX.

DIAGRAM II. Probable Brain-weight for given Age.
Swedish ♂ Total.
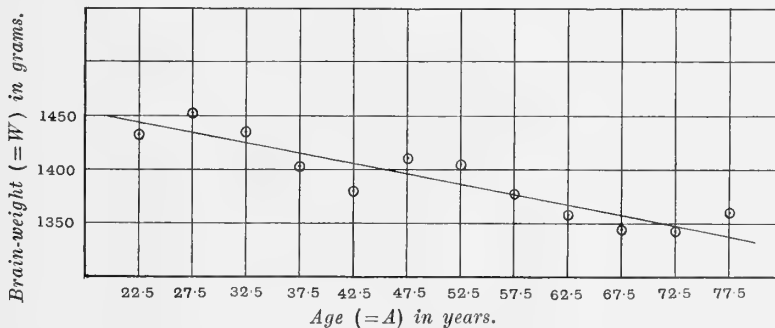
[* The proper test is the probable error of $\eta \sim r$, which we hope will shortly be published. ED.]

DIAGRAM III. Probable Brain-weight for given Age.
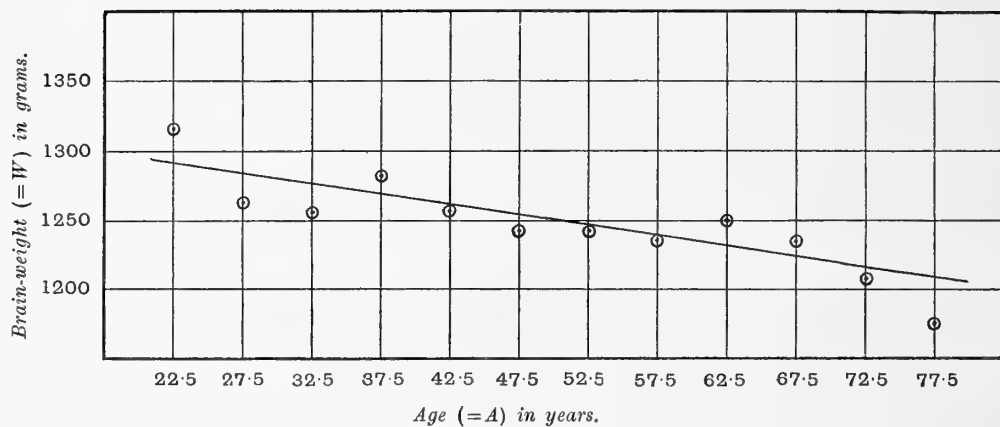
Swedish ♀ Total.

DIAGRAM IV. Probable Brain-weight for given Age.
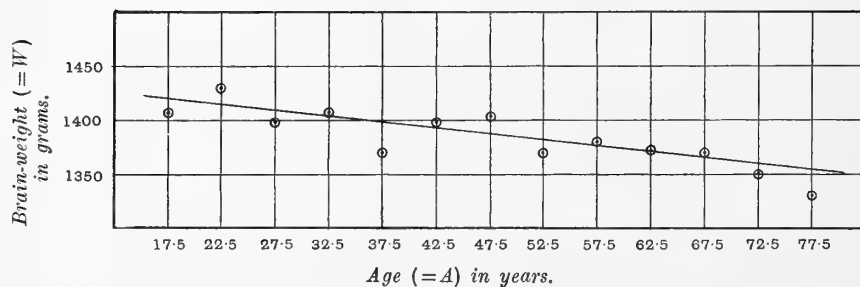
Hessian ♂ Total.

DIAGRAM V. Probable Brain-weight with given Age.

Hessian ♀ Total.
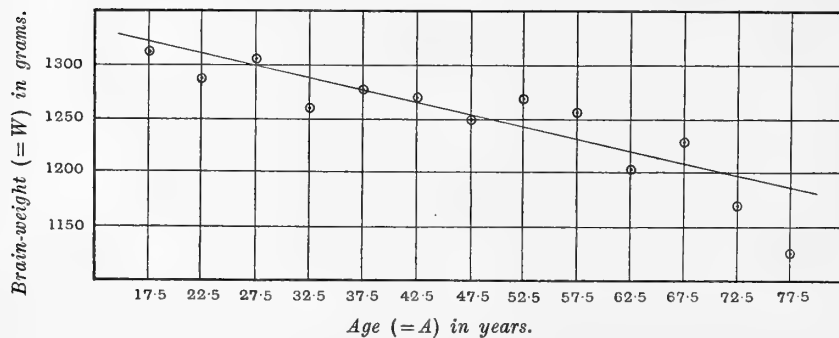
DIAGRAM VI. Probable Brain-weight with given Age.
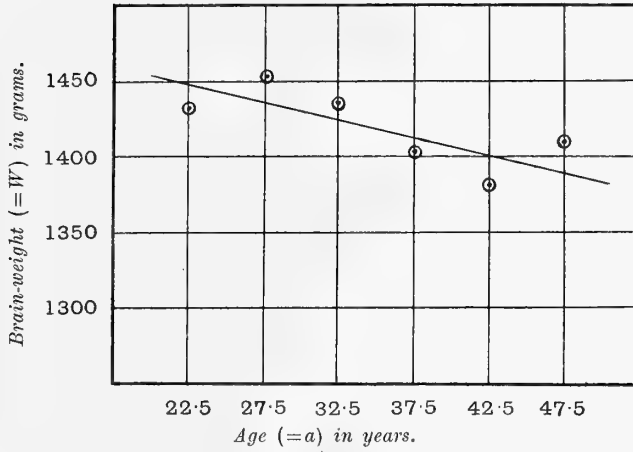
Swedish ♂ Young.

DIAGRAM VII. Probable Brain-weight with given Age.

Swedish ♀ Young.
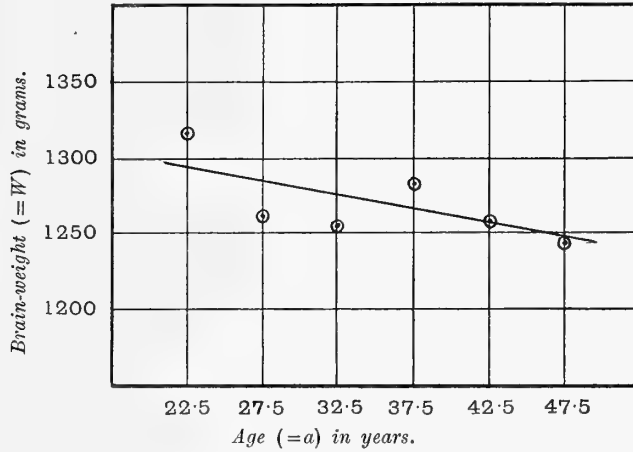
DIAGRAM VIII. Probable Brain-weight for given Age.
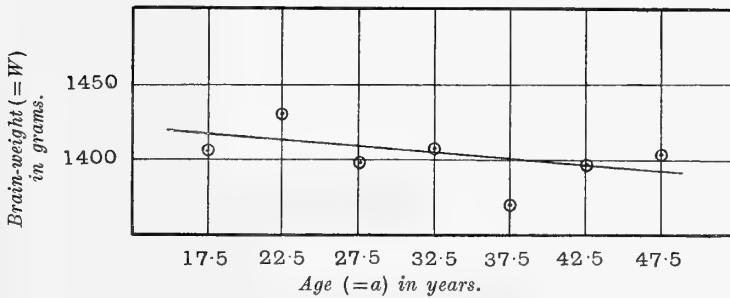
Hessian ♂ Young.

DIAGRAM IX. Probable Brain-weight for given Age.

Hessian ♀ Young.

Age (= a) in years.
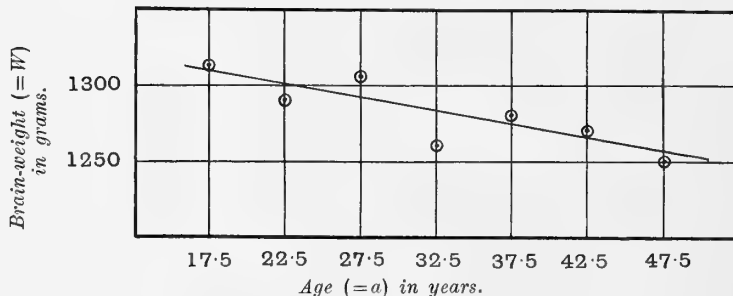
These diagrams show clearly, I think, that the regression of brain-weight on age in these cases is sensibly linear within the ages 15 to 80, so far as can be judged on the material available. The regression diagrams for the other two racial groups show the same relation. Obviously no simple curve will represent the systems of points shown in these diagrams II to IX better than the straight line does. So then, until we have very much larger samples of material to work with, we shall be justified in assuming for practical purposes linearity of regression between brain-weight and age, and in considering the deviations of the means of the arrays from the regression line to be due to errors incident to sampling in lots of less than 500 individuals.

The equations to the regression lines for these diagrams may now be given. For convenience in practical use I have put these equations in the form of "characteristic" equations. The significance of the letters used is as follows: W denotes the probable brain-weight in grams of an array of individuals of age type A or a. The ages included by A are 15 or 20 to 80; by a 15 or 20 to 50, or in other words the equations containing A are deduced from the "total" series and those containing a from the "young" series. Σ is the standard deviation of the array having the probable W as its mean.

(1)	Swedish	♂	$W = 1487 \cdot 783 - 1 \cdot 939\,A,$	$\Sigma = 102 \cdot 972$
(2)	,,	♂	$W = 1501 \cdot 411 - 2 \cdot 372\,a,$	$\Sigma = 107 \cdot 582$
(3)	,,	♀	$W = 1326 \cdot 475 - 1 \cdot 549\,A,$	$\Sigma = 97 \cdot 969$
(4)	,,	♀	$W = 1340 \cdot 438 - 1 \cdot 988\,a,$	$\Sigma = 104 \cdot 329$
(5)	Hessian	♂	$W = 1439 \cdot 734 - 1 \cdot 119\,A,$	$\Sigma = 111 \cdot 087$
(6)	,,	♂	$W = 1432 \cdot 497 - \cdot 847\,a,$	$\Sigma = 111 \cdot 014$
(7)	,,	♀	$W = 1361 \cdot 067 - 2 \cdot 259\,A,$	$\Sigma = 95 \cdot 512$
(8)	,,	♀	$W = 1339 \cdot 298 - 1 \cdot 735\,a,$	$\Sigma = 100 \cdot 445$

The foregoing constants bring out a number of interesting and important facts regarding the relation of brain-weight to age. In the first place it is clear that diminution in brain-weight with advancing years takes place very slowly. In 10 years the Swedish males lose 19·39 grs. and the Hessian males 11·19 grs.*

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* In order to economize space I have not tabulated the regression coefficients separately. It will be understood of course by anyone wishing to refer to or use these regression coefficients that the number prefixing a letter in the second member of any of the characteristic equations 1 to 36 is the regression coefficient (either gross or partial) of brain-weight on the character indicated by the letter.

These are comparatively speaking slight changes, amounting respectively to 1·3 °/₀ and ·8 °/₀ of the mean brain-weight for the whole adult period. In the same length of time (10 years) males lose an average of ·34″ in stature or approximately ·5 °/₀ of the mean stature*. The diminution in brain-weight seems thus to be relatively somewhat more rapid than the diminution in stature.

The fact that the regression is linear implies that this diminution of brain-weight with advancing age so far as can be judged from the material here discussed is sensibly equally distributed over the whole adult period. To be sure the slope of the lines is in all but one case (the Swedes) less when the individuals between 20 and 50 are treated separately, but the difference is a small one when the probable error is considered (cf. Table IX or XIV). Splitting the material up in this way of course greatly reduces the already statistically small series, so that the probable errors of the coefficients of correlation for the "young" series become large. It is significant, however, that with a single exception (the otherwise abnormal Bavarian female series) all the coefficients of correlation between brain-weight and age for the "young" series are negative. The evidence presented by each of the four series here discussed, which are representative and have been shown to be statistically trustworthy, and which if not as large as might be desired by the statistician are among the largest ever collected by the neurologist, is mutually accordant, and leads to but one conclusion: namely, that *after age 15—20 there is a steady though very gradual diminution in the weight of the brain with advancing age.* This conclusion I have emphasized because it is not in agreement with that reached by some eminent neurological authorities. I would further emphasize the fact that the conclusion is by no means final in the mind of the biometrician. It simply marks the point to which the material at present available leads. Very much larger series of brain-weighings are needed before the precise form of the regression can be finally determined. But the fact that four distinct series of fair length taken by different observers on different races lead to such closely accordant results as those shown in Table XIV may be taken as very strong presumptive evidence of the essential correctness of the conclusion reached.

It is noteworthy that there is no tendency towards more rapid diminution on brain-weight after age 50 to 60. The rate of diminution shows no such marked changes at this time as has been maintained to occur by many of the workers on the subject.

With the exception of the Swedish series the females in all cases show a higher correlation between brain-weight and age than the males. This difference seems in the case of the Hessians and Bavarians (total series) to be probably significant. According to Powys' (*loc. cit.*) figures the slope of the regression line between stature and age is slightly steeper for the females. It is difficult to assign any reason for this greater correlation between brain-weight and age in

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* Powys, A. O.: "Data for the Problem of Evolution in Man. Anthropometric Data from Australia," *Biometrika*, Vol. I. pp. 30—49.

the female. It has been shown above that there is in general no significant difference between the sexes in respect to variability in brain-weight, so that the usual formula of "low variation and high correlation" connoting stability of type does not hold in this case. It is possible that we have here an expression in a particular case of a greater general "evenness" in the females of the bodily changes accompanying increasing age, which in turn might be due to the generally more even environmental conditions to which women are subjected. It is noteworthy in this connection that the correlation in respect to duration of life is generally higher between pairs of female relatives than between pairs of male relatives*.

The linearity of the regression of brain-weight on age is of interest as possibly indicating a fundamental difference in the modes of action of the biological processes of growth on the one hand and senescence on the other. When growth in absolute magnitude of a character is plotted on a base line of age the result is usually a curved line (cf. for example the 12—25 portion of Powys' stature curve, *loc. cit.*), which implies, of course, that the amount of increment in the character in question is not the same for each unit of time. On the other hand in the case of decrease of brain-weight and stature with advancing age the decrement seems to be practically uniform for each unit of time. If these relations should prove to be generally true they would furnish a very interesting field for further study and analysis.

Turning now to the relation of brain-weight to stature, we have in Table XV the values of r, η and Σ_M, for all the brain-weight and stature correlations.

TABLE XV.

Linearity of Regression.   Brain-weight and Stature.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Swedish (Total) ...	·1830 ± ·0320	·3490 ± ·0388	·2439	·3847	·1612 σ	·1618 σ
„ (Young)...	·1796 ± ·0403	·3390 ± ·0530	·2837	·4738	·2196 σ	·3310 σ
Hessian (Total) ...	·1823 ± ·0299	·1828 ± ·0389	·2864	·3215	·2209 σ	·2645 σ
„ (Young)...	·1741 ± ·0383	·1809 ± ·0496	·2714	·3807	·2082 σ	·3350 σ
Bavarian (Total) ...	·1664 ± ·0343	·2236 ± ·0413	·2262	·3270	·1532 σ	·2386 σ
Bohemian(Young) ...	·2034 ± ·0397	·2168 ± ·0557	·2419	·3591	·1309 σ	·2863 σ

Again, the values of η and Σ_M are seen to diverge considerably from r and 0 respectively. It is clear that on the whole the regression of brain-weight on stature approaches less closely to linearity than does the regression of brain-weight on age. This is, I think, a somewhat remarkable result, and one not likely to have been foreseen.

---

* Beeton and Pearson : *Biometrika,* Vol. I. p. 60.

In order to determine whether the departure of the regression from strict linearity is due simply to errors of sampling or to a fundamentally different law of relation between the two variables it is again necessary to have recourse to regression diagrams. I have selected eight diagrams which are fairly representative.

DIAGRAM X. Probable Brain-weight for given Stature.

Swedish ♂ Total.

Stature (=S) *in centimetres.*
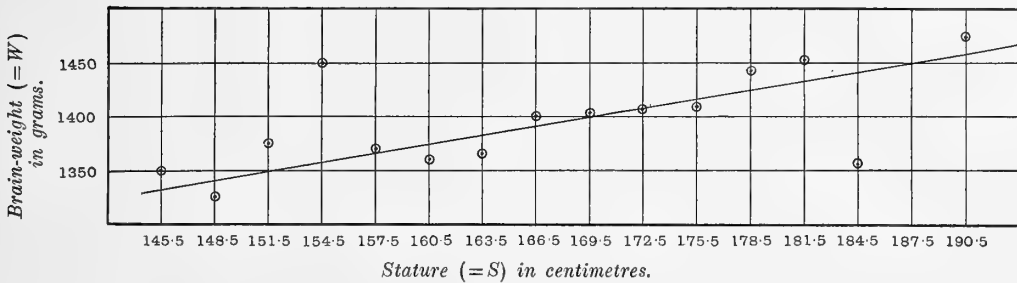
DIAGRAM XI. Probable Brain-weight for given Stature.

Swedish ♀ Total.

Stature (=S) *in centimetres.*
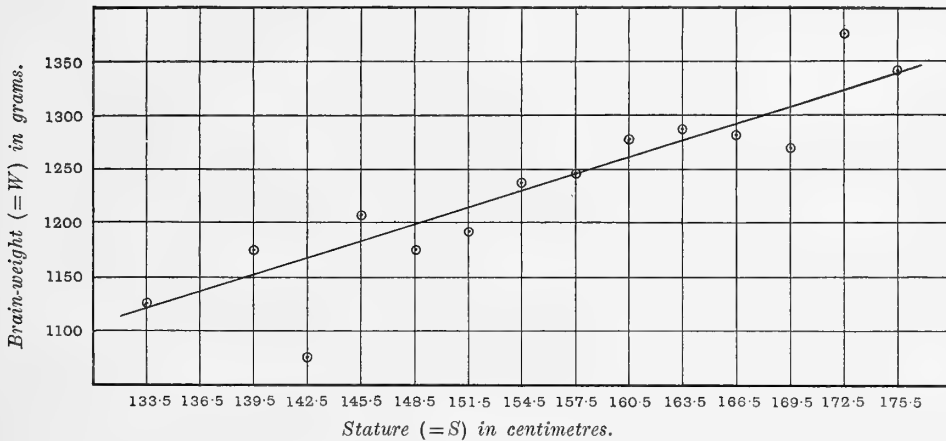
DIAGRAM XII. Probable Brain-weight for given Stature.

Swedish ♂ Young.

Stature (=s) *in centimetres.*

8—2
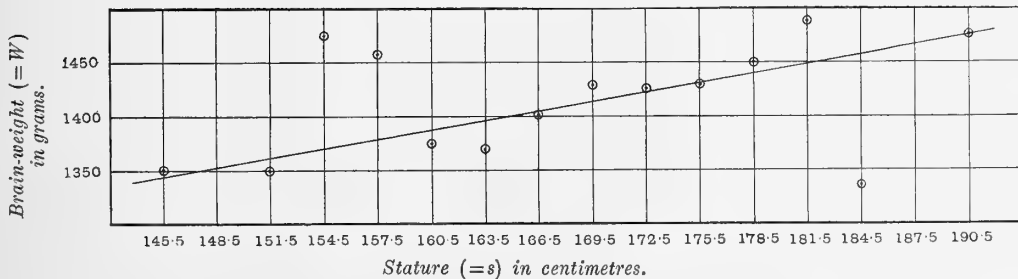
DIAGRAM XIII. Probable Brain-weight for given Stature.
Swedish ♀ Young.

Stature (=*s*) *in centimetres.*
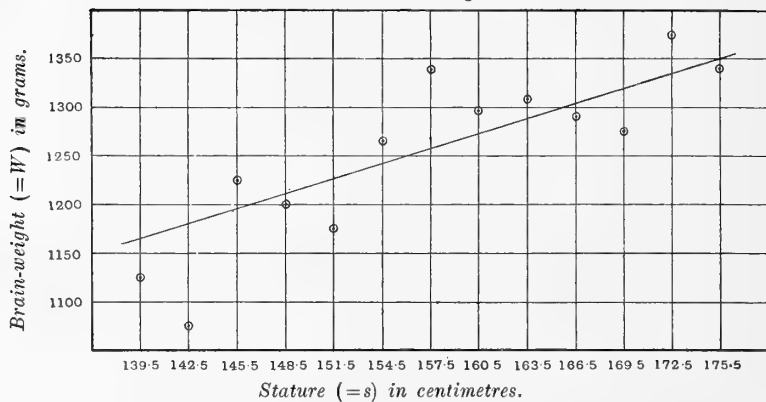
DIAGRAM XIV. Probable Brain-weight for given Stature.
Hessian ♂ Total.

Stature (=*S*) *in centimetres.*
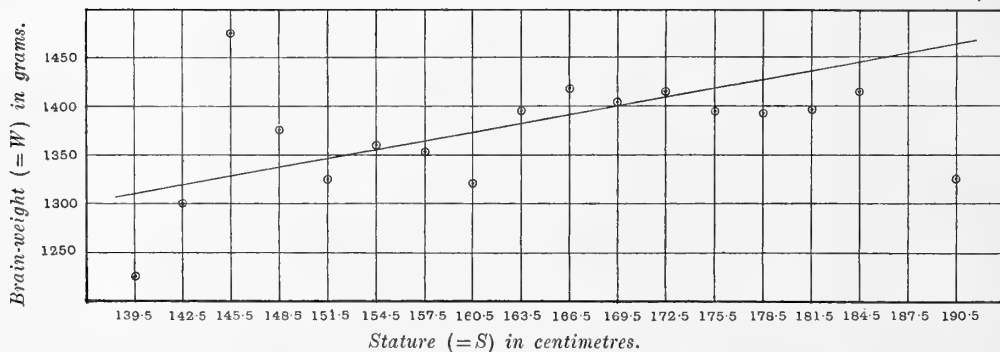
DIAGRAM XV. Probable Brain-weight for given Stature.
Hessian ♀ Total.

Stature (=*S*) *in centimetres.*
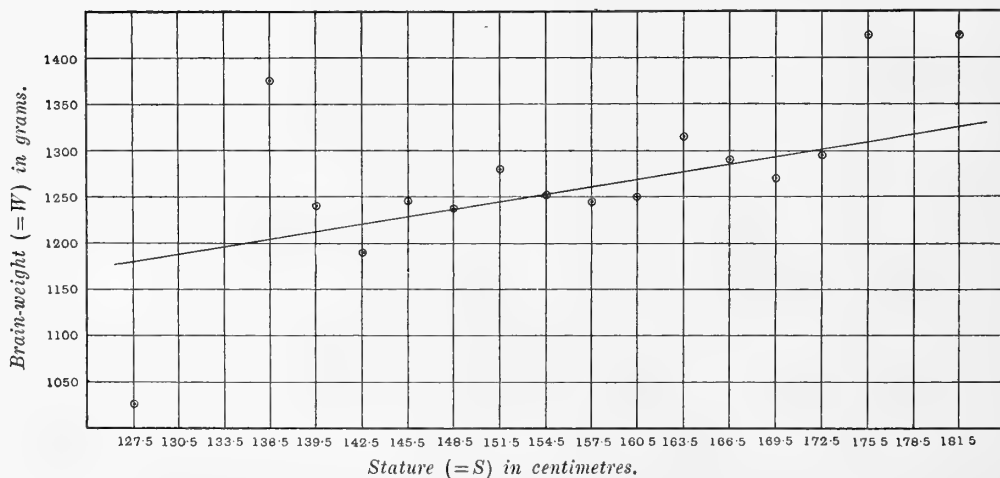
DIAGRAM XVI. Probable Brain-weight for given Stature.
Hessian ♂ Young.

Stature (=s) in centimetres.
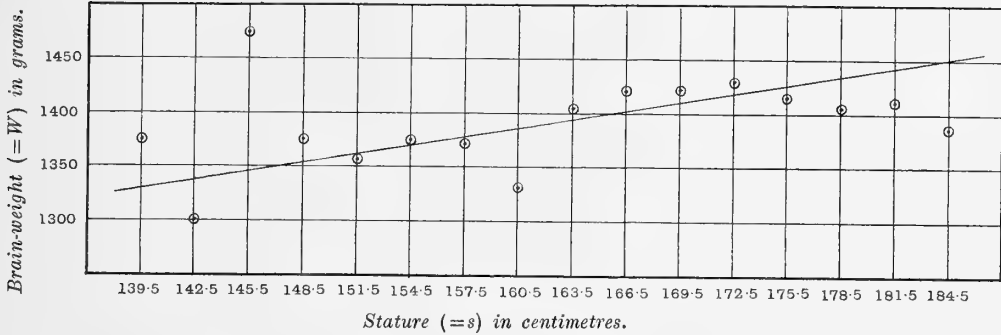
DIAGRAM XVII. Probable Brain-weight for given Stature.
Hessian ♀ Young.

Stature (=s) in centimetres.
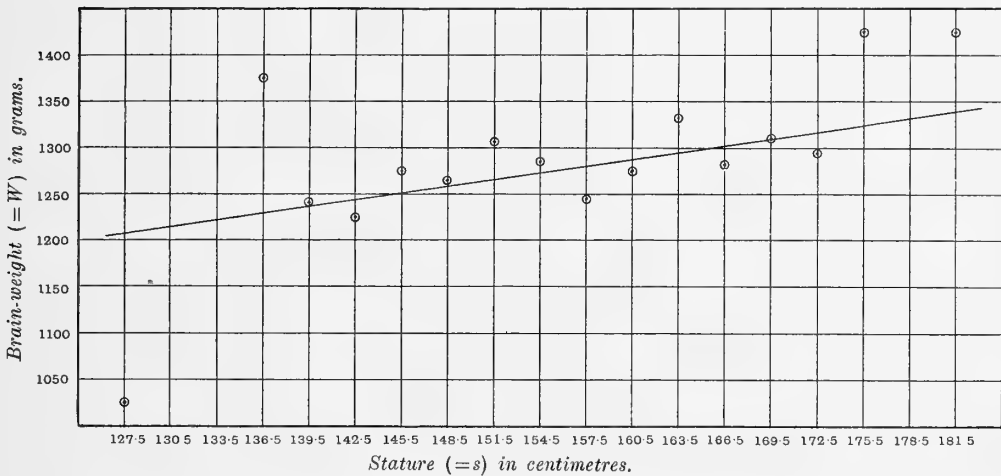
It is evident from these diagrams that a straight line represents the regression relation between brain-weight and stature better than any simple curve would. So that we are justified in concluding, as in the case of age, that until we have very much larger collections of data than are at present available we can do no better than proceed on the assumption that the regression of brain-weight on stature is linear. In fact, in these samples there is no evidence that there is any tendency towards anything but a linear relation between the two variables. The deviations of the means of the arrays from the regression line are only such as would reasonably be expected to arise when we deal with relatively small samples. In connection with all of the regression diagrams in this paper it should be kept in mind that the outlying points are based on single individuals, or at most only on comparatively few individuals. Hence deviations of these outlying points from the regression line have very little or no significance as regards the general trend of the results. In fact it would really be better if

the outlying points were left out entirely in the graphical representations. The non-biometrical reader should understand that each plotted point carries weight in proportion to the fraction which the number of individuals in the array on which it is based is of the total number of individuals. With this caution in mind the substantial linearity of the regression through the bulk of the observations, which alone are of importance, becomes evident.

The regression equations showing numerically the relation between brain-weight and stature, may now be given. In these equations W denotes the probable brain-weight of an array of individuals of mean stature S or s in centi-metres. As before, the equations involving S are based on the "total" series, and those involving s on the "young" series. Σ is the standard deviation of the array having the probable mean brain-weight W.

$$
\begin{array}{llll}
(9) & \text{Swedish} \ \male & W=915 \cdot 054+2 \cdot 859\,S, & \Sigma=104 \cdot 533 \\
(10) & \text{''} \ \male & W=919 \cdot 374+2 \cdot 914\,s, & \Sigma=107 \cdot 405 \\
(11) & \text{''} \ \female & W=421 \cdot 994+5 \cdot 234\,S, & \Sigma=\ 94 \cdot 421 \\
(12) & \text{''} \ \female & W=451 \cdot 643+5 \cdot 121\,s, & \Sigma=\ 99 \cdot 294 \\
(13) & \text{Hessian} \ \male & W=913 \cdot 592+2 \cdot 857\,S, & \Sigma=110 \cdot 787 \\
(14) & \text{''} \ \male & W=950 \cdot 214+2 \cdot 723\,s, & \Sigma=109 \cdot 628 \\
(15) & \text{''} \ \female & W=834 \cdot 624+2 \cdot 714\,S, & \Sigma=100 \cdot 643 \\
(16) & \text{''} \ \female & W=894 \cdot 031+2 \cdot 460\,s, & \Sigma=100 \cdot 161 \\
(17) & \text{Bavarian} \ \male & W=836 \cdot 667+3 \cdot 127\,S, & \Sigma=118 \cdot 452 \\
(18) & \text{''} \ \female & W=627 \cdot 157+3 \cdot 824\,s, & \Sigma=103 \cdot 562 \\
\end{array}
$$

We may now proceed to the general conclusions to be drawn from the data given above regarding the relation of brain-weight to stature. In the first place it is to be noted that the correlation coefficients between these two variables are for each series of the same general order of magnitude as those between brain-weight and age, though of course positive instead of negative, as they are in that case. Or it may be concluded that during adult life, at least, brain-weight is only a very little more closely related to general size of body, in so far as we may take stature as a measure of this, than to age. When we turn to the regression coefficients, however, it is seen that a unit of stature connotes a somewhat larger change of brain-weight than a unit of age. Thus, in the Swedish males, an increase of 10 cm. in stature connotes an increase in brain-weight of 28·59 gr., while an increase of 10 years in age connotes a decrease of 19·39 gr. in the brain-weight.

As in the correlation of brain-weight with age, the females have uniformly a higher degree of correlation between stature and brain-weight than the males. I am inclined to think that the same general sort of explanation may be given in this case as in that of the brain-weight and age correlations. The correlation between brain-weight and stature and the regression of brain-weight on stature are sensibly the same for the early adult period of life as for the total adult period. Without any question, I think, this correlation is to be regarded in its origin as a growth correlation, the relation between the two variables being

practically fixed at the end of the growth period. After that time the relation undergoes no marked change throughout the remainder of life, since both stature and brain-weight regress on age at not far from the same rate.

Finally, the combined effect of stature and age on the weight of the brain may be examined. This relationship can best be examined through the medium of multiple regression equations of the form:

$$x_1 = \beta_{12} x_2 + \beta_{13} x_3,$$

in which β_{12} and β_{13} are the partial regression coefficients of x_1 on its associated variables x_2 and x_3, it being understood that x_1, x_2 and x_3 here stand for deviations from the means of the respective variables. For our material these equations, when reduced to absolute magnitudes, take the following forms, the significance of the letters being the same as in the regression equations given before.

(19)	Swedish	♂	$W = 1091 \cdot 021 + 2 \cdot 288\,S - 1 \cdot 755\,A,$	$\Sigma = 101 \cdot 819$
(20)	,,	♂	$W = 1080 \cdot 715 + 2 \cdot 362\,s - 1 \cdot 856\,a,$	$\Sigma = 106 \cdot 477$
(21)	,,	♀	$W = 561 \cdot 433 + 4 \cdot 679\,S - 1 \cdot 078\,A,$	$\Sigma = 93 \cdot 066$
(22)	,,	♀	$W = 533 \cdot 407 + 4 \cdot 854\,s - 1 \cdot 092\,a,$	$\Sigma = 98 \cdot 923$
(23)	Hessian	♂	$W = 942 \cdot 154 + 2 \cdot 989\,S - 1 \cdot 181\,A,$	$\Sigma = 108 \cdot 992$
(24)	,,	♂	$W = 926 \cdot 586 + 3 \cdot 107\,s - 1 \cdot 318\,a,$	$\Sigma = 108 \cdot 889$
(25)	,,	♀	$W = 1005 \cdot 607 + 2 \cdot 244\,S - 2 \cdot 173\,A,$	$\Sigma = 94 \cdot 261$
(26)	,,	♀	$W = 974 \cdot 382 + 2 \cdot 345\,s - 1 \cdot 829\,a,$	$\Sigma = 98 \cdot 883$

These equations give the probable mean brain-weight of an array of adult individuals of either sex, having any given age (in years) and mean stature (in centimetres). They are, I believe, the first multiple regression equations relating brain-weight to stature, age and sex, to be published. They are of interest from several points of view. Examples of their practical use have been given earlier in the paper (pp. 25 and 48) in connection with the discussion of racial differences in mean brain-weight and later in connection with the sex differences. They afford a means whereby it is possible to make scientific comparisons of the mean brain-weight of different races, since by their use the probable brain-weight of a group of Swedes or Hessians having the same mean stature and age as the sample of the race with which comparison is to be made can be ascertained. We can in effect reduce both races to the same stature-age base and then examine the brain-weight differences. It would seem that these equations ought to prove very useful to anthropologists, who frequently wish to make comparisons of this kind. They are based on the two sets of brain-weighings which I believe to be on the whole the most reliable and trustworthy at present available. These equations also make it possible to determine the effect of indirect selection of stature and age on brain-weight.

As a practical example of this last use of these equations let us consider the following problem: By how much would it be necessary to modify the stature of a group of Swedish males, having a mean age of 45 years, in order that they might have the same mean brain-weight as a group of Bohemian males of the same mean

age ? The mean brain-weight of the Bohemian males with a mean age of 45·699 is 1454·839 gr. From equation (19) we have then

$$1454·839 = 1091·021 + 2·288 \, S - (1·755 \times 45·699)$$

in which S is the mean stature of the selected group of Swedes. Solving we find

$$S = 194·06,$$

or, in other words, our group of Swedes would have a *mean* height of 6 ft. 4·4 in. ! Now as a matter of fact the Swedish males with a mean age of 45·02 years have a mean stature of 169·79 cm. In order then to get from the adult male Swedish population a group having the same mean brain-weight as the adult male Bohemian population, by a selection of stature alone, it would be necessary to raise the mean stature of the Swedes 24·3 cm. or roughly $9\frac{1}{2}$ inches. Of course it is not for a moment to be supposed that evolution in brain-weight proceeds by the indirect selection of stature alone. Indeed when it is remembered that the mean stature of Bohemian males is only 169·4 cm. the example just cited shows how little the indirect selection of stature has to do with the matter. In the discussion of evolutionary problems, however, it is important to know just how much or how little effect each factor involved is capable of producing. It is in this direction of measuring the relative effect of different combinations of factors that regression equations such as those given above have great value.

It is apparent from equations (19) to (26) that stature and age in a measure compensate one another so far as their effect on brain-weight is concerned. The influence of stature is however in all cases greater. It is of some interest to know in the particular cases how great an increase in stature compensates a given increase in age and *vice versâ*. In order to show this I have prepared the following table.

An increase of 10 cm. in stature is compensated in its effect on brain-weight by an increase in age in the

Swedish males of 13·0 years ⎫
Swedish females of 43·4 „ ⎬ Total series only.
Hessian males of 25·3 „ ⎪
Hessian females of 10·3 „ ⎭

10. *Brain-weight and Body-weight.*

In only one of the four series of brain-weighings studied (the Bavarian) was body-weight tabulated in the returns. Recent workers on the subject have left this character out of their data on the general ground that it is of very doubtful significance or reliability in post-mortem returns from a general hospital population. Certainly it is true that it is influenced to a very marked degree by the

conditions preceding death. Yet as Donaldson* has pointed out if we could get this character free of disturbing ante-mortem effects it would furnish the best measure of the general "size" of the body. Since "size" of the body is one of the things to which brain-weight should first of all be related I decided to include in this study a consideration of the correlation of brain-weight with body-weight. I was fully aware of the weakness of the material and expected much worse results than those actually obtained. A glance at Table I will recall the facts as to the distribution of the frequency of body-weight. The means are of course lower than they would be for the living population of the same race and age. This is to be expected. The variation is large but there is very good accord between males and females in this respect. Turning now to the correlations I have arranged in Table XVI the values of r, η and Σ_M for the correlation between brain-weight and body-weight.

TABLE XVI.

Linearity of Regression. Brain-weight on Body-weight.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Bavarian (Total)	·1671 ± ·0343	·2260 ± ·0412	·1845	·3189	·0782 σ	·2250 σ

The result for the males is very satisfactory, indicating a quite reasonable approach to linearity of regression. The female series does not give so good a result, but it will be recalled that throughout the work this Bavarian female series has been found to be somewhat abnormal. The regression lines are shown in Diagrams XVIII and XIX.

DIAGRAM XVIII. Probable Brain-weight for given Body-weight.

Bavarian ♂ Total.

Body-weight $(=W_B)$ *in kilograms.*
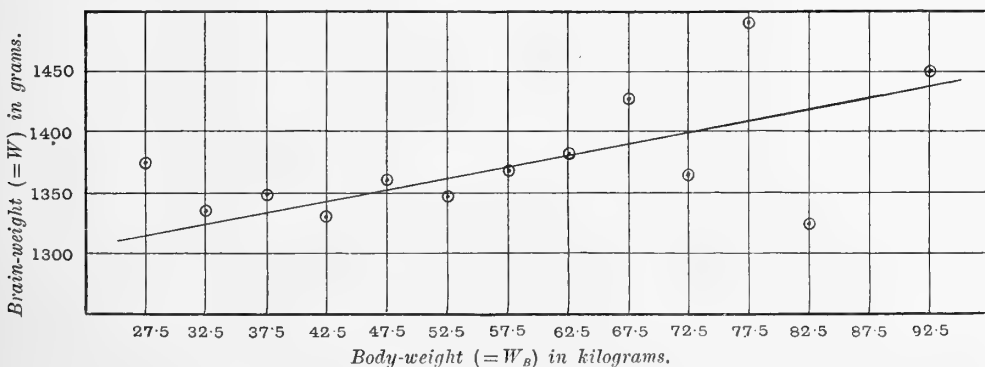
* H. H. Donaldson: *The Growth of the Brain.*

DIAGRAM XIX.   Probable Brain-weight for given Body-weight.
Bavarian ♀ Total.

Body-weight $(=W_B)$ *in kilograms.*
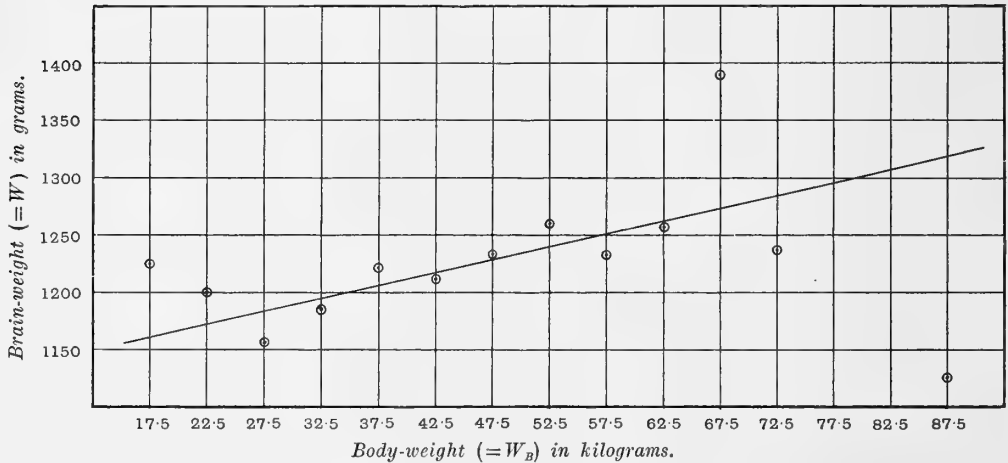
Through the great bulk of the observations the regression is evidently linear*. The male series comes very close to linearity.

The equations to the regression lines may now be given. In addition to the equation showing the regression of brain-weight on body-weight I have included multiple regression equations giving the probable brain-weight for given types of body-weight and stature together. The significance of the letters is as follows: W denotes the probable brain-weight of an array having a mean body-weight in kilograms W_B, or body-weight W_B and stature S. Σ is the standard deviation of the array.

(27)	Bavarian	♂	$W=1263\cdot308+1\cdot886\,W_B,$	$\Sigma=118\cdot439$
(28)	,,	♀	$W=1121\cdot621+2\cdot265\,W_B,$	$\Sigma=103\cdot495$
(29)	,,	♂	$W=\ 917\cdot748+2\cdot234\,S+1\cdot355\,W_B,$	$\Sigma=117\cdot711$
(30)	,,	♀	$W=\ 741\cdot078+2\cdot646\,S+1\cdot593\,W_B,$	$\Sigma=102\cdot432$

The correlation of brain-weight with body-weight is of about the same degree as the correlation of the former with the stature, but the regression approaches more closely to linearity in the case of body-weight. As in the other cases the female shows the higher correlation.

The absolute increase of brain-weight with every increase of 10 kilos. in body-weight is for the males 18·86 gr. and for the females 22·65 gr.   Equations (29) and (30) show that in each case stature contributes absolutely more to an increase in brain-weight than does body-weight. The exact proportions are as follows: in order to bring about a change in brain-weight equal to that produced by an increase of stature of 10 cm. the body-weight would have to be increased 16·5 kilos. in the males, and 16·6 kilos. in the females.   The greater dependence of the brain-weight on bodily characters in the female as compared with the male is very well shown in equations (29) and (30).

* Cf. pp. 61, 62.

11. *Brain-weight and Skull Characters.*

As was to be expected the correlation of brain-weight with the skull length and breadth gives the highest values for the coefficients of any of the correlations dealt with. In fact one would predict on general grounds, I think, a somewhat closer degree of correlation between these characters than is actually found in the Bohemian statistics. It is likely, however, that the values here obtained (Table XVII) are lower than they should be on account of the methods used in measuring skull length and breadth. As has been fully described above (p. 18) these measurements were taken on the horizontal cut surfaces after the dome of the skull had been removed by sawing around it. This is clearly not a very precise way in which to get skull measurements. In order to show so far as may be what has been the effect of this procedure I have prepared the following tables comparing the various constants of the Bohemian skull length and breadth series with other well-known cranial series. Taking first the means we have:

Race	Skull Length		Skull Breadth	
	♂	♀	♂	♀
Bohemians	176·547	170·142	149·841	144·953
Whitechapel English*...	189·06	180·36	140·67	134·68
Altbayerisch†	180·58	173·45	150·47	143·98
Würtemburger† ...	179·48	172·74	147·88	142·90

The means give reasonably accordant values considering that the Bohemians are a markedly brachycephalic race. Turning next to the coefficients of variation we have:

Race	Skull Length		Skull Breadth	
	♂	♀	♂	♀
Bohemians	4·173	3·233	4·441	3·815
Whitechapel English‡...	3·31	3·45	3·75	3·54
Bavarian §	3·37	3·57	3·89	3·39
French‖	3·97	3·65	4·21	3·67
Naqada¶...	3·17	3·14	3·29	3·45
Aino §	3·20	3·08	2·76	2·68

* Macdonell, *Biometrika*, Vol. III. pp. 208, 209.
† Alice Lee, *Phil. Trans.* Vol. 196, A, pp. 225—264.
‡ Macdonell, *loc. cit.* p. 220.
§ Lee, *loc. cit.*
‖ Unpublished reduction of Measurements in Broca's MS. by C. D. Fawcett.
¶ Miss Fawcett, *Biometrika*, Vol. I. pp. 438 and 456.

The Bohemian females are fairly accordant in variability with the other series in both length and breadth, but the males are apparently somewhat more variable. This greater variability in the Bohemian male series is due I believe not to less homogeneous material but to the manner in which the measurements are made. The coefficients of correlation between length and breadth may next be considered.

Race	♂	♀
Bohemians	·4006	·1954
Whitechapel English* ...	·240	·350
French†	·089	− ·042
German‡	·286	·488
English Criminals § ...	·402	—
English Middle Classes §...	·345	—
Aino‡	·432	·376
Naqada†	·344	·143

The results here are somewhat chaotic, but clearly the difference in degree of correlation between Bohemian males and females is too great to be entirely normal. It is probable that the male coefficient is somewhat too high and the female too low. The agreement between the Bohemian male coefficient and Macdonell's value for measurements on the heads of living English criminals is striking. We should expect measurements made as the Bohemian were, to agree better with measurements on the living head than with those from the skull.

The practical result to which we come is that conclusions from the Bohemian data as to the correlation between brain-weight and skull length and breadth must be only tentative until they can be tested by other less objectionable material.

We may now examine in more detail the results as to the correlation of brain-weight with these skull characters. In Table XVII are given the values of r, η, and Σ_M.

TABLE XVII.

Linearity of Regression. Brain-weight and Skull Characters.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Bohemians, Skull Length	·5482 ± ·0273	·3604 ± ·0465	·5660	·4032	·1408 σ	·1808 σ
,, ,, Breadth	·4655 ± ·0306	·5041 ± ·0399	·5498	·5368	·2926 σ	·1845 σ

The four regression diagrams concerned are Diagrams XX to XXIII.

* Macdonell, *loc. cit.* p. 233.   † C. D. Fawcett, *loc. cit.*
‡ Lee, *loc. cit.*
§ Macdonell, *Biometrika*, Vol. I. pp. 181 and 188, "Measurements on living head and not skull."

Diagram XX. Probable Brain-weight for given Skull Length.

Bohemian ♂ Young. 20—59 years.

Skull Length (=L) in millimetres.
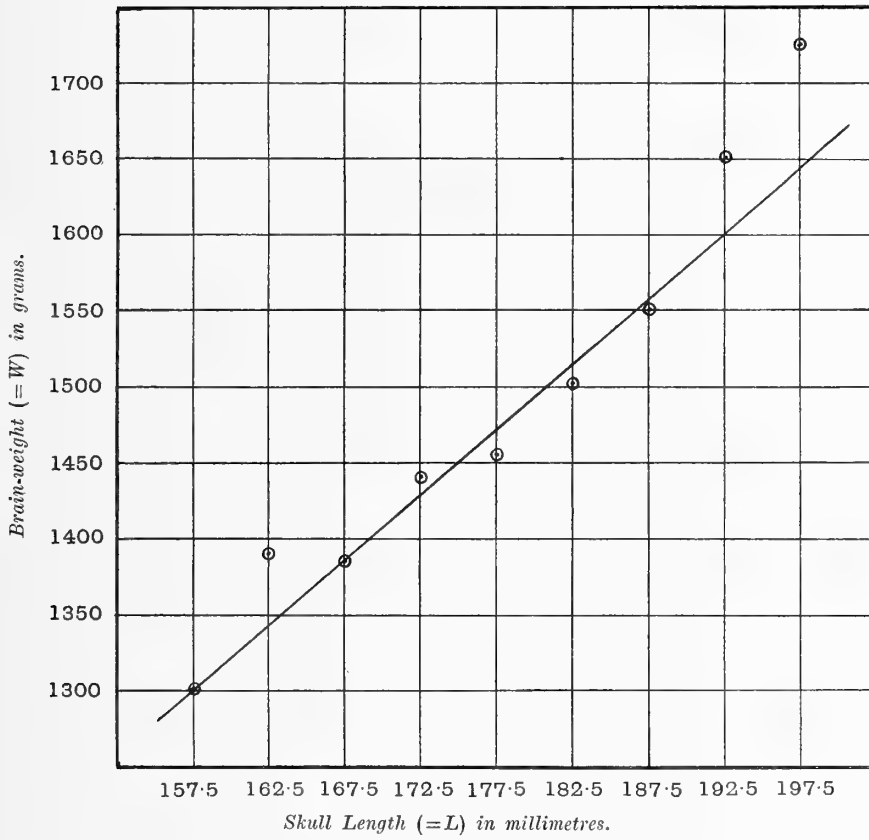
DIAGRAM XXI.   Probable Brain-weight for given Skull Breadth.
Bohemian ♂ Young.   20—59 years.

Skull Breadth (= B) in millimetres.
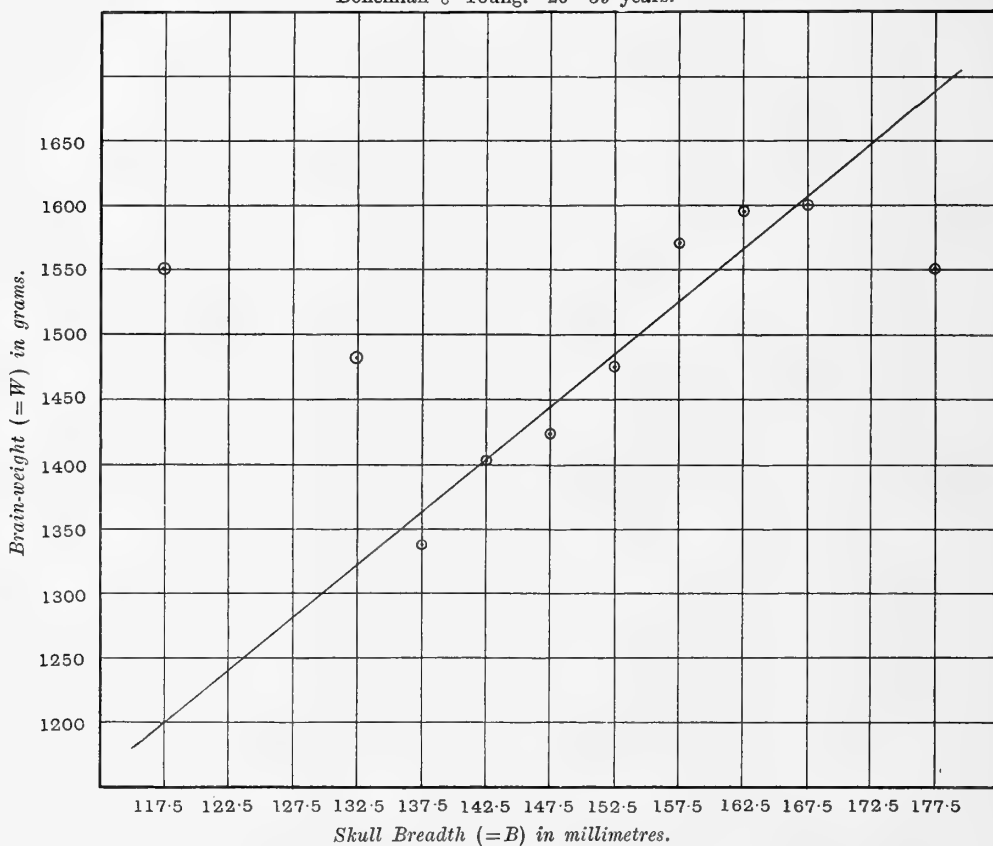
DIAGRAM XXII.   Probable Brain-weight for given Skull Length.
Bohemian ♀ Young.   20—59 years.

Skull Length (= L) in millimetres.
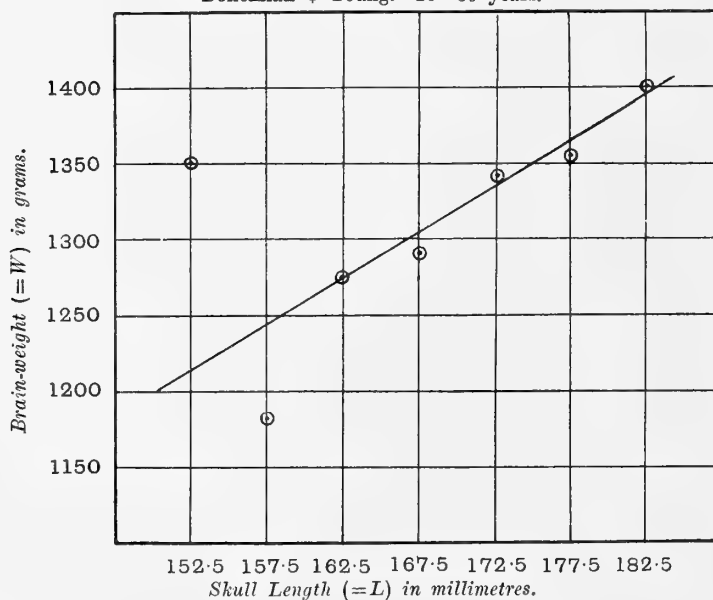
DIAGRAM XXIII. Probable Brain-weight for given Skull Breadth.
Bohemian ♀ Young. 20—59 years.
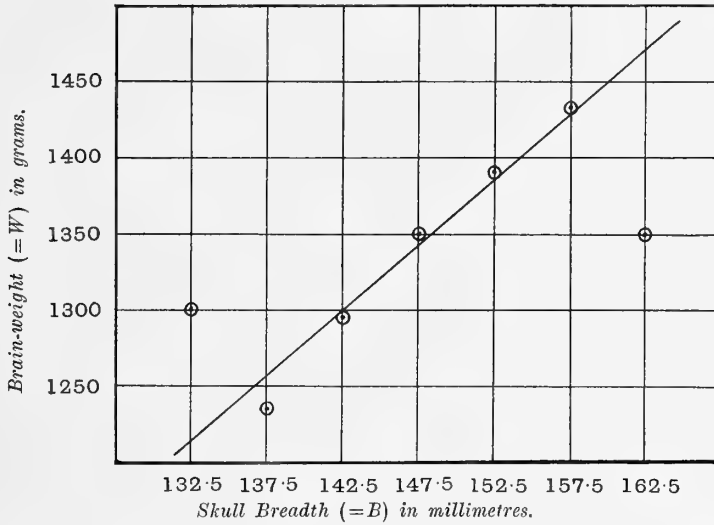
Through the bulk of the observations the regression is very evidently linear. Diagram XXI shows that the considerable deviations of η from r and Σ_M from zero in the case of the male brain-weight and skull breadth regression are really due to the inclusion in the material of five individuals which are to be regarded as either erroneously measured or recorded, or else abnormal or extremely rare normal cases (cf. Table 31, Appendix).

The regression equations follow. In these equations W denotes the probable brain-weight in grams of an array of individuals having a mean skull length L and skull breadth B in millimetres.

(31)	Bohemian	♂	$W=8{\cdot}589\,L- 52{\cdot}650,$	$\Sigma=\ 96{\cdot}548$
(32)	„	♂	$W=8{\cdot}076\,B+253{\cdot}596,$	$\Sigma=102{\cdot}170$
(33)	„	♀	$W=6{\cdot}215\,L+264{\cdot}265,$	$\Sigma=\ 88{\cdot}466$
(34)	„	♀	$W=8{\cdot}646\,B+\ 68{\cdot}434,$	$\Sigma=\ 81{\cdot}908$
(35)	„	♂	$W=6{\cdot}751\,L+\ 5{\cdot}082\,B-489{\cdot}649,$	$\Sigma=\ 99{\cdot}745$
(36)	„	♀	$W=4{\cdot}696\,L+\ 7{\cdot}766\,B-602{\cdot}994,$	$\Sigma=\ 77{\cdot}893$

One rather curious point may be noted here in passing. As the figures stand a smaller probable error will be made in estimating the mean brain-weight of males from a knowledge of skull length alone (equation 31) than if both length and breadth of skull are used. This apparent paradox arises for the following reason[*]. The standard deviation of the array in the regression of x_1 on its associated variables x_2 and x_3 is given by

$$\Sigma = \sigma_1 \sqrt{1 - R_1^2},$$

where

$$R_1^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}.$$

When the regression is of x_1 on x_2 alone the standard deviation of the array is

$$\Sigma = \sigma_1 \sqrt{1 - r_{12}^2}.$$

[*] Cf. Yule, *loc. cit.*

Now the condition under which we shall make a smaller standard error in estimating x_1 from the two variables x_2 and x_3 than in estimating it from x_2 only, is evidently that

$$(1 - R_1{}^2) < (1 - r_{12}{}^2)$$

or that

$$R_1{}^2 > r_{12}{}^2.$$

Now in the case noted above $R_1{}^2 < r_{12}{}^2$, the figures being $\cdot 2534 < \cdot 3005$.

Returning to the main line of the discussion, the results as to the correlation of brain-weight with skull characters are not altogether satisfactory, as they are apparently not entirely in accord with themselves. One of the points here on which information is most desired is whether brain-weight is more closely correlated with length or breadth of skull. According to our results brain-weight is more closely correlated with length of skull in the male and breadth of skull in the female. This seems a somewhat anomalous result as there is no apparent reason why one should not expect the sexes to show the same relation in the matter. However, Macdonell's (*loc. cit.*) figures for the correlation of skull capacity with length and breadth show a similar relation but in the opposite direction. He finds for the Whitechapel English skulls that the correlation between capacity and length is lower than the correlation between capacity and breadth in the males, and higher in the females. Other skull series do not, however, show this reversed relationship between the sexes. With the data at present available one cannot reach any general conclusion as to the relative closeness of correlation between brain-weight and skull length and breadth. It is very much to be hoped that future collectors of brain-weight statistics will include in their returns such head measurements as length, breadth, height and circumference taken before the cranial cavity is opened, and by standard anthropometrical methods.

A comparison of the values of the coefficients of correlation between brain-weight and skull length and breadth and between skull capacity and these characters is of interest. I have adapted the following table from Macdonell's[*] Table XIII.

Race	Brain-weight and Skull Length		Brain-weight and Skull Breadth	
	♂	♀	♂	♀
Bohemians ...	$\cdot 5482 \pm \cdot 0273$	$\cdot 3604 \pm \cdot 0465$	$\cdot 4655 \pm \cdot 0306$	$\cdot 5041 \pm \cdot 0399$
	Skull Capacity and Length		Skull Capacity and Breadth	
English[*] ...	$\cdot 597 \pm \cdot 051$	$\cdot 691 \pm \cdot 040$	$\cdot 631 \pm \cdot 048$	$\cdot 646 \pm \cdot 044$
Naqada [†] ...	$\cdot 501 \pm \cdot 054$	$\cdot 599 \pm \cdot 039$	$\cdot 434 \pm \cdot 058$	$\cdot 532 \pm \cdot 044$
German [‡] ...	$\cdot 515 \pm \cdot 050$	$\cdot 687 \pm \cdot 037$	$\cdot 672 \pm \cdot 037$	$\cdot 706 \pm \cdot 034$
Aino [‡] ...	$\cdot 893 \pm \cdot 016$	$\cdot 663 \pm \cdot 053$	$\cdot 561 \pm \cdot 053$	$\cdot 502 \pm \cdot 070$

[*] Macdonell, *loc. cit.* p. 235.   [†] Fawcett, *loc. cit.*   [‡] Lee, *loc. cit.*

The brain-weight correlations with two exceptions are smaller than the corresponding skull capacity correlations. A portion at least of this difference is probably to be accounted for on the ground that brain-weight fluctuates as a result of causes which do not at all affect skull capacity. Possibly the whole of the difference is to be accounted for in this way, but until there is available a more reliable collection of material on the brain-weight side the question cannot be definitely settled. Until such material is available we shall be justified, I think, in assuming that the values of the coefficients for the correlation between brain-weight and skull characters given by the male series probably represent more nearly the normal relation than do those of the female series.

Making all due allowance for the shortcomings of the material the following points are clearly brought out: (*a*) Brain-weight is positively correlated with both skull length and skull breadth. (*b*) The correlations here are higher than any of the other correlations of brain-weight with bodily characters. (*c*) The regression of brain-weight on these skull characters is approximately linear. (*d*) Indirect selection of skull characters is capable of modifying brain-weight to a considerable extent.

The use of equations (31) to (36) in selection problems has been illustrated above (p. 49). One other example may be given because it bears very directly on a question of perennial interest in certain quarters. The problem to which I have reference may be stated for our present purpose in this form: Does dolichocephaly imply a greater brain-weight than brachycephaly or *vice versâ**? Mere inspection of equations (35) and (36) shows that neither dolichocephaly nor brachycephaly will have a great advantage over the other in respect to brain-weight, but the matter can be made somewhat more *anschaulich* by taking a particular example. Consequently I have chosen the following figures. In Macdonell's Tables I and II (*loc. cit.*) are given the values for cephalic index, skull-length and skull-breadth for a dolichocephalic race (the Whitechapel English series) on the one hand and a brachycephalic race (the Altbayerisch) on the other. Those values are as follows:

	♂		♀
English ...	Cephalic index = 74·34	English ...	Cephalic index = 74·73
Altbayerisch ...	„ „ = 83·20	Altbayerisch ...	„ „ = 83·10
English ...	Length = 189·06	English ...	Length = 180·36
„ ...	Breadth = 140·67	„ ...	Breadth = 134·68
Altbayerisch ...	Length = 180·58	Altbayerisch ...	Length = 173·45
„ ...	Breadth = 150·47	„ ...	Breadth = 143·98

* Evidently the real essence of the question, so far as it has what might be called "humanistic" interest, rests on the assumption of a reasonably high positive correlation between brain-weight and intellectual ability. "Brains" not brain-weights are the things! Up to this point I have carefully avoided any discussion of the validity of such an assumption as that mentioned, deeming it entirely irrelevant to the main purpose of the paper. The point is discussed in the concluding section.

Now using equations (35) and (36) for males and females respectively we have for the probable mean brain-weights of arrays having the types of length and breadth of the English and Altbayerisch series respectively, the following values:

♂	♀
English B.W.　　＝1501·580 Altbayerisch B.W.＝1494·135	English B.W.　　＝1289·901 Altbayerisch B.W.＝1329·676

In the males the dolichocephalic and brachycephalic arrays have practically equal brain-weights, while in the females the brachycephalic array gives a brain-weight about 40 grs. higher than the dolichocephalic. But, as has been pointed out above, the male equations probably come much nearer representing the normal relations than do the female. On the whole, then, there seems to be little or no evidence that either dolichocephaly or brachycephaly is associated with large brain-weight.

It should perhaps be noted that no significance is to be attached to the absolute values of the brain-weights calculated from the English and Altbayerisch skull types, as indicating the probable brain-weight of those races. The differences only are of significance. The measurements of these skulls were taken only for the sake of convenience; hypothetical dolichocephalic and brachycephalic skull means would have served the purpose as well. The brain-weights obtained are of course the probable mean brain-weight of arrays of *Bohemians* having the specified skull characters. Intra-racial regression equations can of course only be used for inter-racial prediction when the regression of all characters on one another is the same in both races, which is a condition never likely to be exactly fulfilled in practice. Of the races and characters discussed in this paper the Hessians and Swedes in brain-weight, stature, and age come nearest to fulfilling this condition.

12. *The Weight of the Cerebrum.*

Up to this point the discussion has had to do entirely with the variation and correlation of the whole brain or encephalon. In the manuscript material placed at my disposal by Prof. Donaldson (*vide supra*, p. 15) data were given from which it was possible to determine the variation in the weight of the cerebral hemispheres, and the correlation of this character with stature and age for English males. The results obtained from this material may now be examined. It should be stated that the records of weight were in ounces, and of stature in inches in the original material. The units of grouping and the distribution of the frequencies may be seen in Tables 45 and 46 of the Appendix. After the constants had been calculated in ounces and inches they were transferred to the metric system in order to facilitate comparison with the other parts of the paper. The values of the constants are exhibited in Table XVIII.

TABLE XVIII.

Variation in the Weight of the Cerebrum. English Males.

Boyd-Marshall Data.

Character	Mean			S. D.	C. of V.
	No.		Table		
Weight of cerebrum	308	1184·940 ± 4·859 gr.	45	126·438 ± 3·436 gr.	10·670 ± ·293
Stature	308	171·602 ± ·268 cm.	45	6·976 ± ·190 cm.	4·065 ± ·111
Age	308	49·481 ± ·645 yr.	46	16·770 ± ·456 yr.	—

For the correlation of weight of cerebrum with age I find from Table 46

$$r = -\,·1412 \pm ·0377,$$

and for the correlation of the weight of cerebrum with stature from Table 45

$$r = \quad ·1202 \pm ·0379.$$

These values bring out the following points. The cerebral hemispheres are markedly more variable in weight as judged by the coefficient of variation than is the entire encephalon. This greater variation may denote a really greater variability of this part of the brain, or it may be due to the variable element which enters as a result of the separating of these organs from the rest of the brain. It seems to me likely that it is in part at least real, for the following reasons: first, the cerebral hemispheres have attained relatively enormous development late in their phylogenetic history; and again from the functional standpoint the cerebrum is the most variable part of the brain.

The correlations of the weight of the cerebrum with age and stature are both somewhat lower than the corresponding values for the other series of males falling within the ages 20 to 80. The differences can hardly be considered as significant however in view of the probable errors.

For the coefficient of regression of brain-weight on age I find $b_1 = -\,1·065$, and on stature $b_1 = 2·178$. By comparing these with the regression coefficients for the entire encephalon on the same characters in the series previously discussed it becomes evident at once to what a great extent changes in the total brain-weight are due to changes in the weight of the cerebrum. Unfortunately it is not possible to get relative figures here on account of lack of knowledge of the regression of the total brain-weight on stature and age for the English series. Of course the weight of the cerebrum forms a large part (ca. 87 per cent. for the English series) of the total brain-weight. The problem here is to determine whether the changes in the weight of the cerebrum cause less or more than their proportional part (say 87 per cent.) of the changes in total brain-weight due to stature and age changes. I have tried a number of approximate methods

of getting at this question with the data available, but none of them led to any result sufficiently well grounded to warrant taking space for its publication. Putting all the results so obtained together they seemed to indicate the following conclusion, which is of course subject to modification when further material is available: viz., that changes in stature affect all parts of the brain equally (i.e., in the proportion of their absolute masses) while changes in age have a relatively greater effect on the cerebral hemispheres than on the remainder of the brain (cerebellum, pons, and medulla).

The final conclusions regarding variation and correlation in the weight of the cerebrum may be summed up as follows: the cerebrum is somewhat more variable in weight than is the entire encephalon, and this character is slightly less closely correlated with age and stature. The values of the variation and correlation constants are, however, as is to be expected, of the same general order of magnitude as those for the total brain-weight.

13. *Concluding Remarks.*

In concluding this paper I wish to call attention to what seem to me to be some of the broader aspects of the work. There are at least two general results of the work which alone justify its being done, I think. The first and most important is that the paper contributes reduced material to the existing collection of biometric data on man. In the nature of the case such a collection grows slowly, but every increase in it means a definite, although it may be small, step in advance in our knowledge regarding the fundamental problems of anthropology. I regard as the second most important result the fact that it may now fairly be said that the essential trustworthiness of the most important of the existing collections of brain-weight statistics has been demonstrated, and that consequently reasoning by statistical methods on the problems involved is not of necessity lacking in validity. The agreement in the statistical constants from four series of data so divergent in their origin as those treated in this paper cannot reasonably be held to be fortuitous. It can mean, I think, but one thing, namely, that the same kind of general lawfulness underlies the variation and correlation of brain-weight and the variation and correlation in other characters of the organism With this conclusion presumably no student of brain-weight would disagree, but some of the most eminent students have disagreed with the converse proposition that it is possible to gain a knowledge of the nature of this lawfulness by statistical methods applied to large masses of material when brain-weight is the thing concerned. This sceptical attitude owes its origin, I think, to the nature of the statistical methods which have hitherto been applied to the problem. Brain-weight statisticians have erred in two directions in the handling of their material. On the one hand, following the much-to-be-condemned practice in other fields of anthropology, entirely heterogeneous material has been grouped together to attain large total numbers. The culminating example here is, perhaps, Topinard's series of the brain-weights of 13,000 Europeans. Secondly, certain brain-weight workers

have attempted to apply to the individual results gained by somewhat doubtful statistical analysis of mass data, and *vice versâ*, what is much worse, have argued that the relations found to hold in a single individual might be considered to hold for the mass. As an example here might be cited Marshall's* attempt to correct the brain-weights of De Morgan, Thackeray, and others for stature and age. The results detailed in this paper will indicate to the biometrician at least how dangerous both of these methods of procedure are with such material as brain-weight statistics. Of course both disregard fundamental canons of general statistical theory, but the chance of grave error becomes very much greater when the character under consideration shows on the one hand definite and clearly marked types in different races, and on the other hand low correlations†. With the application of adequate statistical methods it is possible to reach definite and significant results regarding the weight of the brain. When we have biometric constants for a considerable number of long series of brain-weight statistics from different races we shall be able to advance our knowledge greatly with reference to the general problems of the evolution of man's brain.

While the discussion of such problems can at the present time admittedly lead to only tentative results, still I think it will be worth while to examine for a moment the bearing of our results on a single evolution problem. Progressive evolution through natural selection may be brought about in any character either by the direct selection of that particular character or by the indirect selection of other characters correlated with it. As we proceed in the analysis of the problem the proportional effect of each of these two sets of factors must be quantitatively determined. A first approximation to such a determination has been here made

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* *Loc. cit., supra*, p. 18.

† In this connection Weigner's recent paper "Ein Beitrag zur Bedeutung des Gehirngewichts beim Menschen," *Anatomische Hefte*, Bd. 23, pp. 69—109 is decidedly open to criticism. But when one, after very briefly analyzing the weighings of 69 male and 66 female brains (*sic*) from individuals ranging in age from 1½ to 82 years, by the most superficial of statistical methods, feels justified in drawing the following sweeping conclusions *inter alia*, what can biometry offer that will be in any way effective? Weigner's three principal conclusions as to fact are (*loc. cit.* p. 108):

1. "Das Gehirngewicht steht in keinem direkten Zusammenhang mit dem Alter, und wenn auch die angegebenen Zahlen sich zu Gunsten dieses Zusammenhanges verwerten einen, verlieren dieselben an der Bedeutung, da durch dieselben nicht der Beweis geführt werden kann, wie sich Gehirngewicht bei einem und demselben Individuum in verschiedenen Altersperioden verhält;

2. zwischen dem Gehirngewichte und der Körperlänge lässt sich keine bestimmte Proportion feststellen;

3. die Durchschnittszahl hat bloss einen relativen Wert, und zwar deshalb, weil die das Gehirngewicht repräsentierenden Zahlen durch die Wägung eines zufällig uns gerade zur Verfügung stehenden Materials gewonnen sind und das aus demselben bestimmte arithmetische Mittel keine konstante Grösse, sondern eine frei bewegliche Reihe von Zahlen uns darstellt."

It should be said that the material on which these conclusions are based is apparently within the limits of its probable errors entirely normal, and shows an approximately linear regression of brain-weight on age during adult life. This being the case the very positive conclusions seem to take on the character of extreme dogmatism. If 1. were true, by the same token no life insurance company could fix upon a premium rate for the insurance of lives of individuals, aged 21 say, which would enable it to carry on its business justly and fairly, because no life insurance company knows when any one individual aged 21 will die. Yet life insurance companies seem to be struggling on!

for brain-weight and "size of body." It has been shown (p. 66) that of a given change in mean brain-weight between 25 and 30 per cent. can be brought about by a selection of "size of body," measuring this both by stature and weight of body. As it is fair to assume that natural selection has acted on "size of body" we thus reach some idea of how much of an effect such selection has had on the weight of the brain.

A vital point at issue is as to what are the underlying causes of brain-weight differences. We have seen that there are definite racial types in brain-weight apart from differences in the other bodily characters which could be studied in this connection. To take a concrete example, we find that groups of male and female Bohemians having the same mean length and breadth of skull have probable mean brain-weights differing by about 69 grams, the female mean being lower than the male, of course. To what is this difference to be ascribed? Evidently there are a considerable number of possible factors which may enter into the matter. In the first place these males and females would still differ in stature by a certain amount so that a part of the remaining brain-weight difference would be accounted for in this way, but probably less than 25 per cent. Selecting too the same skull size would undoubtedly reduce the gross sexual difference in stature to some extent. The mean age of the arrays is essentially the same, so that nothing is gained from this source. To account for the still remaining difference there are at least the following possible factors:

1. The specific gravity of the female brain may be lower.

2. A unit increase in skull length and breadth may be associated with a greater change in skull height in the male than in the female.

3. The walls of the female skull may be thicker than in the male.

4. The female brain may fill the skull cavity less closely than does the male. (The opposite has been stated to be the case. Cf. Donaldson, *loc. cit.* p. 118.)

5. The ventricles of the brain may be proportionately larger in the female than in the male brain.

With the material at present available it is impossible to determine which of these possible factors really play the important rôle. It is my purpose merely to call attention to the kind of problem that confronts one at this point in the analysis of brain-weight constants. It at least will serve to show some of the directions in which more definite information is desired.

Donaldson* has well said: "The search for correlation between the size and form of the brain and the degree of the intelligence has interested all who have worked on this organ, and although it might be designated as the psychologist's standpoint, it has, from the very first, been in some measure before the minds of all." This point has so far been avoided in this paper for the reason that it seemed to me desirable to settle the question of the purely physical variations and

* *Loc. cit.* p. 85.

correlations before making any attempt on the much more difficult matter of brain-weight and intelligence correlation. I have, however, one point to add to the discussion of this already much debated subject. It follows rather simply from the theory of multiple correlation and regression that if the correlation coefficients r_{12} and r_{13} measuring the correlation between characters x_1 and x_2, and x_1 and x_3, are known, then it is possible to determine the limits between which the value of the coefficient of correlation r_{23} between x_2 and x_3 must lie. Without going into the details of the theory involved, which is given in convenient form by Yule*, it may be said that since

$$(r_{23} - r_{12}r_{13})^2 \text{ is not } > 1 + r_{12}{}^2 r_{13}{}^2 - r_{12}{}^2 - r_{13}{}^2,$$

r_{23} must lie between the limits

$$r_{12}r_{13} \pm (\sqrt{1 + r_{12}{}^2 r_{13}{}^2 - r_{12}{}^2 - r_{13}{}^2}).$$

Now, very fortunately, Pearson and his co-workers† at University College have determined for Cambridge undergraduates the correlation between "intelligence" as denoted by place in degree examinations and certain physical characters whose correlation with brain-weight we also know. Evidently then we have the material for getting at an approximation of the limits of the correlation between brain-weight and intelligence except that the two correlations from which the limits of the third are to be determined are not from the same material. This of course introduces an element of error, but as the coefficients of correlation of brain-weight with other characters seem to be fairly constant from race to race, a very serious error will probably not be made in assuming that the values of these correlations would not differ greatly in the English from what they are for continental races or from one another in two different groups of English. Leaving for the moment this question of possible error, let us examine the figures, first taking this problem: Assuming that Cambridge undergraduates have the weight of the cerebrum correlated with stature to roughly the same degree as is shown in Boyd's English data, what will be the limits of the correlation between weight of cerebrum and intelligence? The necessary data are as follows:

$r_{12} = -\cdot0056$ correlation coefficient between stature and intelligence.

$r_{13} = \quad \cdot12 \qquad ,, \qquad ,, \qquad ,, \qquad ,, \qquad ,,$ weight of cerebrum.

The limits of r_{23}, or of the coefficient of correlation between brain-weight and intelligence, are roughly $+\cdot98$ and $-\cdot99$. Now r_{12} is really insignificant, the conclusion the authors draw being that (*loc. cit.* p. 107), "stature is not correlated with place in degree examinations." Putting $r_{12} = 0$ the limits of r_{23} are $\pm \sqrt{1 - r_{13}{}^2}$. Or the practical conclusion we reach is that, so far as the known correlations of weight of cerebrum and intelligence respectively with stature give evidence, the correlation between weight of cerebrum and intelligence may have any value

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* *Loc. cit.*

† Lee, A. Lewenz, M.A., and Pearson, K.: "On the Correlation of the Mental and Physical Characters in Man, Part II." *Roy. Soc. Proc.* Vol. LXXI. pp. 106—114, 1902.

between 0 and ± 1. There is no evidence from this source as to whether the correlation is positive or negative. Suppose we take still another case, stating the problem this time in this way: Assuming that the Cambridge undergraduates have brain-weight correlated with skull length to roughly the same degree as is shown by Bohemian males, what will be the limits of the correlation between brain-weight and intelligence? The necessary coefficients are:

$r_{12} = \cdot 0861$ correlation coefficient between skull length and intelligence.

$r_{13} = \cdot 5482$　　　　,,　　　　　,,　　　　　,,　　,,　　,,　　and brain-weight.

The limits of r_{23} or the correlation between brain-weight and intelligence, are $+ \cdot 7$ and $- \cdot 6$, or practically we reach much the same result as before. Now the Bohemian coefficient, as we have seen, probably is in error in the direction of being too small, or the probability is that the limits of r_{23} are still wider than the figures indicate. As to the reliability of these limits this may be said: in order to fix correctly the sign of the correlation between brain-weight and intelligence, both the coefficients between intelligence and physical characters and brain-weight and the same physical characters would have to be greater than $\sqrt{\cdot 5}$ or $\cdot 707$. Now I think it is reasonably evident from what we already know, that for civilized man at least there is no likelihood whatever of ever finding values as high as $\cdot 707$ for the coefficients concerned. Or in other words the degree of the correlation between brain-weight and intelligence is indeterminate, with the probability that it is sensibly equal to zero. That is, brain-weight and intelligence in the sense of mental capacity are probably not sensibly correlated. All the inferential evidence when subjected to careful scrutiny leads to the same conclusion, I think.

In closing I may perhaps be permitted to point out what seem to me to be the most promising lines for future investigation of the problems regarding the weight of the brain. In the first instance it seems to me highly desirable to transfer the problem for a time at least from man, where in the nature of the case all the fundamental records must be vitiated from the fact that we cannot weigh the brains of any considerable number of normal men instantly killed. As a consequence of this impairment of the data resulting from complex ante-mortem conditions the *absolute* values of certain of the biometric constants concerned cannot in the case of man be proven to be sensibly identical with what we should consider necessarily normal values. To be sure, such a sensible identity cannot, on the other hand, be disproven, but a state in which one can neither prove nor disprove is scientifically not altogether satisfactory. By working with animals it is possible to weigh any number of brains of instantly killed normal individuals. From records so obtained the values of the important biometric constants like the coefficients of correlation can be satisfactorily determined. In the hands of Prof. Donaldson this line of work is yielding results of the greatest significance and value. For continuing the work on the human side the primary need is for more and larger collections of brain-weights in which close attention is paid to the racial homogeneity of the material and to the recording of other subsidiary facts

as well as brain-weight. In comparison with the present series of brain-weighings a series which should record the following facts would be almost ideal.

1. Brain-weight, by the standard method.
2. Race. In this the more essential detail given the better.
3. Country of residence during adult life.
4. Occupation (as an index of social status).
5. Immediate cause of death.
6. Chronic diseases of adult life.
7. Stature.
8. Body-weight.
9. Head length.
10. Head breadth.
11. Head height.
12. Maximum horizontal circumference of head.
13. Age.
14. Sex.

Measurements all made before skull is opened, and with hair removed at points of contact.

Such a list at first sight appears formidable from the practical standpoint, but evidently the things which would give the greatest difficulty are 2 to 6 inclusive. The head measurements could be made with little trouble, and a relatively small expenditure of time. This list is presented with the hope that any future workers who may be about to undertake the great labour involved in obtaining a large mass of human brain-weight statistics will at least consider the points raised. The few additional facts would take but little more time in the collection, and they would greatly enhance the value of the completed series. I would especially call attention to the need for larger series of brain-weight statistics than those we now have. In order to do really "close" statistical work on the subject it is desirable that we have larger arrays of individuals of given age and stature types in order that the regressions may be smoothed and the brain-weights in more absolutely homogeneous material may be investigated.

Finally it should be said that the present paper is, from the biometrical standpoint, only a "first" study of the problems of brain-weight correlation. Much yet remains to be done, and as will have been apparent to the reader could be done with the present material. One thing especially which might be done is to separate the material into a third "old" group, comprising the individuals falling in age between 50 and 80, and treat this group separately. I very much doubt, however, in view of the apparent substantial linearity of the regression of brain-weight on age through the whole of adult life, and considering the statistically small number of individuals which in each case would have been available, whether the value of the results so obtained would have been in any way commensurate with the labour involved. Even a biometrician must stop somewhere. When we have much larger collections of brain-weight statistics to work with an old age division of the material will be at least experimentally justified.

14. *Summary.*

Some of the more important general conclusions of a study of five series of brain-weighings, representing Swedish, Hessian, Bavarian, Bohemian, and English sub-races of man and including altogether the weights of 2100 adult male and 1034 adult female brains, may be summarised as follows:

1.   There are definite racial types in brain-weight.   The differences between racial groups in this character are only in part to be accounted for by differences in other characters of the body.

2.   The series studied exhibit a very fair degree of homogeneity.

3.   In respect to variability the characters, brain-weight and skull capacity are sensibly equal.   The coefficients of variation for brain-weight are intermediate in value between those which have been determined for skeletal characters in man on the one hand, and those for the weights of the viscera and for various physiological characters on the other hand.

4.   Having regard to the size of the present series we conclude that variation in weight of the brain may for practical purposes be considered to follow the "normal" law of the distribution of errors.   If the mean and the mode do not exactly coincide the mean will be slightly greater than the mode in brain-weight frequency distributions.

5.   The correlation of brain-weight with age, stature and body-weight, is in all cases low.

6.   The correlation of brain-weight with skull length and skull breadth is, in comparison with the other characters studied, fairly high, and is for both skull length and breadth positive.

7.   The sexual differences in mean brain-weight are practically constant in all the races studied, whether considered absolutely or relatively.

8.   Only a part of the sexual difference is to be accounted for by differences between the sexes in other bodily characters.

9.   The sexes are equally variable in respect to brain-weight.

10.   The weight of the brain tends to be more highly correlated with other characters in the female than in the male.

11.   The correlation of brain-weight with age is negative.   The regression of brain-weight on age is, so far as can be determined from our present series, linear throughout the period of life comprised between the ages 20 to 80.   In other words, there is a steady decline in the weight of the brain with advancing age, beginning at about the twentieth year and continuing throughout adult life.

12. The correlation of brain-weight with stature and with body-weight is positive and in each case the regression is linear.

13. The weight of the cerebrum follows the same laws of variation and correlation as does the weight of the entire brain, but exhibits a somewhat greater relative variability.

14. The results as to variation and correlation in brain-weight are closely accordant for all the races studied.

15. The indirect selection of bodily characters can modify the mean weight of the brain to a considerable extent. The amount of this effect has been measured in certain cases.

16. There is no evidence that brain-weight is sensibly correlated with intellectual ability. The limits of this correlation have been shown to be not closer than 0 and ± ·6.

17. So far as can be determined from present material dolichocephaly has associated with it no advantage in mean brain-weight over brachycephaly and *vice versâ*.

18. There is great need for further large and homogeneous collections of brain-weight statistics. When these are available for a considerable number of races it will be possible to pass from intra-racial to inter-racial problems.

APPENDIX OF MEASUREMENTS.

Correlation Tables.

The fundamental correlation tables on which the results of this paper are based have been placed together here at the end for convenience in reference. The tables are numbered consecutively in Arabic numerals, and each bears its own explanation. A word may be said as to the plan of arrangement. The racial order is

Swedes	Tables	1	to	12	inclusive.
Hessians	„	13	„	24	„
Bohemians	„	25	„	34	„
Bavarians	„	35	„	44	„
English	„	45 and 46			„

Under each race the order of arrangement is in each case for the tables involving brain-weight as one of the variables to precede the subsidiary tables involving only the related characters. The tables of males precede those for females, and the "total" series precede the "young."

TABLE 1.

Swedes. Brain-weight and Stature. " Total " Series. Males.

Stature (centimetres).

Brain-weight (grams).	144–146	147–149	150–152	153–155	156–158	159–161	162–164	165–167	168–170	171–173	174–176	177–179	180–182	183–185	186–188	189–191	Totals
1100—1149	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
1150—1199	—	—	1	—	—	2	3	1	—	2	2	—	—	—	—	—	10
1200—1249	—	—	1	—	—	4	2	5	5	1	1	1	—	1	—	—	21
1250—1299	—	1	—	—	4	3	5	8	8	6	3	3	—	3	—	—	44
1300—1349	1	—	—	—	2	4	4	7	11	9	7	6	2	—	—	—	53
1350—1399	1	1	—	—	1	7	17	21	11	13	6	5	3	—	—	—	86
1400—1449	—	—	1	1	—	2	9	10	13	13	8	11	2	2	—	—	72
1450—1499	—	—	1	1	1	4	6	8	13	8	9	5	1	2	—	1	60
1500—1549	—	—	—	—	1	—	1	2	4	5	7	4	4	—	—	—	28
1550—1599	—	—	—	—	1	3	—	6	1	4	2	6	2	—	—	—	25
1600—1649	—	—	—	—	—	—	—	3	4	4	—	1	—	—	—	—	12
1650—1699	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	3
1700—1749	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	2	2	3	2	10	29	47	72	71	66	45	44	14	8	—	1	416

TABLE 2.

Swedes. Brain-weight and Stature. " Total " Series. Females.

Stature (centimetres).

Brain-weight (grams).	132–134	135–137	138–140	141–143	144–146	147–149	150–152	153–155	156–158	159–161	162–164	165–167	168–170	171–173	174–176	Totals
900— 949	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
950— 999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1000—1049	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
1050—1099	—	—	—	1	—	3	—	1	1	1	1	1	—	—	—	9
1100—1149	1	—	1	—	2	3	2	6	4	4	2	2	1	—	—	28
1150—1199	—	—	—	—	1	2	3	8	7	1	3	3	2	—	—	30
1200—1249	—	—	1	—	2	3	6	9	11	3	12	4	2	—	—	53
1250—1299	—	—	—	—	—	3	1	8	5	9	3	7	2	—	2	40
1300—1349	—	—	—	—	—	—	—	4	8	6	6	5	1	—	—	30
1350—1399	—	—	—	—	1	—	1	5	3	3	5	2	2	1	—	23
1400—1449	—	—	—	—	—	—	—	1	1	1	3	3	1	—	—	10
1450—1499	—	—	—	—	—	—	—	1	—	2	2	—	—	—	1	6
1500—1549	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
1550—1599	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	1	—	2	1	6	14	14	44	40	30	38	28	11	1	3	233

TABLE 3.

Swedes. Brain-weight and Stature. "Young" Series. Males.

20—50 years.

Stature (centimetres).

Brain-weight (grams).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
1100—1149	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
1150—1199	—	—	—	—	—	1	2	—	—	1	—	—	—	1	—	—	4
1200—1249	—	—	1	—	—	2	1	4	2	1	1	—	—	1	—	—	13
1250—1299	—	—	—	—	1	1	2	5	3	2	—	2	—	2	—	—	18
1300—1349	1	—	—	—	—	4	4	5	7	3	6	5	—	—	—	—	35
1350—1399	1	—	—	—	—	4	11	12	6	7	6	4	2	—	—	—	53
1400—1449	—	—	—	—	—	1	3	6	8	10	5	8	1	1	—	—	43
1450—1499	—	—	1	1	—	2	5	7	10	3	5	4	—	1	—	1	40
1500—1549	—	—	—	—	1	—	1	1	3	3	7	2	3	—	—	—	21
1550—1599	—	—	—	—	1	3	—	3	1	3	2	4	2	—	—	—	19
1600—1649	—	—	—	—	—	—	—	2	4	4	—	1	—	—	—	—	11
1650—1699	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	3
1700—1749	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	2	—	2	1	3	18	29	46	45	38	32	32	8	5	—	1	262

TABLE 4.

Swedes. Brain-weight and Stature. "Young" Series. Females.

20—50 years.

Stature (centimetres).

Brain-weight (grams).	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	Totals
900— 949	—	—	—	—	1	—	—	—	—	—	—	—	—	1
950— 999	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1000—1049	—	—	—	—	—	1	—	—	—	—	—	—	—	1
1050—1099	—	1	—	1	—	—	—	—	—	—	—	—	—	2
1100—1149	1	—	1	1	—	2	2	2	1	2	—	—	—	12
1150—1199	—	—	1	—	2	3	5	—	1	2	2	—	—	16
1200—1249	—	—	2	2	1	2	5	3	6	1	2	—	—	24
1250—1299	—	—	—	2	—	5	2	5	3	5	2	—	2	26
1300—1349	—	—	—	—	—	2	3	4	4	2	1	—	—	16
1350—1399	—	—	1	—	1	3	2	1	3	1	1	1	—	14
1400—1449	—	—	—	—	—	1	—	1	3	2	1	—	—	8
1450—1499	—	—	—	—	—	1	—	2	2	—	—	—	1	6
1500—1549	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1550—1599	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	1	1	5	6	5	20	19	18	23	16	9	1	3	127

TABLE 5.

Swedes. Brain-weight and Age. "Total" Series. Males.

Brain-weight (grams).

Age (years)	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	1700—1749	Totals
20—24	—	—	—	1	6	5	2	7	2	3	1	—	—	27
25—29	—	—	—	1	4	10	8	4	2	2	3	2	1	37
30—34	—	1	2	—	5	13	8	6	5	3	4	1	—	48
35—39	1	2	1	6	4	8	9	9	4	4	1	—	—	49
40—44	—	1	9	6	10	9	10	5	5	4	2	—	—	61
45—49	—	—	1	4	6	8	6	9	3	3	—	—	—	40
50—54	—	—	1	7	4	15	7	9	4	2	1	—	—	50
55—59	—	1	3	4	9	6	7	6	1	2	—	—	—	39
60—64	—	4	1	7	2	4	8	3	1	2	—	—	—	32
65—69	—	1	2	4	3	2	3	2	1	—	—	—	—	18
70—74	—	—	—	2	—	4	—	—	—	—	—	—	—	6
75—79	—	—	1	2	—	2	4	—	—	—	—	—	—	9
Totals	1	10	21	44	53	86	72	60	28	25	12	3	1	416

TABLE 6.

Swedes. Brain-weight and Age. "Total" Series. Females.

Brain-weight (grams).

Age (years)	900—949	950—999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	Totals
20—24	—	—	—	—	—	1	2	6	5	—	2	2	—	—	18
25—29	—	—	—	—	1	7	3	—	1	3	1	—	—	1	17
30—34	1	—	—	1	3	—	2	8	—	3	—	2	—	—	20
35—39	—	—	—	—	2	4	5	4	5	2	2	2	—	—	26
40—44	—	—	1	—	4	3	7	5	3	5	2	—	—	—	30
45—49	—	—	—	1	2	1	5	3	2	1	1	—	—	—	16
50—54	—	—	—	2	4	1	6	1	2	3	2	—	—	—	21
55—59	—	—	—	2	4	5	6	1	6	4	—	—	—	—	28
60—64	—	—	—	—	1	5	8	5	4	2	—	—	—	—	25
65—69	—	—	—	1	1	1	3	4	2	—	—	—	—	—	12
70—74	—	—	—	1	4	2	4	2	—	—	—	—	1	—	14
75—80	—	—	—	1	2	—	2	1	—	—	—	—	—	—	6
Totals	1	—	1	9	28	30	53	40	30	23	10	6	1	1	233

TABLE 7.

Swedes. Brain-weight and Age. " Young" Series. Males.

Brain-weight (grams).

Age (years).	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	1700—1749	Totals
20—24	—	—	—	1	6	5	2	7	2	3	1	—	—	27
25—29	—	—	—	1	4	10	8	4	2	2	3	2	1	37
30—34	—	1	2	—	5	13	8	6	5	3	4	1	—	48
35—39	1	2	1	6	4	8	9	9	4	4	1	—	—	49
40—44	—	1	9	6	10	9	10	5	5	4	2	—	—	61
45—49	—	—	1	4	6	8	6	9	3	3	—	—	—	40
Totals	1	4	13	18	35	53	43	40	21	19	11	3	1	262

TABLE 8.

Swedes. Brain-weight and Age. " Young" Series. Females.

Brain-weight (grams).

Age (years).	900—999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	Totals
20—24	—	—	—	—	—	1	2	6	5	—	2	2	—	—	18
25—29	—	—	—	—	1	7	3	—	1	3	1	—	—	1	17
30—34	1	—	—	1	3	—	2	8	—	3	—	2	—	—	20
35—39	—	—	—	—	2	4	5	4	5	2	2	2	—	—	26
40—44	—	—	1	—	4	3	7	5	3	5	2	—	—	—	30
45—49	—	—	—	1	2	1	5	3	2	1	1	—	—	—	16
Totals	1	—	1	2	12	16	24	26	16	14	8	6	—	1	127

TABLE 9.

Swedes.   Stature and Age.   "Total" Series.   Males.

Stature (centimetres).

Age (years)	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
20—24	—	—	—	—	—	—	3	1	5	4	7	3	2	1	—	1	27
25—29	—	—	—	—	—	1	1	7	4	6	6	10	1	1	—	'	37
30—34	—	—	1	—	1	2	5	9	12	6	6	4	—	2	—	—	48
35—39	1	—	—	1	—	5	4	12	9	6	3	6	2	—	—	—	49
40—44	—	—	1	—	1	6	8	12	9	11	5	5	3	—	—	—	61
45—49	1	—	—	—	1	4	8	5	6	5	5	4	—	1	—	—	40
50—54	—	2	—	—	1	2	9	8	10	6	3	4	3	2	—	—	50
55—59	—	—	—	—	2	2	3	7	6	9	5	2	2	1	—	—	39
60—64	—	—	1	—	3	3	4	7	3	8	1	1	1	—	—	—	32
65—69	—	—	—	—	1	2	1	1	5	3	2	3	—	—	—	—	18
70—74	—	—	—	—	—	1	—	1	1	1	1	1	—	—	—	—	6
75—79	—	—	—	1	—	1	1	2	1	1	1	1	—	—	—	—	9
Totals	2	2	3	2	10	29	47	72	71	66	45	44	14	8	—	1	416

TABLE 10.

Swedes.   Stature and Age.   "Total" Series.   Females.

Stature (centimetres).

Age (years)	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	Totals
20—24	—	—	—	—	—	—	—	1	3	4	6	2	1	—	1	18
25—29	—	—	1	—	—	—	—	3	4	1	2	2	4	—	—	17
30—34	—	—	—	—	—	3	2	3	1	2	—	7	—	1	1	20
35—39	—	—	—	—	—	2	2	4	3	6	6	2	1	—	—	26
40—44	—	—	—	—	4	1	—	5	5	4	5	3	2	—	1	30
45—49	—	—	—	1	1	—	1	4	3	1	4	—	1	—	—	16
50—54	—	—	—	—	—	2	2	4	2	3	3	3	2	—	—	21
55—59	1	—	—	—	—	3	3	3	5	6	4	3	—	—	—	28
60—64	—	—	1	—	—	—	2	6	7	—	4	5	—	—	—	25
65—69	—	—	—	—	1	—	1	6	1	1	2	—	—	—	—	12
70—74	—	—	—	—	—	3	1	3	3	1	2	1	—	—	—	14
75—79	—	—	—	—	—	—	—	2	3	1	—	—	—	—	—	6
Totals	1	—	2	1	6	14	14	44	40	30	38	28	11	1	3	233

TABLE 11.

Swedes. Stature and Age. "Young" Series. Males.

Stature (centimetres).

Age (years).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
20—24	—	—	—	—	—	—	3	1	5	4	7	3	2	1	—	1	27
25—29	—	—	—	—	—	1	1	7	4	6	6	10	1	1	—	—	37
30—34	—	—	1	—	1	2	5	9	12	6	6	4	—	2	—	—	48
35—39	1	—	—	1	—	5	4	12	9	6	3	6	2	—	—	—	49
40—44	—	—	1	—	1	6	8	12	9	11	5	5	3	—	—	—	61
45—49	1	—	—	—	1	4	8	5	6	5	5	4	—	1	—	—	40
Totals	2	—	2	1	3	18	29	46	45	38	32	32	8	5	—	1	262

TABLE 12.

Swedes. Stature and Age. "Young" Series. Females.

Stature (centimetres).

Age (years).	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	Totals
20—24	—	—	—	—	—	1	3	4	6	2	1	—	1	18
25—29	1	—	—	—	—	3	4	1	2	2	4	—	—	17
30—34	—	—	—	3	2	3	1	2	—	7	—	1	1	20
35—39	—	—	—	2	2	4	3	6	6	2	1	—	—	'26
40—44	—	—	4	1	—	5	5	4	5	3	2	—	1	30
45—49	—	1	1	—	1	4	3	1	4	—	1	—	—	16
Totals	1	1	5	6	5	20	19	18	23	16	9	1	3	127

TABLE 13.

Hessians.　Brain-weight and Stature.　"Total" Series.　Males.

Stature (centimetres).

Brain-weight (grams)	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
1000—1049	—	—	—	—	—	1	—	2	—	—	1	—	—	1	—	—	—	—	4
1050—1099	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	3
1100—1149	—	1	—	—	—	—	—	3	—	—	1	1	—	—	1	—	—	—	7
1150—1199	—	—	—	—	—	—	2	1	1	2	—	2	—	—	—	—	—	—	8
1200—1249	—	—	—	—	1	—	1	3	3	3	1	4	1	1	1	—	—	—	19
1250—1299	—	—	—	—	1	4	6	7	5	8	6	5	5	1	—	1	—	1	49
1300—1349	—	—	—	1	2	4	8	10	14	8	18	8	6	5	—	—	—	1	85
1350—1399	1	—	—	—	1	—	4	6	9	10	11	14	7	3	1	—	—	—	67
1400—1449	—	—	—	1	1	1	4	5	12	13	19	16	8	2	—	3	—	—	85
1450—1499	—	1	1	—	—	—	2	2	9	11	7	17	2	5	2	—	—	—	59
1500—1549	—	—	—	—	—	3	2	4	7	10	14	9	3	3	1	1	—	—	57
1550—1599	—	—	—	—	—	1	1	1	3	4	3	8	3	1	1	—	—	—	26
1600—1649	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	—	3
1650—1699	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
1700—1749	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	2	2	1	2	6	14	30	45	63	73	82	85	35	22	7	5	—	1	475

TABLE 14.

Hessians.　Brain-weight and Stature.　"Total" Series.　Females.

Stature (centimetres).

Brain-weight (grams)	126—128	129—131	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	Totals
950— 999	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1000—1049	1	—	—	—	—	—	—	1	—	1	2	1	—	—	—	—	—	—	—	6
1050—1099	—	—	—	—	1	—	1	1	1	1	4	3	—	—	—	—	—	—	—	12
1100—1149	—	—	—	—	—	1	—	2	3	3	7	3	1	—	1	—	—	—	—	21
1150—1199	—	—	—	—	—	2	—	5	5	9	3	5	1	1	—	—	—	—	—	31
1200—1249	—	—	—	—	1	1	4	5	3	9	19	10	6	2	1	1	—	—	—	62
1250—1299	—	—	—	—	—	1	—	3	4	9	14	9	5	3	5	3	—	—	—	56
1300—1349	—	—	—	—	—	—	1	1	6	8	6	7	4	2	1	—	—	—	—	36
1350—1399	—	—	—	1	—	1	—	3	6	4	5	4	1	1	1	—	—	—	—	27
1400—1449	—	—	—	—	1	—	1	2	4	1	4	3	3	1	—	1	1	—	1	23
1450—1499	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	3
1500—1549	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	3
Totals	1	—	—	1	3	7	7	23	32	46	65	45	25	10	9	5	1	—	1	281

TABLE 15.

Hessians. Brain-weight and Stature. " Young" Series. Males.

15—50 years.

Stature (centimetres).

Brain-weight (grams)	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	Totals
1000—1049	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	2
1500—1099	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
1100—1149	—	1	—	—	—	—	—	1	—	—	1	1	—	—	1	—	5
1150—1199	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	2
1200—1249	—	—	—	—	—	—	1	1	1	2	—	2	—	—	—	—	7
1250—1299	—	—	—	—	1	2	4	3	2	7	2	2	3	1	—	1	28
1300—1349	—	—	—	1	—	3	3	6	10	5	10	6	4	4	—	—	52
1350—1399	1	—	—	—	1	—	3	4	4	7	8	11	2	—	—	—	41
1400—1449	—	—	—	1	1	1	2	2	7	5	9	8	3	1	—	3	43
1450—1499	—	1	1	—	—	—	2	2	7	9	6	12	2	2	2	—	46
1500—1549	—	—	—	—	—	2	1	4	3	7	11	6	2	1	—	—	37
1550—1599	—	—	—	—	—	—	1	—	2	4	2	8	3	1	1	—	22
1600—1649	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	3
1650—1699	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
Totals	1	2	1	2	3	8	17	26	36	50	50	58	19	10	4	4	291

TABLE 16.

Hessians. Brain-weight and Stature. " Young" Series. Females.

15—50 years.

Stature (centimetres).

Brain-weight (grams)	126—128	129—131	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	Totals
1000—1049	1	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	3
1050—1099	—	—	—	—	1	—	—	—	—	1	2	2	—	—	—	—	—	—	—	6
1100—1149	—	—	—	—	—	1	—	—	—	1	3	2	—	—	—	—	—	—	—	7
1150—1199	—	—	—	—	1	1	—	3	3	2	3	1	1	—	—	—	—	—	—	15
1200—1249	—	—	—	—	1	1	3	2	2	4	14	4	6	1	—	1	—	—	—	39
1250—1299	—	—	—	—	—	—	—	2	3	5	11	6	2	2	2	3	—	—	—	36
1300—1349	—	—	—	—	—	—	—	1	4	5	1	5	3	2	1	—	—	—	—	22
1350—1399	—	—	—	1	—	1	—	2	4	2	3	3	1	1	1	—	—	—	—	19
1400—1449	—	—	—	—	—	—	1	2	4	1	2	3	3	1	—	1	1	—	1	20
1450—1499	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	3
1500—1549	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	3
Totals	1	—	—	1	3	4	4	13	20	22	41	26	20	7	4	5	1	—	1	173

TABLE 17.

Hessians. Brain-weight and Age. "Total" Series. Males.

Brain-weight (grams).

Age (years)	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	1700—1749	Totals
15—19	—	—	1	—	—	2	8	4	10	5	4	2	—	—	—	36
20—24	—	1	1	1	—	2	7	9	6	15	10	5	1	—	—	58
25—29	—	—	1	—	1	6	14	6	6	8	7	1	1	1	—	52
30—34	—	—	1	—	5	2	4	5	9	5	6	3	—	1	—	41
35—39	—	—	—	1	—	6	5	2	6	1	1	2	—	—	—	24
40—44	1	—	—	—	1	4	6	14	1	6	5	4	—	—	—	42
45—49	1	—	1	—	—	6	8	1	5	6	4	5	1	—	—	38
50—54	1	1	—	2	6	6	8	2	17	3	8	—	—	—	1	55
55—59	—	—	1	2	—	5	6	10	8	3	5	1	—	—	—	41
60—64	1	1	—	—	2	1	5	6	12	2	3	—	—	—	—	33
65—69	—	—	—	2	1	4	8	4	3	4	3	1	—	—	—	30
70—74	—	—	1	—	3	2	4	2	1	1	1	2	—	—	—	17
75—79	—	—	—	—	—	3	2	2	1	—	—	—	—	—	—	8
Totals	4	3	7	8	19	49	85	67	85	59	57	26	3	2	1	475

TABLE 18.

Hessians. Brain-weight and Age. "Total" Series. Females.

Brain-weight (grams).

Age (years)	950—999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	Totals
15—19	—	—	1	—	2	4	2	—	—	7	1	—	17
20—24	—	—	1	—	1	5	9	1	2	1	1	1	22
25—29	—	—	1	1	2	4	3	6	4	4	—	1	26
30—34	—	—	—	3	2	6	2	—	3	1	—	1	18
35—39	—	—	3	—	4	5	5	6	6	2	1	—	32
40—44	—	1	—	2	1	6	10	7	2	2	—	—	31
45—49	—	2	—	1	3	9	5	2	2	3	—	—	27
50—54	—	—	—	2	3	8	4	4	5	1	—	—	27
55—59	—	—	2	3	2	5	3	5	3	2	—	—	25
60—64	—	—	2	3	2	3	5	1	—	—	—	—	16
65—69	—	—	1	—	4	4	5	2	—	—	—	—	16
70—74	1	2	1	5	3	3	3	2	—	—	—	—	20
75—79	—	1	—	1	2	—	—	—	—	—	—	—	4
Totals	1	6	12	21	31	62	56	36	27	23	3	3	281

TABLE 19.

Hessians. Brain-weight and Age. "Young" Series. Males.

Brain-weight (grams).

Age (years).	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	Totals
15—19	—	—	1	—	—	2	8	4	10	5	4	2	—	—	36
20—24	—	1	1	1	—	2	7	4	6	15	10	5	1	—	58
25—29	—	—	1	—	1	6	14	6	6	8	7	1	1	1	52
30—34	—	—	1	—	5	2	4	5	9	5	6	3	—	1	41
35—39	—	—	—	1	—	6	5	2	6	1	1	2	—	—	24
40—44	1	—	—	—	1	4	6	14	1	6	5	4	—	—	42
45—49	1	—	1	—	—	6	8	1	5	6	4	5	1	—	38
Totals	2	1	5	2	7	28	52	41	43	46	37	22	3	2	291

TABLE 20.

Hessians. Brain-weight and Age. "Young" Series. Females.

Brain-weight (grams).

Age (years).	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	Totals
15—19	—	1	—	2	4	2	—	—	7	1	—	17
20—24	—	1	—	1	5	9	1	2	1	1	1	22
25—29	—	1	1	2	4	3	6	4	4	—	1	26
30—34	—	—	3	2	6	2	—	3	1	—	1	18
35—39	—	3	—	4	5	5	6	6	2	1	—	32
40—44	1	—	2	1	6	10	7	2	2	—	—	31
45—49	2	—	1	3	9	5	2	2	3	—	—	27
Totals	3	6	7	15	39	36	22	19	20	3	3	173

TABLE 21.

Hessians. Stature and Age. "Total" Series. Males.

Stature (centimetres).

Age (years)	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
15—19	1	2	1	2	1	4	4	4	4	3	5	2	2	1	—	—	—	—	36
20—24	—	—	—	—	—	1	—	6	8	9	18	10	2	4	—	—	—	—	58
25—29	—	—	—	—	—	—	6	7	8	8	8	11	3	1	—	—	—	—	52
30—34	—	—	—	—	1	1	—	2	6	9	2	12	3	1	2	2	—	—	41
35—39	—	—	—	—	—	—	2	—	3	5	4	6	2	1	—	1	—	—	24
40—44	—	—	—	—	1	1	4	5	3	6	7	9	3	1	1	1	—	—	42
45—49	—	—	—	—	—	1	1	2	4	10	6	8	4	1	1	—	—	—	38
50—54	1	—	—	—	1	1	3	3	8	11	4	11	6	4	1	1	—	—	55
55—59	—	—	—	—	—	—	3	3	4	4	11	5	5	4	1	—	—	1	41
60—64	—	—	—	—	—	1	1	4	4	2	9	7	3	2	—	—	—	—	33
65—69	—	—	—	—	1	3	3	2	6	3	4	3	2	2	1	—	—	—	30
70—74	—	—	—	—	1	1	1	5	4	2	3	—	—	—	—	—	—	—	17
75—79	—	—	—	—	—	—	2	2	1	1	1	1	—	—	—	—	—	—	8
Totals	2	2	1	2	6	14	30	45	63	73	82	85	35	22	7	5	—	1	475

TABLE 22.

Hessians. Stature and Age. "Total" Series. Females.

Stature (centimetres).

Age (years)	126—128	129—131	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—183	Totals
15—19	—	—	—	—	2	—	—	3	3	1	3	1	2	—	1	1	—	—	—	17
20—24	—	—	—	—	—	—	—	—	—	6	7	3	4	—	1	—	1	—	—	22
25—29	—	—	—	1	—	1	—	—	7	2	5	2	5	1	1	—	—	—	—	26
30—34	—	—	—	—	—	1	—	1	2	2	5	2	2	1	—	2	—	—	—	18
35—39	—	—	—	—	1	2	1	4	1	4	7	7	4	—	—	1	—	—	—	32
40—44	—	—	—	—	—	—	1	4	3	3	7	7	1	3	1	—	—	—	1	31
45—49	1	—	—	—	—	—	2	1	4	4	7	4	2	2	—	—	—	—	—	27
50—54	—	—	—	—	—	—	—	5	4	4	8	4	—	2	—	—	—	—	—	27
55—59	—	—	—	—	—	—	2	—	—	6	6	6	1	2	2	—	—	—	—	25
60—64	—	—	—	—	—	—	—	1	1	5	3	3	2	—	1	—	—	—	—	16
65—69	—	—	—	—	—	1	1	—	3	3	5	2	—	1	—	—	—	—	—	16
70—74	—	—	—	—	—	2	—	3	4	4	2	3	2	—	—	—	—	—	—	20
75—79	—	—	—	—	—	—	—	1	—	2	—	1	—	—	—	—	—	—	—	4
Totals	1	—	—	1	3	7	7	23	32	46	65	45	25	10	9	5	1	—	1	281

TABLE 23.

Hessians. Stature and Age. "Young" Series. Males.

15—50 years.

Stature (centimetres).

Age (years).	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	Totals
15—19	1	2	1	2	1	4	4	4	4	3	5	2	2	1	—	—	36
20—24	—	—	—	—	—	1	—	6	8	9	18	10	2	4	—	—	58
25—29	—	—	—	—	—	—	6	7	8	8	8	11	3	1	—	—	52
30—34	—	—	—	—	1	1	—	2	6	9	2	12	3	1	2	2	41
35—39	—	—	—	—	—	—	2	—	3	5	4	6	2	1	—	1	24
40—44	—	—	—	—	1	1	4	5	3	6	7	9	3	1	1	1	42
45—49	—	—	—	—	—	1	1	2	4	10	6	8	4	1	1	—	38
Totals	1	2	1	2	3	8	17	26	36	50	50	58	19	10	4	4	291

TABLE 24.

Hessians. Stature and Age. "Young" Series. Females.

15—50 years.

Stature (centimetres).

Age (years).	126—128	129—131	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	Totals
15—19	—	—	—	—	2	—	—	3	3	1	3	1	2	—	1	1	—	—	—	17
20—24	—	—	—	1	—	—	—	—	—	6	7	3	4	—	1	—	1	—	—	22
25—29	—	—	—	1	—	1	—	—	7	2	5	2	5	1	1	1	—	—	—	26
30—34	—	—	—	—	—	1	—	1	2	2	5	2	2	1	—	2	—	—	—	18
35—39	—	—	—	—	1	2	1	4	1	4	7	7	4	—	—	1	—	—	—	32
40—44	—	—	—	—	—	1	4	3	3	7	7	1	3	1	—	—	—	—	1	31
45—49	1	—	—	—	—	—	2	1	4	4	7	4	2	2	—	—	—	—	—	27
Totals	1	—	—	1	3	4	4	13	20	22	41	26	20	7	4	5	1	—	1	173

TABLE 25.

Bohemians. Brain-weight and Stature.
" Young " Series. Males. 20—59 years.

Brain-weight (grams).

Stature (centimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	Totals
	—	—	—	1	—	—	—	—	1
150—154	—	1	3	—	1	—	—	—	5
155—159	1	1	4	4	4	1	—	—	15
160—164	—	5	15	22	9	2	1	—	54
165—169	—	3	15	24	17	7	—	—	66
170—174	—	5	17	21	21	2	2	1	69
175—179	—	3	7	9	10	5	2	—	36
180—184	—	1	—	7	4	2	—	1	15
185—189	—	—	2	—	2	—	—	—	4
190—194	—	—	—	—	—	1	—	—	1
Totals	1	19	63	88	68	20	5	2	266

TABLE 26.

Bohemians. Brain-weight and Stature.
" Young " Series. Females. 20—59 years.

Brain-weight (grams).

Stature (centimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
130—134	—	—	1	—	—	—	1
135—139	—	—	—	1	—	—	1
140—144	—	—	—	2	—	—	2
145—149	—	2	7	5	2	—	16
150—154	1	—	11	6	4	—	22
155—159	2	4	19	14	2	1	42
160—164	—	3	6	14	7	2	32
165—169	—	—	2	5	4	—	11
170—174	—	—	2	2	1	—	5
175—179	—	—	—	—	1	—	1
Totals	3	9	48	49	21	3	133

TABLE 27.

Bohemians. Brain-weight and Age.
" Total " Series. Males.

Brain-weight (grams).

Age (years).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	Totals
20—29	—	4	12	28	21	9	2	1	77
30—39	—	5	12	17	14	6	3	1	58
40—49	1	10	24	27	23	6	—	—	91
50—59	—	3	19	31	19	4	1	—	77
60—69	—	4	17	11	9	4	1	—	46
70—79	—	1	14	8	—	—	—	—	23
Totals	1	27	98	122	86	29	7	2	372

TABLE 28.

Bohemians. Brain-weight and Age.
" Total " Series. Females.

Brain-weight (grams).

Age (years).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
20—29	1	2	28	26	13	2	72
30—39	—	2	9	14	7	3	35
40—49	1	4	8	10	7	—	30
50—59	1	2	14	6	2	1	26
60—69	—	4	10	5	—	—	19
70—79	—	5	4	4	2	—	15
Totals	3	19	73	65	31	6	197

TABLE 29.

Bohemians. Brain-weight and Skull Length. Males. 20—59 years.

Brain-weight (grams).

Skull length (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	1800—1899	Totals
155—159	—	—	1	1	—	—	—	—	—	2
160—164	—	—	2	6	4	2	—	—	—	14
165—169	1	—	9	10	18	3	1	—	—	42
170—174	—	—	5	19	28	11	4	1	—	68
175—179	—	—	4	19	29	23	4	—	—	79
180—184	—	—	—	10	19	23	8	1	—	61
185—189	—	—	—	1	2	12	4	—	—	19
190—194	—	—	—	—	1	2	3	4	—	10
195—199	—	—	—	—	—	1	1	—	2	4
Totals	1	—	21	66	101	77	25	6	2	299

TABLE 30.

Bohemians. Brain-weight and Skull Length. Females. 20—59 years.

Brain-weight (grams).

Skull length (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
150—154	—	1	—	—	—	1	2
155—159	—	2	1	—	—	—	3
160—164	1	1	18	4	3	—	27
165—169	—	4	16	9	3	1	33
170—174	1	2	18	33	14	3	71
175—179	—	—	6	4	7	—	17
180—184	—	—	—	4	1	1	6
Totals	2	10	59	54	28	6	159

TABLE 31.

Bohemians. Brain-weight and Skull Breadth. Males. 20—59 years.

Brain-weight (grams).

Skull breadth (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	1800—1899	Totals
110—114	—	—	—	—	—	1	—	—	—	1
115—119	—	—	—	—	—	—	—	—	—	—
120—124	—	—	—	—	—	—	—	—	—	—
125—129	—	—	—	—	—	—	—	—	—	—
130—134	—	—	—	—	2	1	—	—	—	3
135—139	—	—	4	2	1	1	—	—	—	8
140—144	1	—	8	19	21	7	1	1	—	58
145—149	—	—	8	21	32	17	—	—	—	78
150—154	—	—	1	22	35	26	11	—	—	95
155—159	—	—	—	2	9	14	11	2	2	40
160—164	—	—	—	—	1	6	2	2	—	11
165—169	—	—	—	—	—	3	—	1	—	4
170—174	—	—	—	—	—	—	—	—	—	—
175—179	—	—	—	—	—	1	—	—	—	1
Totals	1	—	21	66	101	77	25	6	2	299

TABLE 32.

Bohemians. Brain-weight and Skull Breadth. Females. 20—59 years.

Brain-weight (grams).

Skull breadth (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
130—134	—	—	4	1	1	—	6
135—139	2	4	9	5	—	—	20
140—144	—	6	28	19	7	—	60
145—149	—	—	14	20	6	4	44
150—154	—	—	4	6	11	1	22
155—159	—	—	—	2	3	1	6
160—164	—	—	—	1	—	—	1
Total	2	10	59	54	28	6	159

TABLE 33.

Bohemians.   Skull Length and Skull Breadth.   Males.

20—59 *years.*

Skull length (millimetres).

Skull breadth (millimetres).	155—159	160—164	165—169	170—174	175—179	180—184	185—189	190—194	195—199	Totals
110—114	—	—	—	1	—	—	—	—	—	1
115—119	—	—	—	—	—	—	—	—	—	—
120—124	—	—	—	—	—	—	—	—	—	—
125—129	—	—	—	—	—	—	—	—	—	
130—134	—	—	2	—	—	1	—	—	—	3
135—139	—	1	3	3	1	—	—	—	—	8
140—144	—	6	12	14	8	17	1	—	—	58
145—149	2	5	14	17	24	10	5	1	—	78
150—154	—	2	10	25	30	20	6	2	—	95
155—159	—	—	1	5	11	11	5	4	3	40
160—164	—	—	—	3	5	1	—	2	—	11
165—169	—	—	—	—	—	1	2	1	—	4
170—174	—	—	—	—	—	—	—	—	—	—
175—179	—	—	—	—	—	—	—	—	1	1
Totals	2	14	42	68	79	61	19	10	4	299

TABLE 34.

Bohemians.   Skull Length and Skull Breadth.   Females.

20—59 *years.*

Skull length (millimetres).

Skull breadth (millimetres).	150—154	155—159	160—164	165—169	170—174	175—179	180—184	Totals
130—134	—	1	1	—	3	1	—	6
135—139	1	1	4	4	7	2	1	20
140—144	—	1	15	13	25	5	1	60
145—149	1	—	3	12	20	6	2	44
150—154	—	—	4	3	11	2	2	22
155—159	—	—	—	1	4	1	—	6
160—164	—	—	—	—	1	—	—	1
Totals	2	3	27	33	71	17	6	159

TABLE 35.

Bavarians. Brain-weight and Stature. "Total," Short Series. Males.

Stature (centimetres).

Brain-weight (grams).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—189	Totals
1000—1049	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	2
1050—1099	—	—	—	—	—	1	1	—	2	—	—	—	—	—	—	4
1100—1149	—	—	—	—	1	—	—	1	2	—	—	—	—	—	—	4
1150—1199	—	1	—	1	2	3	—	1	1	1	1	—	—	—	—	11
1200—1249	—	—	1	2	3	6	6	11	4	1	2	1	1	—	—	38
1250—1299	—	—	—	1	4	7	6	9	13	5	5	1	—	—	—	51
1300—1349	—	—	—	3	6	10	7	18	11	3	—	4	—	—	—	62
1350—1399	—	—	1	3	5	6	12	13	16	7	8	1	1	1	—	74
1400—1449	1	—	2	1	2	5	4	14	10	5	2	2	1	—	—	49
1450—1499	—	—	—	1	4	—	5	8	5	6	2	2	—	—	—	33
1500—1549	—	—	—	—	2	1	—	5	4	1	6	1	—	—	1	21
1550—1599	—	—	—	—	1	—	—	—	3	1	1	2	—	—	—	8
1600—1649	—	—	—	—	—	—	2	3	1	—	—	—	—	—	—	6
1650—1699	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	2
Totals	1	1	4	12	31	39	44	83	72	31	28	14	3	1	1	365

TABLE 36.

Bavarians. Brain-weight and Stature. "Total," Short Series. Females.

Stature (centimetres).

Brain-weight (grams).	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—177	Totals
800— 849	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
850— 899	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
900— 949	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
950— 999	—	—	—	—	—	1	—	1	2	—	—	—	—	—	—	4
1000—1049	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
1050—1099	—	—	—	—	1	3	2	2	4	2	—	—	—	—	1	15
1100 —1149	—	—	—	1	3	6	7	6	7	3	2	—	—	—	—	35
1150—1199	—	—	—	1	2	4	9	4	15	4	2	—	—	—	1	42
1200—1249	—	1	1	—	4	5	7	9	10	8	1	1	—	—	—	47
1250—1299	1	—	—	—	6	6	13	5	9	7	3	—	1	—	—	51
1300—1349	—	—	—	—	1	4	1	6	1	3	1	1	1	—	—	19
1350—1399	—	—	—	—	—	1	1	1	4	6	1	—	—	—	—	14
1400—1449	—	—	—	—	—	—	—	1	2	1	—	—	—	—	—	4
1450—1499	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
1500—1549	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	3
1550—1599	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	1	1	2	17	31	41	36	54	37	11	3	2	—	3	241

TABLE 37.

Bavarians. Brain-weight and Age. "Total," Long Series. Males.

Brain-weight (grams).

Age (years)	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	Totals
20—24	—	—	—	1	3	3	6	10	5	1	3	—	—	—	32
25—29	—	—	—	—	6	7	8	15	8	5	—	3	2	—	54
30—34	—	1	—	2	7	12	10	18	9	7	2	6	3	1	78
35—39	—	2	1	4	10	14	22	14	16	7	8	3	1	2	104
40—44	—	1	—	—	7	5	4	11	13	8	2	—	1	—	52
45—49	—	—	1	—	5	6	9	10	3	6	3	1	1	—	45
50—54	—	—	—	1	5	10	7	14	9	4	3	2	3	—	58
55—59	—	—	1	1	3	4	8	8	6	1	3	—	—	2	37
60—64	—	—	—	2	6	2	2	6	4	3	1	—	1	—	27
65—69	1	—	—	2	1	3	6	4	1	1	—	—	—	—	19
70—74	—	—	1	2	—	5	2	2	2	1	—	—	—	—	15
75—79	1	—	—	1	1	3	—	1	—	1	—	—	—	—	8
Totals	2	4	4	16	54	74	84	113	76	45	25	15	12	5	529

TABLE 38.

Bavarians. Brain-weight and Age. "Total," Long Series. Females.

Brain-weight (grams).

Age (years)	800—849	850—899	900—949	950—999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	Totals
20—24	—	—	—	—	—	6	2	10	9	11	7	1	—	—	—	1	47
25—29	—	—	—	—	—	—	6	13	9	14	3	3	1	1	1	—	51
30—34	—	—	—	—	1	1	6	13	15	7	4	3	1	—	—	—	51
35—39	—	—	—	—	—	6	1	4	9	13	7	3	2	—	—	—	45
40—44	—	—	—	—	—	1	2	2	5	4	2	2	1	—	2	—	21
45—49	—	—	—	1	—	—	5	3	7	6	—	1	—	—	—	—	23
50—54	1	—	—	1	—	3	6	4	6	4	1	2	—	—	—	—	28
55—59	—	—	—	—	—	—	1	1	2	5	—	—	1	—	—	—	10
60—64	—	—	—	1	1	1	4	1	3	2	2	2	—	—	—	—	17
65—69	—	—	1	—	2	1	3	3	1	3	—	1	—	—	—	—	15
70—74	1	—	—	—	—	3	1	1	2	—	—	1	—	—	—	—	9
75—79	—	—	—	1	—	—	4	1	—	—	—	—	—	—	—	—	6
Totals	2	—	1	4	4	22	41	56	68	69	26	19	6	1	3	1	323

TABLE 39.

Bavarians. Brain-weight and Age. " Young," Long Series. Males.

Brain-weight (grams).

Age (years)	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	Totals
20—24	—	—	1	3	3	6	10	5	1	3	—	—	—	32
25—29	—	—	—	6	7	8	15	8	5	—	3	2	—	54
30—34	1	—	2	7	12	10	18	9	7	2	6	3	1	78
35—39	2	1	4	10	14	22	14	16	7	8	3	1	2	104
40—44	1	—	—	7	5	4	11	13	8	2	—	1	—	52
45—49	—	1	—	5	6	9	10	3	6	3	1	1	—	45
Totals	4	2	7	38	47	59	78	54	34	18	13	8	3	365

TABLE 40.

Bavarians. Brain-weight and Age. " Young," Long Series. Females.

Brain-weight (grams).

Age (years)	950—999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	Totals
20—24	—	—	6	2	10	9	11	7	1	—	—	—	1	47
25—29	—	—	—	6	13	9	14	3	3	1	1	1	—	51
30—34	—	1	1	6	13	15	7	4	3	1	—	—	—	51
35—39	—	—	6	1	4	9	13	7	3	2	—	—	—	45
40—44	—	—	1	2	2	5	4	2	2	1	—	2	—	21
45—49	1	—	—	5	3	7	6	—	1	—	—	—	—	23
Totals	1	1	14	22	45	54	55	23	13	5	1	3	1	238

TABLE 41.

Bavarians. Brain-weight and Body-weight. " Total," Short Series. Males.

Body-weight (kilograms).

Brain-weight (grams).	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	Totals
1000—1049	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
1050—1099	—	---	—	2	1	—	1	—	—	—	—	—	—	—	4
1100—1149	—	—	—	2	1	1	—	—	—	—	—	—	—	—	4
1150—1199	—	2	—	4	2	1	1	1	—	—	—	—	—	—	11
1200—1249	—	5	9	7	7	5	3	2	—	—	—	—	—	—	38
1250—1299	—	1	4	12	10	8	7	2	2	3	1	1	—	—	51
1300—1349	—	2	8	16	10	13	7	5	—	1	—	—	—	—	62
1350—1399	1	1	8	9	18	10	11	9	2	3	—	1	—	1	74
1400—1449	—	3	7	5	12	5	4	7	4	2	—	—	—	—	49
1450—1499	—	2	2	7	6	5	6	1	4	—	—	—	—	—	33
1500—1549	—	2	2	2	5	1	3	2	1	1	1	—	—	1	21
1550—1599	—	—	—	1	1	2	1	2	1	—	—	—	—	—	8
1600—1649	—	—	1	2	2	—	1	—	—	—	—	—	—	—	6
1650—1699	—	—	1	—	—	—	—	—	—	—	1	—	—	—	2
Totals	1	18	43	69	76	51	45	31	14	10	3	2	—	2	365

TABLE 42.

Bavarians. Brain-weight and Body-weight. " Total," Short Series. Females.

Body-weight (kilograms).

Brain-weight (grams).	15—19	20—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—90	Totals
800— 849	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
850— 899	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
900— 949	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
950— 999	—	—	1	—	1	—	—	1	—	—	—	1	—	—	—	4
1000—1049	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	2
1050—1099	—	—	2	3	5	3	1	—	1	—	—	—	—	—	—	15
1100—1149	—	1	1	8	7	9	3	3	—	2	—	—	—	—	1	35
1150—1199	—	—	8	6	9	5	8	2	3	1	—	—	—	—	—	42
1200—1249	1	—	—	5	10	6	12	5	5	1	1	1	—	—	—	47
1250—1299	—	1	5	4	11	10	10	6	3	1	—	—	—	—	—	51
1300—1349	—	—	—	1	7	3	2	2	1	2	—	1	—	—	—	19
1350—1399	—	—	2	1	2	3	4	—	1	—	1	—	—	—	—	14
1400—1449	—	—	—	—	2	—	1	—	—	—	—	1	—	—	—	4
1450—1499	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
1500—1549	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	3
1550—1599	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	2	22	28	54	40	42	22	14	8	3	4	—	—	1	241

TABLE 43.

Bavarians. Body-weight and Stature. "Total," Short Series. Males.

Stature (centimetres).

Body-weight (kilograms).

	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	Totals
25—29	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
30—34	—	—	—	2	5	4	1	4	2	—	—	—	—	—	—	18
35—39	1	—	3	—	4	7	7	16	3	2	—	—	—	—	—	43
40—44	—	1	—	7	8	9	5	19	12	4	1	2	1	—	—	69
45—49	—	—	1	1	5	5	17	17	15	3	6	6	—	—	—	76
50—54	—	—	—	—	4	9	4	13	10	7	3	1	—	—	—	51
55—59	—	—	—	1	2	4	3	7	16	6	2	1	2	—	1	45
60—64	—	—	—	1	1	—	4	4	8	4	5	3	—	1	—	31
65—69	—	—	—	—	—	—	1	1	3	3	6	—	—	—	—	14
70—74	—	—	—	—	1	1	1	2	3	—	1	1	—	—	—	10
75—79	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
80—84	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
85—89	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
90—94	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
Totals	1	1	4	12	31	39	44	83	72	31	28	14	3	1	1	365

TABLE 44.

Bavarians. Body-weight and Stature. "Total," Short Series. Females.

Body-weight (kilograms).

Stature (centimetres).

	15—19	20—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—90	Totals
132—134	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
135—137	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
138—140	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
141—143	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
144—146	—	1	3	4	5	1	3	—	—	—	—	—	—	—	—	17
147—149	—	1	7	5	6	4	6	1	—	1	—	—	—	—	—	31
150—152	—	—	2	9	11	9	5	3	1	1	—	—	—	—	—	41
153—155	—	—	2	2	12	5	5	3	4	1	1	1	—	—	—	36
156—158	—	—	2	5	12	11	11	8	3	—	—	2	—	—	—	54
159—161	—	—	1	1	5	8	8	7	6	1	—	—	—	—	—	37
162—164	—	—	2	—	2	1	2	—	—	2	1	—	—	—	1	11
165—167	—	—	—	1	—	—	—	—	—	1	—	1	—	—	—	3
168—170	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
171—173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174—177	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	3
Totals	1	2	22	28	54	40	42	22	14	8	3	4	—	—	1	241

TABLE 45.

English.   Weight of Cerebrum and Stature.   Males.

Stature (inches).

	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	Totals
28	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
29	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2
30	—	—	—	—	1	—	—	—	—	1	1	1	—	—	—	4
31	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
32	—	—	—	—	2	—	—	1	—	—	—	—	—	—	—	3
33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
34	—	—	—	1	—	1	2	—	1	—	—	—	1	—	2	8
35	—	—	—	—	—	1	—	4	4	—	1	—	1	—	1	12
36	—	—	—	—	—	—	2	—	2	2	2	—	3	—	1	12
37	—	—	—	—	—	1	—	4	1	3	2	2	1	—	—	14
38	1	—	1	—	—	1	2	1	5	6	4	3	3	—	—	27
39	—	—	1	—	1	1	1	2	5	—	2	3	2	—	—	18
40	—	—	—	—	—	—	1	3	4	4	5	2	2	—	1	22
41	—	—	1	1	—	1	3	2	2	4	3	3	5	3	2	30
42	—	—	—	—	—	—	1	4	4	6	6	2	3	1	—	27
43	—	—	—	—	—	1	2	2	4	6	4	2	2	1	4	28
44	—	—	—	—	—	1	1	2	4	2	4	2	5	2	3	26
45	—	—	2	—	—	—	—	—	6	2	2	1	1	2	—	16
46	—	—	1	1	—	—	—	—	2	4	1	3	—	2	3	17
47	—	—	—	—	1	—	3	3	2	2	1	—	1	1	—	14
48	—	—	—	—	—	1	—	2	—	2	5	1	1	—	1	13
49	—	—	—	—	—	—	1	1	1	1	—	1	1	—	—	6
50	—	—	—	—	1	—	—	—	—	—	1	—	—	1	—	3
51	—	—	—	—	—	—	—	—	—	1	1	2	—	—	—	4
Totals	2	—	6	3	6	9	20	31	47	47	44	28	34	13	18	308

(left margin) Weight of Cerebrum (ounces).

TABLE 46.

English.   Weight of Cerebrum and Age.   Males.

Weight of Cerebrum (ounces).

	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	Totals
20—29	1	1	1	—	—	—	1	1	1	2	2	2	2	6	1	6	5	—	6	2	6	1	—	2	49
30—39	—	—	1	—	—	—	3	2	3	2	3	3	4	4	3	5	4	5	4	4	2	3	—	—	55
40—49	—	—	2	1	1	—	2	2	—	3	6	6	4	4	3	3	6	3	1	2	3	2	2	2	58
50—59	—	—	—	—	—	—	—	—	4	2	2	4	8	4	6	4	4	2	4	4	1	—	—	—	49
60—69	—	1	—	—	1	—	2	3	2	2	6	1	3	8	6	4	3	5	1	1	—	—	—	—	49
70—79	—	—	—	—	1	—	—	4	2	3	8	2	1	4	8	6	4	1	1	1	1	—	1	—	48
Totals	1	2	4	1	3	—	8	12	12	14	27	18	22	30	27	28	26	16	17	14	13	6	3	4	308

(left margin) Age (years).

A STUDY OF THE RELATIONS OF THE BRAIN TO THE SIZE OF THE HEAD.

By REGINALD J. GLADSTONE, M.D.

ONE of the main objects of the investigation which forms the subject of this paper, has been to obtain a series of reconstruction formulæ, by which it will be possible, when in possession of certain chief measurements of the head, to predict within the limits of normal variation, the approximate weight of the brain.

The collection of data for this purpose, which has occupied a period of over three years, has been carried out with the permission of the Resident Medical Officer, in the 'post-mortem' room of the Middlesex Hospital, whereas the mathematical treatment, in the following paper, is chiefly the work of Mr J. Blakeman, M.Sc., of University College, London.

Briefly, the method which we have adopted has been to measure the heads of 'post-mortem' room subjects, and afterwards in each case to take out the brain and weigh it. In each case, therefore, a comparison can be made between the outside measurements of the head, and the weight of the brain.

The individuals measured have been a few presumably healthy subjects, who have died an accidental death; the ordinary subjects of post-mortem examination at the Middlesex Hospital, London; and a fairly large proportion of subjects who have died from malignant disease in the Cancer Department of the same Hospital. They may be regarded, therefore, as forming a fairly typical sample of what has been termed a 'hospital population'; containing, however, owing to the inclusion of 'cancer cases,' a rather larger proportion of individuals who have died from wasting diseases than is ordinary. A comparison, however, of the ratio that the brain-weight bears to the outside measurements of the head in a series of cases in which death took place from accident, or acute illness, with cases of death from wasting diseases, has shown that the diminution of brain-weight in the latter, although measurable, is small; and that their inclusion, therefore, with the acute cases does not seriously affect the general result.

No case has been included in which the brain showed a distinctly pathological condition which would have obviously affected its weight; nor have individuals with foreign or Jewish names been included.

For permission to make use of this valuable material, and for assistance in many ways, I am indebted to Dr R. A. Young, Pathologist, and Lecturer on Morbid Anatomy at the Middlesex Hospital, and to Mr W. T. Hillier, Pathological Assistant in the Cancer Department of the Middlesex Hospital.

In each case the following items have been recorded: (*a*) name, (*b*) age, (*c*) sex, (*d*) cause of death, (*e*) remarks: on general condition, degree of emaciation, &c., (*f*) stature, (*g*) measurements of head:

These were : L, the length of the head from the glabella to the occipital point.

B, the maximum transverse diameter of the head above the level of the zygomatic arches.

H, the height of the cranium as indicated by the vertical distance from the biauricular line * to the bregma.

U, the horizontal circumference, taken in a plane passing in front through a point just above the glabella, and behind through the occipital point.

S, the longitudinal or sagittal arc, measured from the glabella, over the vertex to the external occipital protuberance.

Q, the transverse or coronal arc, measured from the tragus of one side, over the vertex to the tragus of the opposite side.

These measurements of the head having been obtained, the scalp was reflected, and ($\mathfrak{L}$) the length, ($\mathfrak{B}$) the breadth, ($\mathfrak{H}$) the height, and ($\mathfrak{U}$) the circumference were measured on the bared skull. The vault of the skull was then removed, and the brain taken out and weighed in the usual manner, without removing the pia and arachnoid membranes†, the weight being recorded in grammes‡.

The longitudinal and transverse diameters of the head have been taken with a Flower's craniometer, made by Aston and Mander, of 25 Old Compton Street,

---

* Taken at the *centre* of the auricular orifices.

† The pia and arachnoid membranes vary considerably in their total weight and an increase in their weight accompanies an increase in age; thus according to Broca, the weight of the 'pia' at different ages in males, averages

20—30 years ...	... 45 grammes
31—40 ,, ...	... 50 ,,
60 ,, ...	... 60 ,,

The variations ranged between 38 and 130 grammes. The mean weight of the 'pia' in the case of 133 females was 48·7 grammes, and of 273 males 55·8 grammes. The term 'pia' as used by Broca obviously includes both the pia and arachnoid membranes.

‡ The weight of the brain in ounces was in a large number of cases also ascertained, and recorded in the Hospital Reports; and a reference to these reports proved to be a useful means of verifying the recorded weight in grammes, of certain cases in which there was a considerable deviation from the usual relation between the brain-weight, and the outside measurements of the head.

London, and the vertical diameter by an instrument, Fig. 1, similar to the one which I have previously described and figured in the *Report of the Proceedings of the Anatomical Society*, November, 1901 ; the instrument which we have used for the post-mortem measurements has, however, been improved by the adoption of a suggestion made by Mr J. Gray, namely, the substitution of a vertical screw to act upon the indicator, for the rack and pinion with which the first instrument was fitted. Any slipping of the measuring rod, from the instrument working loose, is thus avoided.
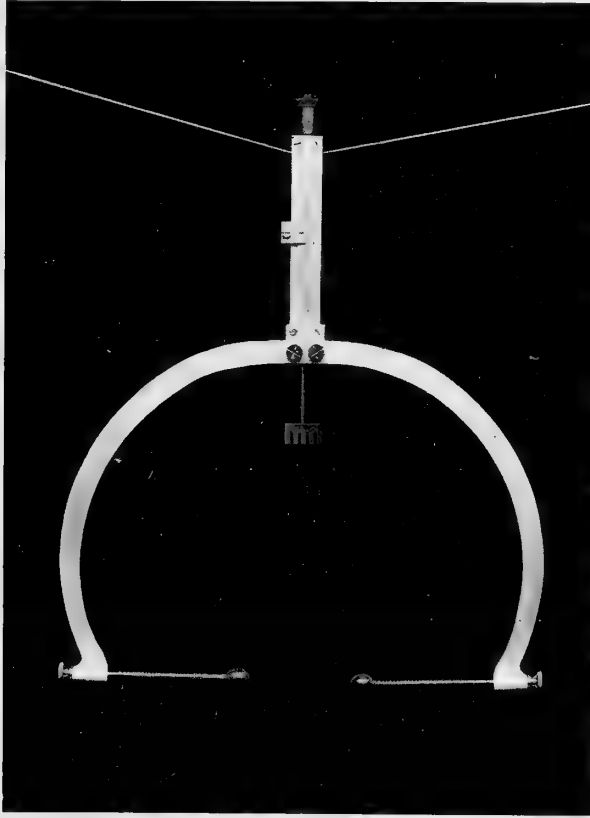
Fig. 1. Instrument for measuring the vertical height of the head, from the biauricular line to the vertex.

The circumference and the longitudinal and transverse arcs have been taken with a Chesterman's steel tape-measure.

Although the longitudinal and transverse arcs were measured, and are preserved in each case, we have not made use of them in constructing any of the tables or formulæ. The diameters being in our opinion the more trustworthy measurements, since they are less influenced by variations in the amount and thickness of the hair, and because the points between which the measurements are

taken are in the case of the diameters much more precise. In many individuals the external occipital protuberance, even when the muscles of the neck are thoroughly relaxed, is difficult to localize. This is the case in the living subject, and still more in the cadaver, so much so that in some subjects it is impossible to determine its exact position. Moreover, when prominent and easily felt, there may be as much as 1 cm. difference between the measurement taken by one individual and that by another, according to whether the top or bottom of the projection has been taken as the starting point. The small tubercle on the tragus from which the measurement is taken in recording the transverse arc is also relatively to the centre of the external auditory meatus, a somewhat variable point. I regard these measurements, therefore, as less reliable than the diameters. The measurement of the arcs and circumference has this important advantage, however—they can
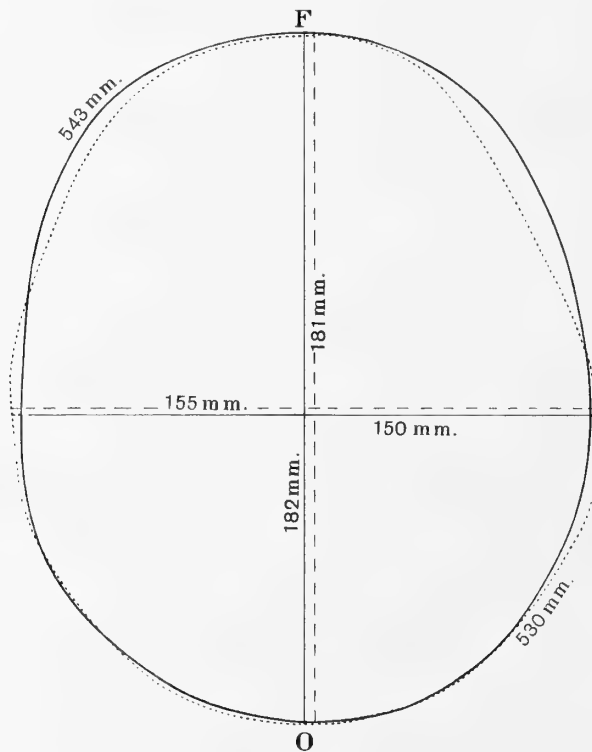
Fig. 2.

be taken by a tape measure, which can be carried in the waistcoat pocket, and they can thus be more readily ascertained by travellers who would be unable to carry a more bulky instrument. Moreover the measurement of the longitudinal and transverse arcs, and of the horizontal circumference, will in some cases give a truer estimate of size than the greatest diameters taken through the principal axes, as will be seen by the accompanying Fig. 2, which shows tracings of the horizontal circumference of two heads, one of which, represented by a continuous line, was narrow in the frontal region, but wide in the parietal, whereas the other, represented

by broken lines, although having a wider frontal region, had a less maximum transverse or interparietal diameter. The total area of the latter is greater than the former, as is indicated by the circumference, which is 543 mm. in the broken tracing as compared with 530 mm. in the continuous. The product of the two principal diameters of the continuous tracing ($L = 181 \times B = 155$) is 28,055, whereas that of the broken curve ($L = 182 \times B = 150$) is 27,300, or 755 less than the continuous. The product of the diameters thus gives in this case an erroneous indication of the area enclosed by the tracings[*]. We have accordingly made use of the horizontal circumference, although discarding the longitudinal and transverse arcs for the reasons stated above.

The correlation between brain-weight and the outside measurements of the *head* is naturally not nearly so close as that which exists between the outside measurements of the *skull* and the capacity of the *skull*. For in addition to variations in the thickness and shape of the skull, we have to deal with variations in the thickness of the scalp and hair, and also with the very considerable variations in the amount of space which exists between the surface of the brain and the internal surface of the skull. Moreover the weight of the brain is modified by variations in density[†], the size of the cerebral ventricles[‡], and by the amount of blood contained in its vessels.

The weight of the brain also, as is well known, varies considerably with age, a progressive decrease of weight and most probably also of size taking place after middle age. This decrease, however, appears according to both Boyd's[§] and Vierordt's[||] statistics, to commence at a much earlier age, the highest average brain-weights for different ages occurring between 14 and 20 in both sexes.

The brain in children and in youth is both actually and relatively to the body weight and stature very large. According to Vierordt's statistics, the average brain-weight of 35 boys, between 12 and 16 years of age inclusive, was as much as 1423 grammes; and in a table published by the same author, showing the average brain-weight for each year from birth up to the age of 25, the highest average brain-weights, 1490 grm. ♂, and 1345 grm. ♀, occur at the ages of 15 and 14 respectively; whereas, according to Boyd's statistics, the average brain-weights of adult British subjects (mostly of the lower classes), are :

<div align="center">

Males between 20 and 40 ...... 1360 grm. or 48 oz.

Females „ 20 and 40 ...... 1230 grm. or $43\frac{1}{2}$ oz.

</div>

[* The problem really is which gives the better *average* result. It may be worth noting that the volume of an ellipsoid is proportional to the diametral product, but not to the product of the three circumferences. ED.]

† According to Professor Donaldson's statistics, "the average specific gravity of the entire encephalon should be for the adult male 1·0363, and for the adult female 1·0360." H. H. Donaldson: *Growth of the Brain*, p. 95.

‡ "The cast of the ventricles as made by Welcker displaces 26 cm.³ of water so that the fluid filling such a cavity would weigh a trifle over 26 grammes." H. H. Donaldson : *Growth of the Brain*, p. 87. The specific gravity of the cerebro-spinal fluid is about 1·009.

§ See table compiled from the observations of R. Boyd, *Phil. Trans.* 1860, in Quain's *Anatomy*, Vol. III. Part I. p. 178.

|| Table 17, p. 104, H. H. Donaldson: *The Growth of the Brain*.

The maximum, average and minimum weights of the brain, in four groups, arranged according to age and sex, of the ' post-mortem ' room subjects of the Middlesex Hospital, may be seen in the following table :

The weight of the brain is expressed in grammes, and each group contains 50 subjects.

Age	Males			Females		
	Max.	Aver.	Min.	Max.	Aver.	Min.
20—46 ...	1635	1370·5	1207	1520	1223·5	1027
46 upwards ...	1588	1316·1	1120	1408	1195·7	955

It will be noticed that in passing from the younger to the older groups there is a decrease in the average brain-weight of 54·4 grm. in the male, and of 27·8 grm. in the female, the mean decrease for both male and female being approximately 40 grammes.

This result corresponds very closely with the table compiled from the observations of R. Boyd, and published in Quain's *Anatomy*, Vol. III. Part I. p. 178, in which it is shown that " the brain is absolutely heavier between 14 and 20 years of age than at any other period of life, and that at the age of 80 it has lost about 90 grammes, or rather more than 3 ozs., i.e., $\frac{1}{15}$ of its total weight."

Not only is there a marked change in brain-weight corresponding with different periods of life, but there are also considerable variations in the thickness of the skull and its coverings.

The scalp is thin in infants and young children, and a sensible attenuation of the scalp occurs in old age, which is probably largely due to atrophy of the hair follicles following the loss of hair.   The amount, although small, is measurable, and is sufficient to affect the general result of a statistical investigation.   The amount of atrophy may be seen by a comparison of the figures in the following tables, which show the average differences between the diameters of the head and the diameters of the skull as measured before and after the scalp has been turned down in order to remove the ' skull cap.'

18 *Males between* 20 *and* 46 *years of age.*

<div align="center">

mm.

Height of head minus height of skull = 4·05.

Length   „   „   length   „   = 8·47.

Breadth   „   „   breadth   „   = 8·27.

</div>

27 *Males from* 46 *years upward.*

<div align="center">

mm.

Height of head minus height of skull = 3·79.

Length   „   „   length   „   = 7·25.

Breadth   „   „   breadth   „   = 7·40.

</div>

17 *Females from* 20—46 *years of age.*

                                                   mm.
Height of head minus height of skull=3·82.
Length   „   „   length   „   =7·52.
Breadth   „   „   breadth   „   =7·88.

27 *Females from* 46 *years upward.*

                                                   mm.
Height of head minus height of skull=3·50.
Length   „   „   length   „   =7·12.
Breadth   „   „   breadth   „ .   =6·98.

It will be observed that the scalp is thinner in female subjects than in males and that there is a diminution in the thickness of the scalp, which amounts to :

♂ : 0·26 mm. ♀ : 0·32 mm. at the vertex.
  1·22 mm.   0·40 mm. at the glabella and occipital point together.
  0·87 mm.   0·90 mm. on the two sides.

There is thus a diminution in the length of the principal diameters of the head, attributable to the atrophy attendant on old age, which amounts if the mean of both sexes be taken to :

0·29 mm. in the vertical diameter.
0·81 mm.   „   longitudinal   „
0·885 mm.   „   transverse   „

The thickness of the skull also varies with age, as will be noted on comparing the cut surface of the vault of a skull shown in Plate II. Fig. D, which is a photograph of the vault of the skull of a child about four years old, with that in Fig. C, which is the photograph of the vault of a normal adult skull.

The difference in thickness of the skull of a child, and that of an adult will also be seen on comparing Fig. A, Plate II. with Fig. B on the same plate. Fig. A represents the right half of the skull of a child about five or six years of age; Fig. B the right half of a thick, adult skull. Note also the absence of frontal sinuses in Figs. A and C.

The extent to which the thickness of the skull may be affected by disease, may be seen in Fig. 3, which represents part of the vault of a skull, from a case of ' osteitis deformans *,' the walls of which averaged about one inch in thickness. The increase in thickness of the skull in these cases takes place partly on the outer and partly on the inner surface of the skull, so that although the size of the head is enlarged the capacity of the skull is diminished, and compression exerted on its contents.

The reverse condition is met with in cases of chronic hydrocephalus in which the size of the head is increased by an augmentation in the volume of its contents.

* Specimen 1239. Museum of the Royal College of Surgeons, England.

An extreme case of this disease is seen in Figs. E and F in Plate III., which are photographs of specimen 3878 in the Hunterian Museum of the Royal College of Surgeons, England. It is described in the catalogue as " The skeleton of a man who died with hydrocephalus at the age of 25 years. Its greatest horizontal circumference is 914 cms. ! Besides the enlargement of the frontal and parietal bones by which the greater part of the enlarged cranial cavity is formed rows of Wormian bones from 2·5 to 3·8 cms. in breadth are placed in the whole course of the lambdoid and sagittal sutures, and in great part of the length of the squamous sutures."

Fig. 3.

The enlargement of the head in hydrocephalus is associated with an increase in the amount of the cerebro-spinal fluid, which is found either in the space between the arachnoid and pia mater, outside the brain, within the cerebral ventricles, or in both these situations. In the case of James Cardinal, the skeleton of whom is preserved in the Museum of Guy's Hospital, seven pints of cerebro-spinal fluid were found between the membranes, whereas the ventricles contained one pint. In this case " it appeared that the fluid had been originally contained within the ventricles, but had burst through an opening in the corpus callosum *."

It is obvious that both in the case of *osteitis deformans*, and *hydrocephalus* the size of the brain cannot be gauged by the size of the head.

The enlargement of the head in hydrocephalus is also of interest, as it indicates one means by which the size of the cranium is increased in the course of normal growth, namely by an expanding force acting from within. In the early stages of development the fœtal brain has a smooth even surface, at a later stage from the 8th week of intrauterine life to the 4th month, what are called the ' transitory fissures' make their appearance ; these are infoldings of the thin walls of the

* See description of case in *Tumours, Innocent and Malignant*, p. 448, J. Bland Sutton.

hemisphere vesicles, which appear to be occasioned by the growth of the brain being more rapid than the capsule which confines it. At a still later stage following a more rapid enlargement of the skull many of the early fissures disappear, but they are afterwards replaced by the permanent fissures or sulci of the adult brain. The appearance and disappearance of these fissures seem to indicate that there are two counteracting forces concerned in the growth of the brain, namely an expanding force due to increase in brain matter, and cerebro-spinal fluid, and a passive restraining force exerted by the skull.

In anencephalic monsters the brain is imperfectly developed; the hemisphere vesicles appear at an early stage of development to have ruptured and collapsed, and the cerebro-spinal fluid to have escaped through a triangular aperture, which may often be recognized at the time of birth in the position of the anterior fontanelle. The cerebro-spinal fluid thus passes into the cavity of the amnion, and the amount of fluid within this sac becomes thereby much increased.

In these cases the expanding force concerned in the growth of the skull is absent, and consequently the bones forming the vault of the skull, if they are developed at all, grow only to a sufficient size to cover over the base of the skull, and rudiments of the brain, whereas the bones of the face, and of the skeleton generally, grow to their normal size.

In rickets on the other hand the bones of the skull are abnormally soft and yielding, and apparently do not afford the usual resistance to the expanding pressure from within, and as a result the cranial portion of the skull enlarges, and the well-known square overhanging forehead of a typical rickety child is produced, while the face remains of the normal size and owing to the enlargement of the head above, appears unusually small.

In the course of normal growth the two forces continue to counteract one another until the brain has attained its maximum size. Later a progressive diminution in the weight and probably also of the size of the brain takes place, and the sutures of the skull afterwards, gradually consolidate.

Not only is there the difference mentioned above in the general thickness of the skull at different ages, but there is also a considerable difference in the frontal region, due to the development after puberty of the frontal air sinuses, which tend to increase in size with the advance of age, and which are usually considerably larger in the male than in the female. Their position and the size which they may attain are well seen in Fig. 4, D, and Plate II., Figs. B and C.

It is obvious, therefore, that in any attempt to calculate the weight of the brain age must be taken into account, since it affects not only the brain itself, but also the thickness of the scalp and of the skull. It may be noted, however, that after middle age the variations attributable to age are so far as the thickness of the scalp and the weight of the brain are concerned, in the same direction, namely diminution, so that the proportion between the outside measurements of the head and the brain-weight is not very materially altered. The diminution in brain-
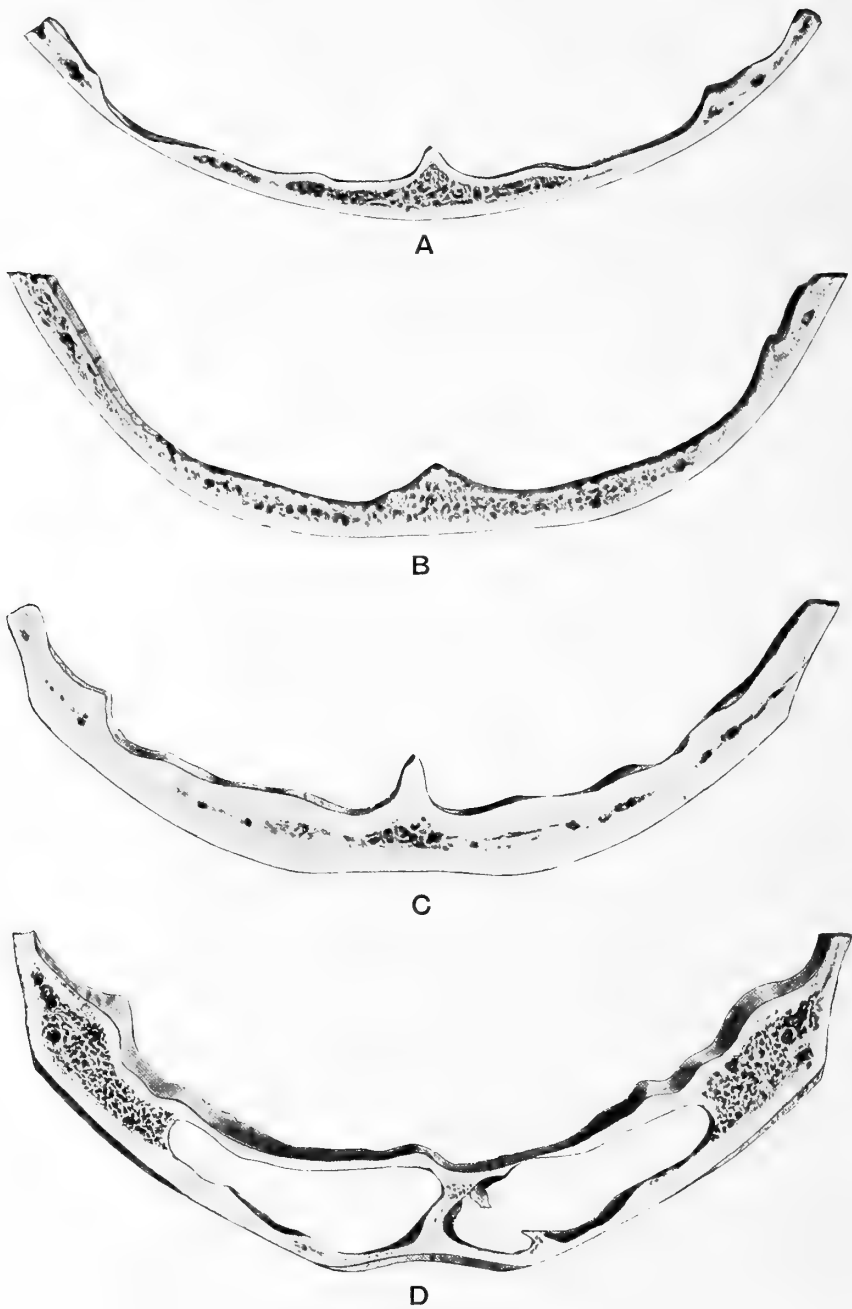
Fig. 4. Four horizontal sections through the frontal bone, showing the condition of the bone at various ages.

    A. Male, aged 9.   Bone thin, no frontal sinuses.
    B.   ,,   ,,   14.   Bone considerably thicker, no frontal sinuses.
    C.   ,,   ,,   32.   Illustrating the occasional absence of the frontal sinuses in an adult.
    D.   ,,   ,,   54.   Unusually large frontal sinuses.

From specimens in the Museum of the Royal College of Surgeons, London.

weight follows very closely the diminution in the size of the head, as is shown by the following table, which gives the proportion between these two quantities in 4 groups of 50, arranged according to age and sex. The size of the head is here indicated by a figure obtained from the product of the three principal diameters $(L \times B \times H)$ expressed in millimetres, the first four figures of this number represent the number of cubic centimetres which would be contained in a rectangular block having the same diameters as the cranial portion of the head, and I shall refer to it subsequently as the 'index of size' of the head, thus:

$$(196) \times (156) \times (138) = 4219488.$$

4219, the 'index of size' of the head, represents cubic centimetres and may therefore be conveniently compared with the weight of the brain expressed in grammes.

Males	H	L	B	Index of size	Brain-weight	Index of Size / Brain-weight	Number in each group
Age, 20—46 ...	134·8	190·8	149·5	3876	1370·5	2·806	50
„ 46 and upwards	132·4	189·8	148·7	3736	1316·1	2·838	50
Females							
Age, 20—46 ...	129·8	183·4	144·9	3449	1223·5	2·818	50
„ 46 and upwards	128·2	182·1	143·5	3350	1195·7	2·801	50

By combining the male with the female groups, we obtain the following ratios between the size of the head and the brain-weight:

$$\text{Age, 20–46} \quad . \quad . \quad . \quad 2\text{·}812$$
$$\text{„ 46 and upwards} \quad . \quad 2\text{·}819$$

In comparing the extreme ends of the series, however, it appears that the diminution in brain-weight, with the advance of age, is more rapid than the diminution in the size of the head; thus taking the mean of 4 ♂ and 3 ♀ cases over 70 years of age, the ratio between size of head and brain-weight is 2·821, whereas an equal number of cases between 20 and 25 years of age give 2·702.

The influence of stature upon the ratio between the outside measurements of the head and the brain-weight is apparently very small; but it has a marked influence on the actual size of the head as compared with the general mean, and a less but measurable influence on the proportion of the size of the head to the body in the individual. In other words, tall men in the aggregate have larger heads than short men, but proportionally to the size of their bodies their heads are considerably smaller than those of short men.

The formula which I have employed to express this relationship of the size of the head to the stature is the following:

$$100 \times \frac{\sqrt[3]{\text{Index of size}}}{\text{Stature in centimetres}} = \text{Capitulo-statural index.}$$

$$\text{Ex.} \quad \frac{\sqrt[3]{3921 \text{ cm.}^3} \times 100}{170 \text{ cm.}} = \frac{15\text{·}77 \text{ cm.} \times 100}{170 \text{ cm.}} = \frac{1577}{170} = 9\text{·}27.$$

It will be seen on referring to the table given below that a considerable diminution of the capitulo-statural index takes place with increase in stature; and also that the size of the head increases with the stature, but that the enlargement of the head does not keep pace with the increase in stature.

Table showing the proportion that the size of the head bears to the stature in groups of individuals arranged according to their height, the degrees of which are expressed in intervals of 3 inches.

Number of cases in each group	Stature		Index of size $(L \times B \times H)$ cm.^{3}	$\sqrt[3]{\text{Index of size}}$ cm.	Capitulo-statural index $\dfrac{\sqrt[3]{\text{Index of size}} \times 100}{\text{Stature}}$
	Inches	cm.			
37	74—72	185·3	4135	16·05	8·66
120	71—69	177·7	4071	15·97	8·98
124	68—66	170·0	3993	15·87	9·00
67	65—63	162·4	3902	15·75	9·69
15	62—60	154·8	3778	15·56	10·05

This table is based upon the measurements of 363 male subjects of all classes in England above 20 years of age, the larger number of these belong to the professional class, so that both the mean stature and mean size of the head are slightly above the general average.

The increase in size of the head, accompanying increase in stature, is shown also in the following chart, which is planned from the same figures as the preceding table:
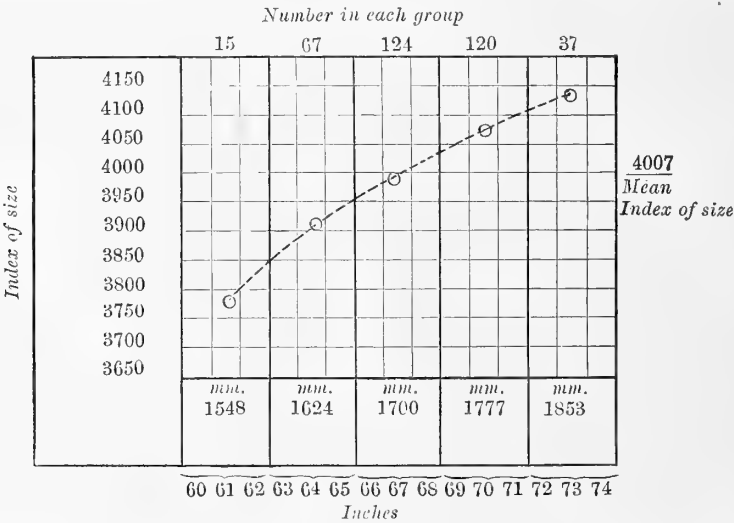
FIG. 5. Chart showing the increase in the size of the head which takes place with increase of stature. The chart is based on the measurements of 363 adult male subjects, above 20 years of age. The individuals have been grouped according to their stature, which is expressed in intervals of 3 inches. The figures indicating the size of the head are shown in the column on the left.

Plate II

E

F
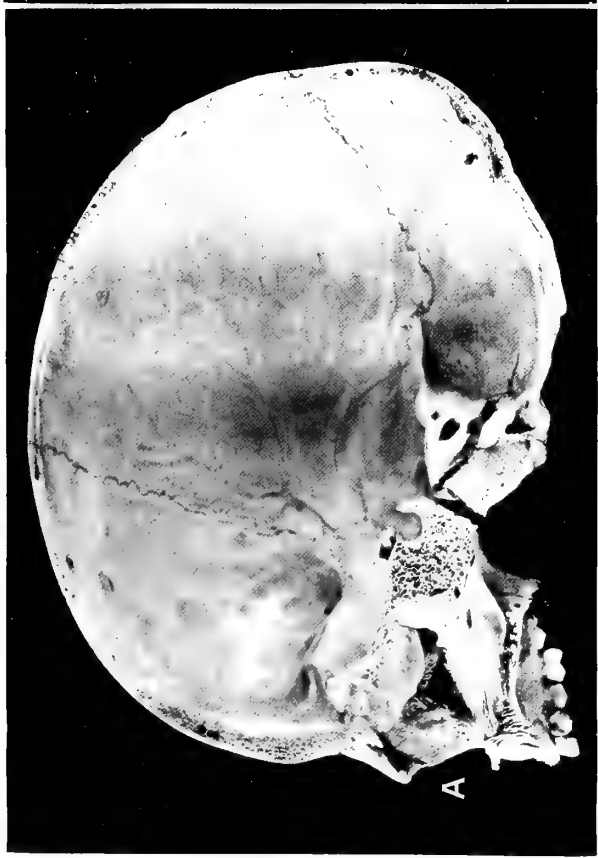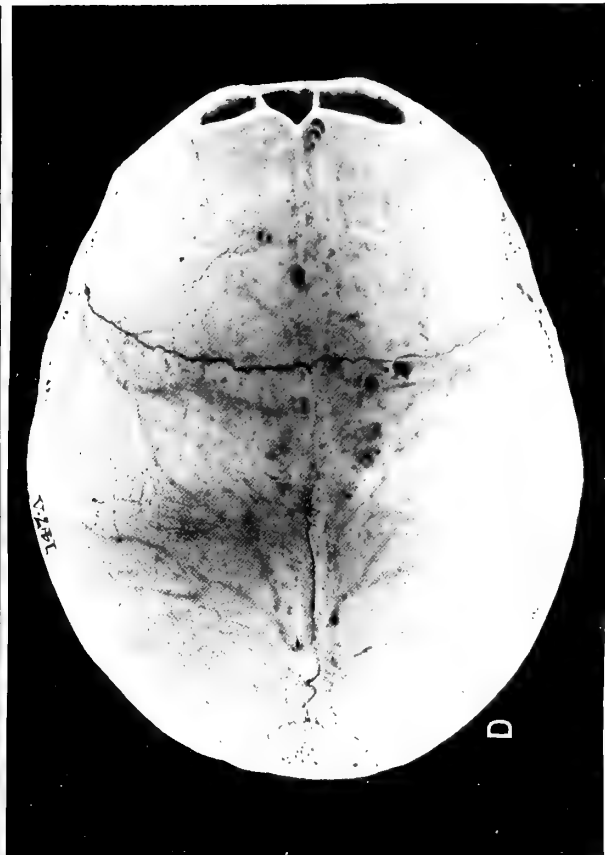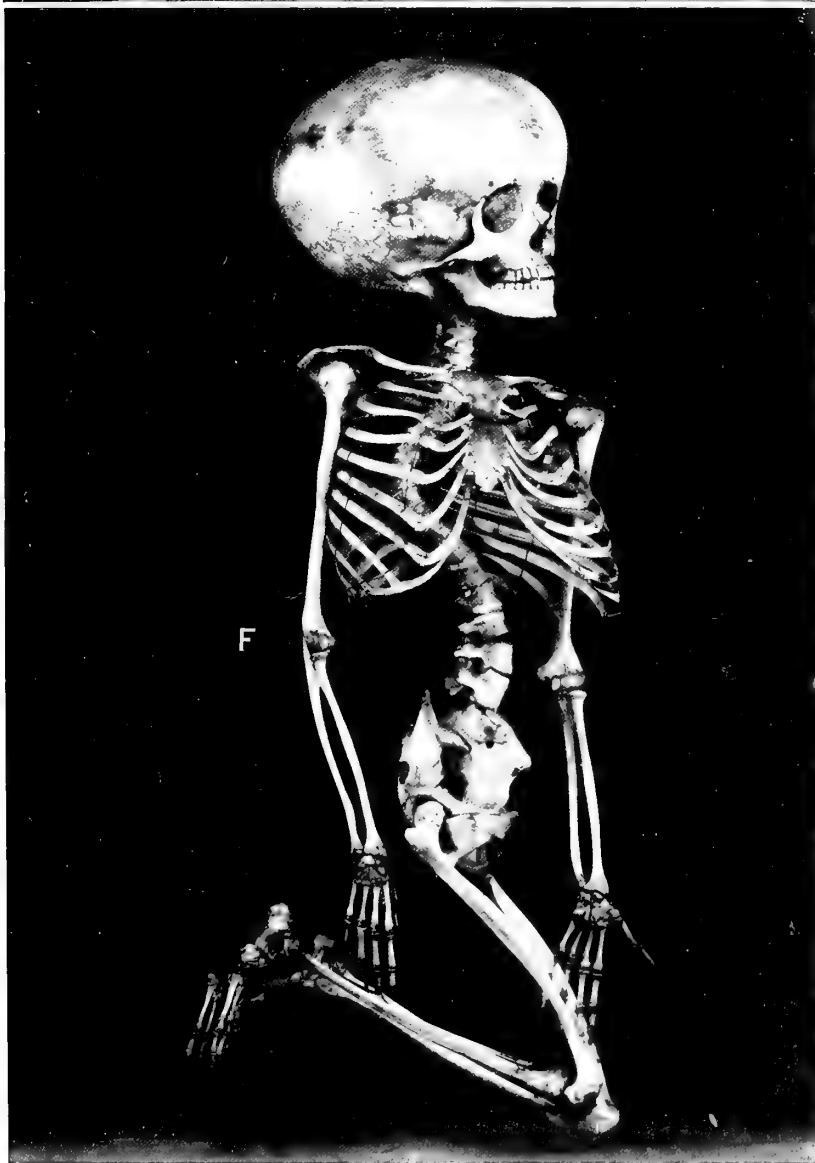
The following table is also of interest, as exemplifying the same law in certain extreme cases, in giants and dwarfs*.

	Stature			Index of size	$\sqrt[3]{\text{Index of size}}$	Capitulo-statural index $\dfrac{\sqrt[3]{\text{Index of size}} \times 100}{\text{Stature}}$
	ft.	in.	cm.			
Irish giant, O'Brian ...	7	7	231·1	4557	16·58	7·17
American giant, Freeman	6	9	205·7	4085	15·99	7·77
363 All classes, British, ♂	5	7	170·0	4007	15·88	9·33
Dwarf from Kiel, ♂ ...	4	0	121 9	2788	14·07	11·55
Dwarf from Holstein, ♂ ...	3	2½	97·8	2699	13·92	14·23

In conclusion I must express my thanks to Mr Freke Field, who has worked out most of the tables in this section of the paper, and to Mr J. W. Stockwell and Mr G. Fairclough, who have taken the majority of the measurements.

DESCRIPTION OF PLATES.

PLATE II. FIG. A. Longitudinal section through skull of a child between five and six years of age.

FIG. B. Longitudinal section through a normal adult skull. From the Anatomical Department, Middlesex Hospital.

FIG. C. Interior of the vault of the skull of an adult male. Note the thickness of the sawn edge of the bone, and the size of the frontal sinuses. From a specimen 417 A, in the Museum of the Royal College of Surgeons, London.

FIG. D. Interior of the vault of the skull of a child, aged about four. Note the thinness of the sawn edge of the bone, and the absence of the frontal sinuses. From a specimen in the Museum of the Royal College of Surgeons, London.

PLATE III. E skull, F skeleton of man of 25 years with hydrocephalus. From specimen 3878 in the Hunterian Museum of Royal College of Surgeons.

* For permission to measure the skeletons of the two giants, which are contained in the Hunterian Museum of the Royal College of Surgeons, England, I am indebted to Professor C. Stewart. In calculating the 'indices of size' of the heads, 4·5 mm. were added to the vertical diameter, and 9 mm. to the longitudinal and transverse diameters, as taken on the bare skull, thus allowing about 0·5 mm. on each diameter for shrinkage of the skull in drying. See table, p. 110, 'Males 20—46 years of age.'

APPENDIX OF MEASUREMENTS.

A. *Male Subjects between* 20 *and* 46. *Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cm.³) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
1	—	—	67	141	200	160	4512	1530	574	80·0	Chronic nephritis. Cardiac degeneration
2	A	39	68	137	182·5	149·5	3738	1297	550	81·9	Accident. Fractures of bones
3	C	35	71	139	201·	152·5	4261	1335	569	75·9	Pulmonary tuberculosis
4	C	35	70	134·5	193	145·5	3777	1282	542	75·4	Malignant growth affecting mouth
5	A	37	68	140	195	153	4177	1590	576	78·5	Accident. Fracture of skull
6	—	34	65	132	186	146	3585	1300	547	78·5	Cerebral hæmorrhage
7	C	38	68	137·5	192·5	143	3785	1400	549	74·2	Pulmonary tuberculosis. Skull thin
8	C	45	69·5	134·5	189	140	3559	1255	535	74·1	Carcinoma of stomach
9	—	45	68	132·5	185·5	147	3613	1355	555	79·2	Pneumonia. Pleural effusion
10	—	40	64	135	196	150·5	3982	1375	565	76·7	Cerebral hæmorrhage
11	C	21	66	132	185	141	3443	1340	537	76·2	Meningitis. Mastoid disease
12	—	42	63	136·5	197	148·5	3993	1380	570	75·4	Cellulitis of upper lip. Septicæmia
13	—	25	69·75	136	188·5	142	3640	1355	558	75·3	Basal meningitis. Pulmonary tuberculosis
14	A	23	66·5	138	198	154	4208	1522	576	77·8	Fracture of skull. "Large pear-shaped head"
15	—	45	71	132	191	152	3832	1208	561	79·6	
16	A	41	60·5	135	194	148	3876	1405	569	76·3	Fracture of skull [hæmorrhage
17	A	28	63·5	130	183	147	3497	1358	551	80·3	"Run over." Rupture of liver. Cerebral
18	C	45	65	130	184·5	144·5	3466	1292	541	78·3	Malignant growth. R. ext. auditory meatus
19	A	20	71·5	126	178	138	3095	1340	517	77·5	Burns. Congestion of brain
20	C	43	68	144	192	160	4424	1400	580	83·3	Fatty liver and kidneys. Delirium tremens
21	C	43	68·5	137	190	149	3878	1357	568	78·4	Tubercular nephritis. Thin skull
22	—	25	68·5	130	197	158	4046	1287	582	80·2	Fatty heart. Congestion of lungs
23	C	29	68·5	137·5	189·5	146	3804	1275	554	77·0	Morbus cordis
24	C	40	68·5	135	182	151	3710	1270	541	83·0	Carcinoma of œsophagus, and stomach
25	A	?	71	142	199	168	4747	1635	602	84·4	Impaction of food in pharynx
26	A	40	72·5	135	195	168	4423	1505	577	86·2	Inguinal hernia. Pulmonary embolism
27	—	29	69·25	135	198	151	4036	1490	564	76·3	Appendicitis. Thick skull. Foreign
28	—	35	70·5	138	193	151	4022	1485	570	78·2	
29	—	27	69·5	135	184	139	3454	1310	537	75·5	Appendicitis. General peritonitis
30	A	20	69	142	200	147	4175	1420	574	73·5	Purulent meningitis. Fracture of nasal bones
31	C	38	68·5	136	182	153	3787	1318	567	84·1	Malignant growth of kidney
32	—	24	69·75	135	190	148	3796	1432	564	77·9	Lymphatic leukæmia. Punctiform hæm. c'blm.
33	—	41	70·5	138	187	159	4103	1364	574	85·0	Lobar pneumonia. Enlgemt. of liver. Foreign
34	C	45	71	137	191	159	4161	1405	584	83·2	Chronic laryngitis. Thin skull. Scalp very thick
35	C	41	67	139	195·5	153	4158	1432	574	78·3	Malignant growth of lip. Metastasis
36	—	37	61	134	191	149	3814	1207	548	78·0	Fracture of tibia. Delirium tremens
37	—	29	65	135	184	142	3527	1375	530	77·2	Cerebral tumour
38	A	36	61·5	131	187	153	3748	1350	560	81·8	" Run over." Fractures of ribs and sternum
39	C	21	61	129	182	142	3334	1236	528	78·0	Pulmonary tuberculosis
40	C	38	62·5	129	192	141	3492	1250	540	73·4	Malignant growth in floor of mouth
41	C	42	61·5	134	194·5	152	3962	1350	558	78·1	Ulceration of larynx
42	C	43	62·5	129	190	143	3505	1320	539	75·3	Carcinoma of omentum. Emaciation
43	C	37	66	138·5	201	155	4315	1525	587	77·1	Cerebral abscess following empyema
44	A	31	67·5	130	190	154	3804	1570	550	81·1	Pulmonary embolism. Inguinal hernia
45	C	34	69	131·5	192	153	3863	1340	563	79·7	Diabetes
46	C	44	69	134	193	156	4034	1422	578	80·8	Cirrhosis of liver. Morbus cordis
47	C	24	69·5	142	192	158	4308	1506	569	82·3	Chronic nephritis. Œdema of lung. Foreign
48	C	28	68·5	124	181	141	3165	1215	510	77·9	Infective endocarditis. Skull very thin
49	C	44	67	134	190	143	3641	1311	545	75·3	Renal calculus
50	A	43	66·5	132·5	191	144	3644	1300	555	75·4	Pneumonia. Well nourished
51	C	44	68·5	130·5	193	154·5	3891	1224	570	80·1	Cirrhosis of liver. Well nourished
52	A	38	68·5	132	193·5	148·5	3793	1350	560	76·7	Fractures of sternum and ribs
53	C	35	70	136	195	161	4270	1335	558	82·6	Aortic regurgitation. Well nourished
54	A	39	69	132	190	162	4063	1390	557	85·3	Meningitis. Bullet in brain 14 years
55	A	36	69	135	193	154	4012	1400	569	79·8	Fall down lift. Well nourished
56	C	43	66	125	182	152	3458	1225	538	83·5	Fibroid degeneration of heart
57	C	42	60·5	134	191	152	3890	1310	549	79·6	Bronchitis and emphysema. Thick scalp

B. *Male Subjects from 46 and Upwards. Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	H	L	B	(Cm.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
58	—	50	68	140	192	155	4166	1560	560	80·7	Fatty degeneration of heart
59	C	47	69	139	190	149	3935	1330	544	78·4	Calculous nephritis. Pericarditis. Pleurisy
60	—	47	67	132	193	144	3669	1222	556	74·6	Pneumonia
61	C	49	64·5	133	190	153	3866	1415	553	80·5	Epithelioma of lower lip [ribs
62	A	50	69	130	180	145	3393	1175	530	89·6	Injuries to head. Fracture of clavicle and
63	C	48	69·5	139	201	159	4442	1330	585	79·1	Tubercular nephritis. Pulmonary tubercu-
64	—	47	69	136·5	201	155	4253	1485	585	77·1	losis. Foreign
65	A	50	72·25	129·5	195	148	3737	1470	568	75·9	Strangulated umbilical hernia
66	A	48	64·5	124·5	182·5	146·5	3329	1135	541	80·3	Cerebral hæmorrhage. L. hemisphere
67	—	47	67	132	183·5	141	3415	1310	548	76·8	
68	—	50	68	122	188	147	3372	1154	550	78·2	Fracture of base of skull
69	C	48	68	141	204	154	4430	1510	583	75·5	Carcinoma of stomach
70	—	49	68·5	139	197	160	4381	1415	589	81·2	Pleurisy, Pericarditis. Skull thick
71	C	48	69·5	138·5	191	151·5	4008	1468	564	79·3	Malignant growth of pharynx
72	—	49	69	135	188	152	3858	1390	563	80·9	Cerebral hæmorrhage into L. ventricle
73	—	48	66	137	188	160	4121	1380	576	85·1	Foreign
74	A	50	69·5	131	196	158	4057	1432	575	80·6	Subdural, and intrapontine hæmorrhage
75	C	69	69·5	136	190	148	3824	1240	560	77·9	Fatty degeneration of heart and kidneys
76	—	?	63	126	184·5	146	3394	1195	540	79·1	Cardiac failure
77	A	58	67	131	183·5	148	3558	1225	540	80·7	Tetanus
78	—	57	69	130·5	184	140	3362	1188	544	76·1	Lymphosarcoma of neck
79	C	67	68·5	137	187·5	153	3930	1252	569	81·6	Cerebral hæmorrhage. Fibroid kidneys
80	A	61	66	136	188	150	3835	1315	552	79·8	Fracture of spine, R. femur, and ribs
81	—	53	69	138	185	150	3830	1245	545	81·1	
82	A	60	68·5	139	190	146	3856	1430	564	76·8	Compound fracture of R. tibia and fibula
83	C	58	70	128	180	141	3249	1279	532	78·3	Malignant disease of R. lung
84	C	55	67·5	131	187	146	3577	1245	572	78·1	Carcinoma of soft palate
85	A	57	67	132	196	152	3933	1309	569	77·6	Fracture of skull
86	—	75	67·5	133	193	150	3850	1412	553	77·7	Burns. Receding forehead
87	—	63	61	125·5	187	141	3309	1120	534	75·4	Cerebral hæmorrhage. Heavy drinker
88	—	69	67·25	128·5	188	141	3406	1220	540	75	Cerebral hæmorrhage
89	A	?	65	130	186	145	3506	1280	538	78	Fracture of base of skull
90	C	52	63·25	132	196	151	3907	1440	552	77	Carcinoma of stomach. Skull thick
91	C	66	71·5	142	194	151	4160	1370	570	77·8	Carcinoma of œsophagus
92	C	67	69·5	128	180	144	3318	1192	531	80·0	Malignant growths in liver. Skull thick
93	—	67	67	138	183	145	3662	1230	524	79·2	Carcinoma of stomach
94	A	53	63	137	191	149	3899	1346	568	78·0	Compound fracture of tibia and fibula
95	C	55	63	132	196	143	3700	1290	551	73·0	Epithelioma of R. cheek
96	A	56	65	133	192	148	3779	1165	558	77·1	Fracture of base of skull
97	C	54	65	130	185·5	144	3473	1240	530	77·6	Epithelioma of tongue. Skull very thin
98	C	59	63	136	182	141	3490	1132	528	77·5	Coma. Alcoholic dementia. Skull thick
99	A	65	65·5	128	183	156	3654	1242	544	85·2	Fracture of skull and ribs. Skull and scalp thin
100	C	59	64	128	190	143	3478	1270	536	75·3	Carcinoma of stomach. Skull and scalp thin
101	C	60	65·5	128	184·5	148	3495	1218	525	80·2	Epithelioma of lips [drinker "
102	A	60	69	130·5	192	153	3834	1430	579	79·7	Fractured ribs. Syncope. "Heavy malt
103	C	60	68	135	194	148	3876	1588	555	76·3	Carcinoma of stomach. Skull and scalp thin
104	C	62	65	130·5	187	150	3661	1320	548	80·2	Chronic nephritis. Alcoholism. Skull thin
105	A	70	67	128	191	148	3618	1290	565	77·5	Rupture of heart. "Dropped dead in street "
106	A	59	67·5	126·5	191	151	3648	1260	558	79·0	Fracture of arm and ribs. Pneumonia
107	A	51	68	140·5	200	143·5	4032	1425	565	71·7	Acute colitis. Peritonitis
108	C	57	66	128·5	185	143	3399	1226	542	77·3	Malignant disease of liver. Emaciated
109	C	51	68·5	133·5	193	152	3916	1360	556	78·8	Malignant disease of glands in neck
110	A	65	70·5	139	203	157	4430	1620	585	77·3	"Brought in dead." Fatty heart. Syncope
111	C	56	66	130	189·5	150	3695	1310	549	79·2	Chronic interstitial nephritis. Cerebral hæm.
112	C	75	72·5	125·5	193	145·5	3524	1250	555	75·3	Chronic interstitial nephritis. Fatty heart
113	C	47	69·5	124	192	150	3571	1295	561	78·1	Carcinoma of tongue
114	C	57	67·25	129	187	149	3594	1290	555	79·7	Carcinoma of tongue. Emaciated

B. *Male Subjects from 46 and Upwards. Middlesex Hospital. (Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cm.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
115	C	62	69·75	123	184	149·5	3383	1290	536	81·2	Carcinoma of liver
116	C	84	50·5	127	190	145	3499	1275	545	76·3	Epithelioma of lip. Scalp thin
117	C	48	61·5	128·5	190	147	3589	1250	549	77·4	Cardiac failure. Pulmonary tuberculosis
118	C	58	68·5	134	189	154	3900	1270	557	81·5	Carcinoma of tongue. Emaciated [& spleen
119	A	63	67·5	136	199	152	4114	1362	588	76·4	Fracture of skull and ribs. Rupture of liver
120	C	52	67	133	191	155	3937	1300	558	81·2	Carcinoma of rectum. Well nourished
121	A	57	66	126	183	147	3399	1173	544	80·3	Meningeal hæmorrhage. Cerebral compression
122	A	52	69	132	192	164	4200	1256	563	85·4	Fracture of scapula and ribs. Foreign. 18 stone
123	—	58	72	134	212	158	4488	1440	585	74·5	Cerebral hæmorrhage. Skull and scalp thick
124	C	63	65	129	188	149	3614	1180	551	79·3	Papilloma of stomach. Emaciated. Foreign
125	A	57	67·5	133	194	157	4051	1306	573	80·9	Cerebral hæmorrhage. Well nourished
126	A	46	69	134	196	144	3782	1350	556	73·5	Pneumonia. Fairly well nourished
127	C	61	64	128	184	144	3391	1125	531	78·3	Malignant ulceration of face. Emaciated
128	—	58	66	126	171	145	3124	1165	534	84·8	Foreign
129	—	55	65	141·5	186	154	4053	1312	558	82·8	Apoplexy. Skull thin. Scalp thick
130	C	50	66	125·5	189	151	3582	1300	547	79·9	Chronic interstitial nephritis. Scalp thin
131	C	52	66	129	196	145	3666	1270	556	74·0	Carcinoma of scrotum. Wasted
132	—	51	67	128	189	146	3532	1335	553	77·2	Tumour of stomach. Well nourished
133	—	63	69	131	198	156	4046	1450	583	78·8	Fracture of skull. Cerebral hæmorrhage
134	—	72	66·5	132	184	151	3667	1310	554	82·1	Fracture of skull. Subdural hæmorrhage

C. *Female Subjects from 20 to 46 Years of Age. Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
135	C	36	—	114	179	140	2857	1027	570	78·2	Cirrhosis of liver. Excess of subarachnoid
136	—	20	63	133·5	184·5	139·5	3436	1235	523	75·6	Cerebro-spinal meningitis [fluid
137	—	35	63·5	132·5	187	153	3791	1260	558	81·8	Cellulitis and œdema of pharynx. Thick
138	—	40	60	126	182	144	3302	1165	532	79·1	Carcinoma uteri. Thin skull [skull
139	—	30?	51	126	178·5	138	3104	1080	498·5	77·3	Uræmia
140	—	33?	58	122·5	181	143	3171	1127	531	79·0	Valvular disease of heart. Emboli in brain
141	—	?	64	137	175	149	3572	1270	532	85·1	Pneumonia. Pericarditis
142	.—	42	62	132	187	143	3530	1252	552	76·5	Fatty heart
143	—	35	66	126	180	140	3175	1200	521	77·8	Rheumatism. Bronchitis
144	—	31	66	131	181	145	3438	1290	539	80·1	Raynaud's disease
145	C	35	64	128	193	158	3903	1334	554	81·9	Carcinoma. Cervix uteri
146	A	23	69	142	176	156	3899	1380	555	88·6	Gun-shot wound in L. breast
147	A	38	65·5	129	185	142·5	3401	1140	542	77·0	Burns of neck and chest
148	—	23	67·5	128	181	141	3267	1243	540	77·9	Septicæmia
149	—	32	70	133	184	141	3451	1340	532	76·6	Pneumonia. Bronchitis
150	C	34	58	126	179	137	3090	1168	505	76·5	Carcinoma mammæ. Skull thin
151	C	37	60	132	186	139	3413	1322	532	74·7	Carcinoma mammæ. Skull and scalp thin
152	—	39	64	129·5	182	141	3323	1249	521	77·5	Broncho-pneumonia
153	—	34	63	134·5	190	144	3680	1321	552	75·8	Pneumonia. Skull very thick
154	—	41	66·5	127·5	183·5	147	3439	1192	533	80·1	Morbus cordis. Skull very thick, scalp thin
155	C	43	66·5	136	187	151·5	3853	1373	565	81·0	Carcinoma mammæ. Skull and scalp thick
156	C	33	59	128	180	137	3156	1170	530	76·1	Carcinoma of stomach. Skull thick behind
157	C	28	64	129	179	142	3279	1265	528	79·3	Intestinal ulceration probably syphilitic
158	C	33	67	132	186	151	3707	1235	560	81·2	Sarcoma of frontal bone
159	A	34	66	138	191	152	4006	1302	546	79·6	Pulmonary embolism. Fibromyoma of uterus
160	C	38	65·5	131	177	141	3269	1241	523	79·7	Carcinoma uteri. Scalp thin. Foreign

C. *Female Subjects from 20 to 46 Years of Age. Middlesex Hospital. (Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres)			(Cm.³) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
				H	L	B					
161	C	45	68	125	178	138	3071	1078	514	77·5	Carcinoma uteri. Thick scalp. Thick bushy hair
162	A	25	67	136	181	153·5	3779	1520	550	84·8	Unresolved lobar pneum. Cerebral congestion
163	—	39	62	133	184	145	3548	1460	535	78·8	Carcinoma uteri. Thick hair. Thin scalp
164	C	36	61	131	179·5	140	3292	1075	519	78·0	Carcinoma uteri. Skull thick, except at sides.
165	C	36	62·5	130	183	147	3497	1280	535	80·3	Carcinoma mammæ [Foreign
166	C	40	60	123·5	174·5	143	3082	1180	513	81·9	Malignant dis. R. side of face. Emaciated
167	—	22	58·5	126	179	144	3248	1250	513	80·4	Diabetic coma. Scalp rather thin
168	C	40	63·5	125	184	146	3358	1190	528	79·3	Carcinoma uteri. Scalp thin
169	C	40	61·5	130·5	193	151	3803	1374	548	78·2	Addison's disease. Skull rather thick [thick
170	C	40	65	128·5	185	150	3566	1306	535	81·1	Tubercular nephritis. Skull very thin. Scalp
171	C	30	63	126	177	141	3145	1202	502	79·7	Malignant disease of R. ear. Skull very thin
172	C	42	64·5	128·5	188	145	3503	1240	539	77·1	Carcinoma mammæ. Skull thick
173	C	34	63·5	131	188	145	3571	1316	540	77·1	Pernicious anæmia. Well nourished
174	C	42	62·5	130·5	189	151	3724	1280	543	79·9	Carcinoma uteri. Skull thin
175	C	39	63·5	126	190	151	3615	1350	555	79·5	Cirrhosis of liver. Ascites
176	C	39	62·5	128	175	143	3203	1180	515	81·7	Carcinoma uteri. Skull thin. Scalp thick
177	C	24	64	132	186	147	3609	1210	560	79·0	Pyonephrosis. Foreign
178	C	44	64·5	128·5	188·5	147	3561	1127	541	78·0	Softening and growth in brain
179	C	24	64·5	136	190	154	3979	1324	552	81·1	Bronchitis. Empyema. Dilated heart
180	C	30	63·5	134	187	141	3533	1210	540	75·4	Diabetic coma. Wasting. Skull thick
181	C	37	63·75	137	187	144	3689	1290	530	77·0	Carcinoma mammæ. Wasted
182	C	33	55	120	179	147	3158	1100	490	82·1	Carcinoma uteri. Emaciated. Alcoholic
183	C	38	67	132	197	154	4005	1280	578	78·2	Calcareous tumour of cerebellum
184	A	26	64·5	128	175	142	3181	1175	545	81·1	Chorea. Well nourished
185	C	25	61·5	129	186	145	3479	1160	542	78·0	Tuberculosis of lungs and intestine
186	C	35	63	129·5	190	148	3642	1205	547	77·9	Pulmonary tuberculosis. Emaciated
187	C	21	66·5	128·5	191	148	3632	1163	541	77·5	Sarcoma of maxilla. Emaciated. Scalp thin

D. *Female Subjects from 46 Years of Age and Upwards. Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres)			(Cm.³) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
				H	L	B					
188	C	46	66	116	185	143	3069	1022	?	77·3	Carcinoma uteri
189	C	50	60	132	187	137·5	3394	1243	528	73·5	Carcinoma of breast, and pleura
190	C	46	66	139	185	144	3703	1350	532	77·8	Malignant growths in peritoneum
191	C	50	63	124	181	141	3165	1237	522	77·9	Epithelioma of scalp and face
192	C	46	67·5	130	183	141	3354	1204	522	77	Carcinoma of urethra
193	C	49	60·5	123	173	141	3000	1090	514	81·5	Carcinoma uteri. Scalp and hair thin
194	C	49	65·5	132	185	151	3687	1355	545	81·6	Cystic kidneys. Uræmia. Skull thin
195	C	48	64	129	185	149	3556	1250	530	80·5	Carcinoma uteri. Skull thin
196	C	47	66·5	120·5	177	130	2773	1076	516	73·4	Carcinoma mammæ. Skull very thick
197	C	53	64	130	168	140	3058	1120	515	83·3	Carcinoma uteri
198	C	56	60	131	187	136·5	3344	1220	531	73·0	Carcinoma mammæ
199	—	64	61·5	132	182·5	145	3493	1240	540	79·7	Diabetes. Gangrene of foot
200	C	69	62·5	125·5	185	142	3297	1220	525	76·8	Morbus cordis
201	C	60	61	130	182	142	3360	1095	515	78·0	Carcinoma uteri. Skull thick
202	C	55	61·5	129	180	139	3228	1235	518	77·2	Chronic bronchitis. Skull thin
203	C	55	59	129	174	146	3277	1105	506	83·9	Bronchitis. Alcoholism. Skull soft and thick
204	—	53	61	133	188	154	3851	1405	577	81·9	Cerebral hæmorrhage. Thick skull and scalp
205	C	60	60·5	126	179	136	3067	1150	518	76·0	Multiple sarcomata. Very thin skull & scalp

D.   *Female Subjects from 46 Years of Age and Upwards.   Middlesex Hospital.   (Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
206	C	54	60·5	129·5	181	157·5	3692	1305	548	87·0	Carcinoma. Metastases in L. hip. Thin skull
207	C	60	64·5	132	179	144	3402	1220	523	80·4	Chronic interstitial nephritis. Scalp thin
208	—	62	62·5	136	192	153	3995	1296	561	79·7	Chr. nephritis. Fatty degen. of Heart. Very thick skull
209	C	53	61	128	180	144	3318	1175	523	80·0	Carcinoma mammæ. Skull thick. Scalp thin
210	C	68	61	120	173	131	2720	955	502	75·7	Carcinoma uteri. Scalp thin
211	C	63	62·5	126·5	172	135	2937	1070	520	78·5	Sarcoma of nose
212	—	51	66	134	183	146	3580	1320	532	79·8	Appendicitis. Hair and skull thin
213	C	78	58·5	121	176	138	2939	1060	509	78·4	Carcinoma recti. Skull very thin
214	C	57	61	123	180	135	2989	1130	525	75·0	Carcinoma mammæ. Emaciated
215	—	68	64	128	188	149	3586	1250	540	79·3	Carcinoma of pancreas. Well nourished
216	C	58	60·5	124	178	143	3156	1225	515	80·3	Carcinoma mammæ. Cerebral abscess
217	C	59	65	127	180	142	3246	1180	524	78·9	Carcinoma mammæ. Skull thin
218	C	49	57	126	181	139	3170	1178	512	76·8	Carcinoma uteri. Fairly nourished
219	C	56	60	122	186	144	3268	1142	541	77·4	Carcinoma. Sigmoid flexure of colon
220	C	58	59	133	182	140	3389	1130	520	76·9	Carcinoma uteri
221	C	62	62·5	128·5	184	143	3381	1185	528	77·7	Carcinoma uteri. Wasted
222	C	67	63	124	175	132	2864	1012	509	75·4	Carcinoma uteri. Well nourished
223	—	47	64	133	185	152	3740	1280	547	82·2	
224	C	64	59	131·5	180	147	3479	1103	532	81·7	Carcinoma mammæ
225	C	47	65	131	192	145	3647	1408	565	75·5	Carcinoma mammæ. Skull rather thick
226	A	74	63	128	191	152	3716	1300	564	79·6	Strang. femoral hernia. Well nourished
227	C	67	62	125	185	142	3284	1246	528	76·8	Cerebral hæm. Interstitial nephritis. Emac.
228	A	50	—	142	191	155	4204	1380	559	81·2	Fall. Rupture of aorta. Fracture of ribs. Fat
229	C	63	63	130	189	152	3735	1350	548	80·4	Malignant dis. of intestines. Very fat. Skull
230	A	74	62	121	186	143	3218	1060	542	76·9	Fractured ribs. Skull and scalp thick [thin
231	C	49	65·5	137	185·5	145	3685	1350	535	78·2	Carcinoma of omentum. Very fat. Skull thin
232	C	46	67	135	196	140	3704	1220	521	71·4	Carcinoma uteri. Emaciated. Skull thick
233	C	46	62·5	126·5	174	146	3214	1110	532	83·9	Pulmonary tuberculosis. Skull thick & spongy
234	C	50	64·5	125	181	150	3394	1215	542	82·9	Carcinoma uteri. Emaciated. Scalp thin
235	C	63	58·5	127	178	143	3233	1104	522	80·3	Carcinoma mammæ. Emaciated. Skull thin
236	C	56	64·5	126·5	184	144	3352	1170	533	78·3	Carcinoma of stomach. Slight wasting
237	C	63	62	128	179	148	3391	1120	537	82·7	Carcinoma uteri. Emaciated. Scalp thin

E.   *Children ♂.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
238	C	2·5 ms.	22	89·5	125	95	1063	450	361	76·0	Broncho-pneumonia
239	A	9 ms.	28	114	155	117	2067	—	—	75·5	Peritonitis
240	—	1 y.–8 m.	37·5	119	168	125	2499	1052	481	74·4	Tubercular meningitis
241	—	2 y.–6 m.	31·75	111	160	131	2327	1015	474	81·9	Acute general tuberculosis
242	—	2 y.–6 m.	33·	121	172	135·5	2820	1120	486	78·8	Broncho-pneumonia. Foreign
243	—	2 y.–8 m.	38	116	162	139	2612	1240	483	85·8	
244	A	3	37	124·5	181	137	3087	1310	514	75·7	Meningitis
245	C	3 y.–6 m.	35·5	121	170	129·5	2664	1095	476	76·2	Septic broncho-pneumonia. Emaciated
246	C	3 y.–10 m.	34	127	174	135	2983	1080	501	77·6	Otitis media. Emaciated
247	—	4	39	127	173	134	2944	1202	501	77·5	—. Skull very thin
248	—	5	37·5	125	170	132	2805	1200	490	77·7	Death by suffocation. Foreign
249	C	7	46	127	175	140	3112	1375	504	80·0	Tubercular cervical glands
250	—	8	43	131·5	192	140	3535	1450	530	72·9	Pericarditis. Head large and long

E. *Children ♂.* *(Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres)			(Cms.³) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
				H	L	B					
251	—	10	52	137·5	187	149·5	3844	1417	539	79·9	Pericarditis. Foreign
252	C	11	55	130	181	145	3412	1412	532	80·1	Morbus cordis [spleen
253	A	12	65	136	183	141	3509	1379	540	77·1	"Run over." Fracture of ribs. Rupture of
254	C	12	48·5	122·5	183	142	3183	1322	528	77·6	Broncho-pneumonia. Pulm. tuberculosis
255	—	12–13	56·5	134	184	143	3526	1454	530	77·7	
256	C	15	62·5	141	190·5	151	4056	1618	547	79·2	Diabetes. Coma
257	C	17	67	129	189	150·5	3669	1095	536	79·6	Morbus cordis. Wasted. Foreign
258	C	18	71	131	172	142	3200	1230	520	82·6	Tubercular meningitis. Foreign
259	C	19	67	143	187	146	3904	1460	556	78·1	Psoas abscess. Tubercular meningitis
260	A	19	67·5	135	191	157	4048	1278	558	82·2	Bullet wound in head
261	A	19	65·5	133	188·5	147·5	3698	1412	540	78·2	Pneumonia
262	—	19	69·5	138	189	149	3886	1450	547	78·8	Cerebral abscess. Meningitis

F. *Children ♀.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres)			(Cms.³) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
				H	L	B					
263	—	3 wks.	21·5	92·5	114	88·5	933	385	324	77·6	Bronchitis. "Brought in dead"
264	C	2·5 ms.	22	94·5	129·5	99	1212	490	369	76·4	Gastro-enteritis. Foreign
265	C	5 ms.	22·5	88·5	129	95	1085	512	368	73·6	Convulsions
266	C	6 ms.	24·5	100	132·5	104·5	1385	503	385	78·8	Malignant growth of orbit (? sarcoma)
267	C	8 ms.	25	118	144	126	2141	960	437	87·5	Rickets. Gastro-enteriti. Foreign
268	C	2 y.–6 ms.	33·5	124	172	132	2815	995	496	76·7	Tuberculosis. Emaciated
269	A	3 y.–6 ms.	38·5	114	164	126	2356	1100	475	76·8	Fracture of base of skull
270	—	7	47	121	169	131	2679	1120	493	77·5	Pulmonary tuberculosis. Meningitis
271	C	7	48	118	174	133	2731	1205	485	76·4	
272	C	15	61·5	128	185	139	3292	1205	518	75·1	Otitis media. Thrombosis of R. lateral sinus
273	C	17	61	125·5	178	143·5	3206	1180	524	80·6	Tubercular meningitis. Skull thick. Emac.
274	C	18	53·5	125	179	135	3021	1147	507	75·4	Mitral regurgitation. Gangrene of hands
275	C	18	66	139	186	151	3904	1419	554	81·2	Chronic otitis media. Cerebellar abscess
276	—	19	65	131	179	141	3306	1245	512	78·8	

A STUDY OF THE BIOMETRIC CONSTANTS OF ENGLISH BRAIN-WEIGHTS, AND THEIR RELATIONSHIPS TO EXTERNAL PHYSICAL MEASUREMENTS.

By J. BLAKEMAN, B.A., M.Sc., assisted by ALICE LEE, D.Sc., and KARL PEARSON, F.R.S.

(1) *Introductory.*

THE purpose of this paper is to present a biometric analysis of the measurements provided by Dr R. J. Gladstone and published in this volume. The conclusions reached are therefore of the same order of validity as the data upon which they are based. An attempt has been made to compare them with the fuller material reduced by Dr Raymond Pearl, and in many points where comparison was possible general confirmation of his conclusions has been obtained. Gladstone's statistical material differs from that used by Pearl in two essential points. It is in the first place more meagre, but in the second place it provides additional measurements which enable us to predict with a moderate degree of accuracy brain-weight from external measurements on the living subject. The characters recorded by Gladstone were the age (A), stature (S), cause of death, the maximum head-length (L), the maximum head-breadth (B), the auricular height (H), the horizontal circumference (U), and the brain-weight (w). The sagittal arc, nasion to inion, and the biauricular transverse arc (Q of our notation) were also measured, but are not considered, as the difficulty of determining the inion, and the impediment formed by the hair, especially in the case of the female subjects, was found to be so great as to make the measurements unreliable. The product $P = L \times B \times H$ was formed, and the present study is concerned with the biometric relations of w, A, S, L, B, H, P, and U. The distribution of the work has been as follows: The whole of the labour of obtaining the multiple regression formula, p. 146, for the males fell to J. Blakeman, the similar formula for the females to A. Lee. All the other statistical and calculating work throughout the paper as well as the preparation of Figures 3—6 is further due to J. Blakeman. K. Pearson is responsible only for suggesting the general lines the paper should take, and preparing the draft of its final form.

In dealing with Gladstone's material a reduction was first made in its amount by

    (a)   the exclusion of a small number of foreigners;

    (b)   the exclusion of those individuals for whom all the seven characters referred to above were not available;

(c) the exclusion of all males under 24 years of age and all females under 20 years of age. The reason for adopting this restriction was that from considerable experience of anthropometric measurements we have found that these ages roughly mark in man and woman a turning-point in the nature of the growth curves. Up to these ages there is a rather rapid increase, after these ages in most characters a slight but continuous decrease.

With the above exclusions we had 117 male and 94 female cases. These numbers are small, but the material is fairly homogeneous and the probable errors have in each case been given. The publication of the actual data (pp. 118—123) has been undertaken largely with the hope that other medical schools may collect similar material so that ultimately proper numbers will be available, large enough for due allowance to be made for the cause of death and the condition at death. No such separate classifications are possible at present.

(2) *On the Special Character of the Present Material.*

The present material is what has been described in this journal* as a "General Hospital Population,"—with perhaps in this case a rather large proportion of cancer cases. It cannot be too often insisted upon that such a population is not a fair sample of the "general population" of a given district. There is a larger amount of what it would be convenient to call "shrinkage" due to illness and defective nourishment. The well-nourished, physically well-developed middle classes are largely absent, and taking the brain-weights of these hospital returns as typical of the "general population"—especially applying them to problems in the relationship of intelligence to physical measurements—is liable to lead to very erroneous conclusions. An illustration of this may be given from Gladstone's classification into acute and chronic cases—the A and C of the second column of the tables. We find for average brain-weights:

TABLE I.

Ages	Cause of Death	No.	Male	No.	Female
20 to 46 ...	Acute	16	1430	5	1303
	Chronic	27	1331	32	1227
Over 46 ...	Acute	22	1318	3	1248
	Chronic	34	1354	40	1178

It will thus be seen that, with the exception of the males over 46, the cause of death makes a very substantial difference in the average brain-weight. The numbers are however far too few to allow us to make any separate classification, but the point is a vital one and must undoubtedly be properly dealt with when greater numbers are forthcoming. The reader will find as he proceeds further that we have other grounds for suspecting that the general hospital population differs in some essential features from the general population.

* Greenwood : *Biometrika*, Vol. III. p. 65.

(3) *On the Means and Variabilities of the Anthropometric Characters of the Present Material.*

Table II. contains the means, standard deviations, and coefficients of variation of all the characters dealt with.

We notice that the woman is *absolutely* more variable than man in four out of the nine characters considered, and *relatively* more variable in eight out of the nine. In the ninth, stature, the difference in variability between the two sexes is well within the probable error of the difference. This is a further illustration of the point that man is not essentially more variable than woman.

We can now make some comparisons between this general hospital population and other English groups. In the first place let us take the head-measurements:

TABLE II. *Means and Variabilities.*

Character	Mean		Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀	♂	♀
Head Length ... (*L*) in mm. ...	190·36 ± ·35	183·20 ± ·39	5·62 ± ·25	5·59 ± ·28	2·95 ± ·13	3·05 ± ·15
Head Breadth ... (*B*) in mm. ...	149·34 ± ·34	144·55 ± ·40	5·41 ± ·24	5·76 ± ·28	3·62 ± ·16	3·99 ± ·20
Auricular Height (*H*) in mm. ...	132·90 ± ·28	129·21 ± ·30	4·51 ± ·20	4·35 ± ·21	3·39 ± ·15	3·37 ± ·17
Cephalic Index ... (100 *B/L*) ...	78·48 ± ·17	78·93 ± ·20	2·76 ± ·12	2·87 ± ·14	3·52 ± ·16	3·64 ± ·18
Horizontal Circumference ... (*U*) in mm. ...	555·79 ± ·99	533·25 ±1·17	15·84 ± ·70	16·83 ± ·83	2·85 ± ·13	3·16 ± ·16
Product ... (*P*=*L*×*B*×*H*) in cm.³ ...	3782·91 ±18·11	3427·03 ±19·68	290·47 ±12·81	282·94 ±13·92	7·68 ± ·34	8·26 ± ·41
Stature ... (*S*) in inches ...	67·16 ± ·19	63·05 ± ·19	3·05 ± ·13	2·79 ± ·14	4·55 ± ·20	4·42 ± ·22
Age ... (*A*) in years ...	49·26 ± ·76	45·90 ± ·95	12·11 ± ·53	13·69 ± ·67	24·60 ±1·15	29·83 ±1·59
Brain-Weight ... (*w*) in grs. ...	1327·69 ±6·49	1224·90 ±6·70	104·03 ±4·59	100·16 ±4·93	7·84 ± ·35	8·18 ± ·40

N.B. In working out all further biometric constants, regression formulae, etc., not the above values but the values as actually calculated to four decimal places were used.

TABLE III.

Results for Cephalic Index.

Group	Male	Female
General Hospital Population ...	78·48	78·93
3000 Criminals*	78·54	—
1000 Cambridge Graduates† ...	79·56	—
2000 School Children ‡	78·92	78·29
100 to 200 British Association §	77·75	78·75

We see that as far as the cephalic index is concerned there is nothing to markedly differentiate our material from other groups of the general population that have been dealt with by other observers.

Turning next to the diameters we have

TABLE IV.

Group	Male			Female		
	L	B	H	L	B	H
General Hospital Population ...	190·4	149·3	132·9	183·2	144·6	129·2
3000 Criminals*	191·7	150·4	—	—	—	—
1000 Cambridge Graduates† ...	193·5	154·0	—	—	—	—
British Association ‖	198·1	155·0	130·9	185·6	147·3	128·4
Congress of Anatomists‖ ...	198·4	157·2	133·1	—	—	—
University College Staff‖ ...	196·4	153·5	134·8	—	—	—
Bedford College Students ‖ ...	—	—	—	189·7	146·8	132·7

These results show very clearly that the general hospital population has a smaller head than the middle class English population.   The only exceptions are the male and female auricular heights for the British Association members.   These were probably not measured with an ear-plug craniometer.   The nearest approach is the criminal population.   Before commenting on this let us consider the stature returns.   We have

* Macdonell: *Biometrika*, Vol. i. p. 177 *et seq.*
† Pearson : *The Chances of Death*, Vol. i. p. 351.
‡ Pearson : *Biometrika*, Vol. iii. p. 140.
§ Pearson : *The Chances of Death*, Vol. i. p. 351.
‖ Lee and Pearson : *Phil. Trans.* Vol. 196 A, p. 251 *et seq.*

TABLE V.

Group	Male Stature	Female Stature
General Hospital Population	67″·2	63″·1
Galton's South Kensington Returns* ...	67″·9	63″·3
3000 Criminals†	65″·5	—
Australian Criminals‡	66″·9	62″·2
Cambridge Students§	68″·9	63″·8
Pearson's Family Measurements‖, Offspring	68″·7	63″·9
„ „ „ Parents	67″·7	62″·5

Thus we see that the general hospital population is shorter than the average of the upper middle classes, approaching closer to the lower middle classes, who would chiefly supply the South Kensington Exhibition returns. It is taller than the criminal classes either in England or New South Wales, and approaches closely to the average of Pearson's "Parent" Group, who must all have been over 40 years of age.

It would accordingly seem that our general hospital population belongs anthropometrically to a class intermediate between the classes from which criminals are drawn and the lower middle class. Its reduced head-measurements are probably due in part to this fact, in part to chronic illness, and in part to a concomitant of class differentiation, namely want of good nourishment and exercise in the period of growth. We might reasonably attribute a great deal to this want of nurture, and it seems as hopeless to settle the problem of correlation between size of head and intelligence by measuring different social classes, as to settle the correlation between stature and intelligence by the same process.

If we turn to age at death we shall get much confirmation of the view that we are dealing with a badly nourished, physically degenerate class. We have

TABLE VI.

Mean Age at Death in Years.

Group	Male	Female
General Hospital Population	49·3	45·9
Society of Friends ¶... ...	60·9	60·8
Peerage and Landed Gentry ¶	62·9	—

* Galton's *Natural Inheritance*, pp. 200—1; Pearson: *The Chances of Death*, Vol. I. p. 311.
† Macdonell: *Biometrika*, Vol. I. p. 191.
‡ Powys: *Biometrika*, Vol. I. p. 44.   § *Biometrika*, Vol. I. p. 191.
‖ Pearson and Lee: *Biometrika*, Vol. II. p. 370.
¶ Pearson and Beeton: *Biometrika*, Vol. I. p. 64.

Here in the latter two groups adults, 21 years and over, have been taken, while in the hospital group among the males 24 was the minimum admitted age. Clearly we have to deal, even allowing for a larger percentage of accidents and an absence of some of the diseases of old age, for a population of lesser constitutional vitality as measured by expectation of life. It is to be noted that the female is shorter-lived than the male[*].

For the horizontal circumference we have no reliable data for comparison. Dr Beddoe gives it[†] for 30 groups, containing various numbers from two to a hundred, and also for a hundred individuals, but in *not one single case* of the groups or of the individuals has he obtained a result as low as our *mean* of 558 mm. On the other hand it is almost exactly the value 560 Lewenz and Pearson[‡] have found for Jeremy Bentham's head, which in other respects is close to the English mean type. There appears to be little doubt accordingly that Dr Beddoe has an individual method of measuring this circumference.

Lastly, turning to the brain-weights, we have, pooling the results of Reid and Peacock, Clendenning, and Sims, the following mean values.

TABLE VII.

English Brain-weights in grms.

Group	♂	♀
Gladstone's Middlesex Hospital Data ...	1328	1224
Reid and Peacock, Clendenning, Sims§ ...	1335	1235

Thus the average brain-weight of an English general hospital population appears to be fairly constant when we compare Gladstone's data with the means of other material. But this is of course no evidence that the general hospital population is at all representative of the general population.

On the contrary, the fact that for the head diameters, for stature, and for constitutional robustness, as measured by duration of life, the hospital population falls below the general population, would lead us to believe that it probably does so in brain-weight; and we should attribute the probable source of this differentiation to want of nourishment and "shrinkage" due to chronic disease.

(4) *Variation Constants.*

Turning first to the head measurements we may compare our results with those for other English groups. This is done in Table VIII.

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[*] On this point see "Assortative Mating in Man," *Biometrika*, Vol. II. p. 488.

[†] *L'Anthropologie*, Vol. XIV. pp. 284, 291—4.

[‡] *Biometrika*, Vol. III. p. 394.

[§] Pearson: *The Chances of Death*, Vol. I. p. 321. Of course the pooling of the results of these different observers is open to considerable criticism.

TABLE VIII.   *Variability of Head Measurements.*

Group	Length				Breadth				Auricular Height			
	S. D.		C. of V.		S. D.		C. of V.		S. D.		C. of V.	
	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
General Hospital Population	5·62	5·59	2·95	3·05	5·41	5·76	3·62	3·99	4·51	4·35	3·39	3·37
English Criminals* ...	6·05	—	3·15	—	5·01	—	3·33	—	—	—	—	—
Cambridge Graduates† ...	6·16	—	3·18	—	5·06	—	3·28	—	—	—	—	—
English Skulls	6·27	6·22	3·31	3·45	5·28	4·77	3·75	3·54	4·28	4·50	3·73	4·12

Very little of a definite kind can be deduced from these rather sparse results. In length the general hospital population seems less variable than either the English criminals or Cambridge graduates, but it is more variable in breadth. If we compare the variability of the diameters of the living head with those of the skull, we see that the head is clearly not definitely more variable than the skull, and accordingly the assumption often made that owing to the presence of the flesh it must be so, is erroneous. The paradox is explicable on the basis of a negative correlation between skull diameters and amount of covering flesh‡, i.e., individuals growing much bone must on the average have less flesh.

Taking next cephalic index we have for English groups:

TABLE VIII *bis.*

Group	Cephalic Index			
	Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀
General Hospital Population ...	2·76	2·87	3·52	3·64
English Criminals§	2·74	—	3·55	—
Cambridge Graduates‖	2·90	—	3·70	—
School Children¶	3·31	3·99	4·20	5·09
British Association**	2·52	2·96	3·24	3·76
English Skulls ††	3·26	2·98	4·38	3·99

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* Macdonell : *Biometrika*, Vol. I. p. 202.   † Fawcett : *R. S. Proc.*

‡ Let σ_l be s.d. of skull length, σ_L, of head length, σ_f of flesh thickness ; then :

$$\sigma_L = \{\sigma_l{}^2 + \sigma_f{}^2 + 2\sigma_l\sigma_f r_{fl}\}^{\frac{1}{2}}$$

or σ_L must be greater than σ_l unless r_{fl}, the correlation between amount of flesh and bone size, be negative.

§ Macdonell : *Biometrika*, Vol. I. p. 191.   ‖ *Idem*, p. 191.

¶ Pearson : *Biometrika*, Vol. III. p. 140.   ** Pearson : *The Chances of Death*, Vol. I. p. 351.

†† Macdonell : *Biometrika*, Vol. III. p. 222.

We see at once that a general hospital population is about equally variable with other English groups, whether of lower grade, like the criminal, or of higher grade, like the middle class, chiefly drawn upon for Cambridge graduates or B.A. members. School children are more variable—probably a phenomenon of growth. But again we see that our skull series is more variable than the head series. For the circumference our comparable data are *nil*; we can only compare with the English skull. This provides:

TABLE IX.

Coefficients of Variation for English Horizontal Circumference.

Group	♂	♀
General Hospital Population ...	2·85	3·16
English Skull *	2·87	2·92

In the matter of the circumference we should therefore conclude that there is equal variability for head and skull measurements.

Stature provides us with more material. We find:

TABLE X.

Variation in Stature in Inches.

Group	Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀
General Hospital Population	3·05	2·79	4·55	4·42
Galton's South Kensington Records † ...	2·55	2·40	3·95	3·79
English Criminals ‡	2·59	—	3·88	—
Cambridge Students †	2·51	2·42	3·64	3·79
Pearson's Family Measurements, Offspring §	2·71	2·61	3·95	4·09
„ „ „ Parents §	2·70	2·39	3·99	3·83

It will be seen that both absolutely and relatively the general hospital population is, in the case of stature, more variable than any of the other English groups. We believe this increased variability in stature to be due to the measurements being taken on the corpse, not on the living subject. As a rule we may say that the coefficient of variation of the living stature of the adults of any race is under 4. Pearl gives twelve cases of this coefficient for corpse measurements in his Table I, p. 23, and in ten of these cases the coefficient of variation is over 4.

* Macdonell : *Biometrika*, Vol. III. p. 222.
† Pearson : *The Chances of Death*, Vol. I. pp. 313 and 311.
‡ Macdonell : *Biometrika*, Vol. I. p. 181.
§ Pearson and Lee : *Biometrika*, Vol. II. p. 370.

In fact the mean values are ♂s 4·11, ♀s 4·39; and we may, we hold, safely conclude that the increased variability does not make a class difference, but is peculiar to measurements made under some difficulties*.

Lastly, we may turn to the variation in age at death:

<div align="center">TABLE XI.</div>

Variation in Duration of Life.

Group	Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀
General Hospital Population ...	12·11	13·69	24·59	29·83
Society of Friends †	17·13	18·73	28·13	30·80
Peerage and Landed Gentry † ...	16·84	—	27·61	—
Yorkshire Tombstones ‡ ...	12·97	16·40	18·97	24·53
Oxfordshire Tombstones ‡ ...	12·70	14·67	18·55	21·68
London Tombstones ‡	13·14	14·06	19·56	21·22

Thus the general hospital population, while more variable in duration of life than the groups taken from tombstone records, which were all of married adults, is less variable than the pedigree records of either the Society of Friends or the Peerage and Landed Gentry, which were simply for persons over 21. We think, therefore, that the general hospital population, while sensibly differentiated from the other classes in actual duration of life, is not sensibly differentiated in the variability of that duration.

We may sum up then the result of the last two sections as follows:

A general hospital population is sensibly differentiated in its average anthropometric characters from other groups of the same population. But there appears to be no markedly significant difference in variation. It would seem, therefore, that observations made on a hospital population may with due precautions be used for problems in variation, but they should not be used for drawing inferences as to the type of the surrounding general population without extreme care.

(5) *On the Relative Magnitude and Variability of English Brain-weights compared with those of other Races.*

Professor Raymond Pearl's reductions enable us to form a fairly extensive table of comparative brain-weights. We have added a few results for stature and skull capacity. In the bulk of cases these are dependent upon few individuals, and are far from reliable, so that they must be treated as merely suggestions until further data are available.

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* See on difference between stature and corpse length Pearson: *Phil. Trans.* Vol. 192 A, p. 191.

† Pearson and Beeton: *Biometrika*, Vol. I. pp. 60 and 64.

‡ *Biometrika*, Vol. II. p. 488.

TABLE XII.　*Comparative Brain-weights, etc.*

Group	Brain-weight in grs.				Skull Capacity in cms.				Corpse Stature in cms.				Ratio of Brain-weight to skull-capacity
	No.	Mean	S. D.	C. of V.	No.	Mean	S. D.	C. of V.	No.	Mean	S. D.	C. of V.	
Bohemians ♂	372*	1455	114	7·8	10(‡)**	1448	70(‡)	—	266*	169	7·3	4·3	1·004 (‡)
,, ♀	197**	1311	97	7·4	—	—	—	—	133*	158	7·2	4·5	—
Swedish ♂	416†	1400	106	7·6	—	—	—	—	416†	170	6·8	4·0	—
,, ♀	233†	1253	101	8·0	—	—	—	—	262†	159	6·7	4·2	—
Hessian ♂	475‖	1392	113	8·1	—	—	—	—	475‖	167	7·2	4·3	—
,, ♀	281‖	1260	102	8·1	—	—	—	—	281‖	157	6·9	4·4	—
Bavarians ♂	559§	1362	114	8·4	100¶	1504	116	7·7	390§§	166	6·7	4·0	·906
,, ♀	347§	1220	103	8·4	100¶	1336	109	8·2	266§§	154	6·6	4·3	·913
English I ♂♀	340††	1335	124	9·2	72§§§	1477	122	8·3	177‖‖	171	7·7	4·6	·904
,, II ♂♀	269††	1235	119	9·7	80§§§	1300	113	8·7	94‖‖	160	7·1	4·4	·905
,, III	117‖‖	1328	104	7·8									·899
,, IV	94‖‖	1225	100	8·2									·942
French I ♂	292***	1325	121	9·2	56§§§	1473	107	7·3	50¶¶	166	—	—	·900
,, ♀	140***	1144	105	9·1	7(‡)‖‖‖	1296	101	7·8	50¶¶	154	—	—	·883

* Matiegka's data reduced by Pearl.

‖ Marchand's data reduced by Pearl.

§ Bischoff's data reduced by Pearson: *The Chances of Death*, Vol. I. pp. 295 and 322.

¶ Ranke's data reduced by Pearson, *loc. cit.* p. 333.

** A few isolated crania taken from the German anthropological catalogues reduced by J. Blakeman.

†† Data of Reid, Peacock, Clendenning and Sims combined and reduced by Pearson, *loc. cit.* p. 321.

§§ Whitechapel data of Macdonell reduced by him, *Biometrika*, Vol. III. pp. 208 and 222.

‖‖ Gladstone's data reduced by Blakeman.

† G. Retzius' data reduced by Pearl. For Pearl's reductions see *Biometrika*, Vol. IV. p. 23.

*** Data of *Société d'Anthropologie de Paris* reduced by Pearson, *loc. cit.* p. 322.

§§§ French crania in Munich Catalogue reduced by Pearson, *loc. cit.* p. 332.

‖‖‖ A few French crania in Barnard Davis' *Thesaurus Craniorum* collected and reduced by Blakeman.

¶¶ Rollet's data reduced by Lee and Pearson, *Phil. Trans.* Vol. 192 A, p. 180.

Now, although this table is very incomplete, and impresses upon one the need for the further collection of material, we can safely, we think, draw one or two conclusions :

(i) The order of racial average brain-weights is very far from the order of average racial intelligence. Nor is the order bettered if we allow in any manner for stature. We think that even allowing for differences in racial stature, it is quite impossible to compare the brain-weights of men of different races, and any consideration of intelligence and brain-weight which proceeds by "pooling" the brain-weights of distinguished men of a great variety of races is fundamentally fallacious *.

(ii) The variability, as measured by the coefficient of variation, of brain-weight is for the three races for which we possess fairly reliable data greater than that of skull capacity. Roughly, the average brain-weight variability is 9, the average skull capacity 8. On the whole the female shows very slightly greater variability in both cases.

(iii) The ratio of brain-weight to skull capacity has an average value of about ·9, and this is almost exactly the value of the ratio of the variation in brain-weight to that in skull capacity. It would appear that the cranial chamber is more closely packed with brain in the case of the female than in that of the male, but the evidence, which is based upon heterogeneous material, is not convincing†.

(iv) The standard deviation of brain-weight in grammes is fairly close to the standard deviation of capacity in cubic centimetres.

(v) Gladstone's results are, as far as the means are concerned, in close agreement with the results obtained by Pearson from mixing the data of a variety of observers. The increased variability, however, of the English Series I shows its real heterogeneity, and not too much stress ought accordingly to be laid on the sensible identity of the mean values. The means of the component groups, indeed, vary widely.

(6) *On the Correlations of the various Characters measured.*

Table XIII gives the values of the correlation coefficients and their probable errors. They are tabled to four decimal places, not because they are worth more than one or two, but because these values must be used if the regression coefficients are to be tested. We propose to consider the separate entries in this table, and test them against such scant data as are available at present from other sources. As there are nine observed characters, each sex would provide 36 coefficients of correlation, but only 17 have been calculated for each case, partly because of the

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* Especially when the next stage is to compare this heterogeneous group with the fairly homogeneous results for a local general hospital population !

† Data are being collected on more homogeneous material, and it is further hoped to test more completely than has hitherto been done the relative densities of different parts of the male and female brain.

TABLE XIII. *Correlations of Brain-weight and other Characters.*

		L ♂	L ♀	B ♂	B ♀	H ♂	H ♀	I=100 B/L ♂	I=100 B/L ♀	U ♂	U ♀	P=L×B×H ♂	P=L×B×H ♀	S ♂	S ♀	A ♂	A ♀	w ♂	w ♀
L	♂	1														−·0751 ± ·0620		·5980 ± ·0401	
	♀		1														−·1526 ± ·0679		·5450 ± ·0489
B	♂			1												−·0910 ± ·0618		·4885 ± ·0475	
	♀				1												−·2059 ± ·0666		·6074 ± ·0439
H	♂					1										−·2147 ± ·0595		·5443 ± ·0439	
	♀						1										−·3334 ± ·0618		·6734 ± ·0380
I	♂							1										·0014 ± ·0624	
	♀								1										·2110 ± ·0665
U	♂									1		·8273 ± ·0197		·3302 ± ·0556		−·1095 ± ·0616		·6227 ± ·0382	
	♀										1		·8067 ± ·0249		·4085 ± ·0580		−·1763 ± ·0674		·6273 ± ·0422
P	♂									·8273 ± ·0197		1		·3116 ± ·0563		−·1714 ± ·0605		·7034 ± ·0315	
	♀										·8067 ± ·0249		1		·3997 ± ·0585		−·2878 ± ·0638		·7493 ± ·0305
S	♂									·3302 ± ·0556		·3116 ± ·0563		1		−·1499 ± ·0610		·2885 ± ·0572	
	♀										·4085 ± ·0580		·3997 ± ·0585		1		−·3217 ± ·0624		·3669 ± ·0602
A	♂	−·0751 ± ·0620		−·0910 ± ·0618		−·2147 ± ·0595				−·1095 ± ·0616		−·1714 ± ·0605		−·1499 ± ·0610		1		−·2559 ± ·0583	
	♀		−·1526 ± ·0679		−·2039 ± ·0666		−·3334 ± ·0618				−·1763 ± ·0674		−·2878 ± ·0638		−·3217 ± ·0624		1		−·3724 ± ·0599
w	♂	·5980 ± ·0401		·4885 ± ·0475		·5443 ± ·0439		·0014 ± ·0624		·6227 ± ·0382		·7034 ± ·0315		·2885 ± ·0572		−·2559 ± ·0583		1	
	♀		·5450 ± ·0489		·6074 ± ·0439		·6734 ± ·0380		·2110 ± ·0665		·6273 ± ·0422		·7493 ± ·0305		·3669 ± ·0602		−·3724 ± ·0599		1

great labour involved (each coefficient being found without grouping), and partly because we realized at once that the product of the diameters and the horizontal circumference had the highest correlations with brain-weight, and that accordingly these, and not the individual diameters or cephalic index, were the appropriate characters to use in the multiple regression equations, if the latter were not to be too unmanageable. The relationship of the head diameters to each other and to stature has already been determined for more copious and better material, and was accordingly hardly worth repeating here*.

(*a*) We will consider first the influence of sex on the observed characters. We note that with two exceptions, the correlations of length of head with brain-weight and the product of the diameters with the horizontal circumference, the female has higher correlations than the male. Even in these cases the difference is within the probable error of the difference. This confirms the result already reached by Pearson and Lee that in civilised races the organs of the women are more highly correlated than those of the men†. It will be remembered that this was attributed to more intense selection in the case of the male.

(*b*) We turn next to the consideration of the influence of age on the other characters. We note at once that all twelve coefficients of correlation are negative, or the average value of each physical character dealt with decreases with age. We conclude that at any rate for a general hospital population, the *physical characters all shrink with age.* While the brain-weight shrinkage is most marked the auricular height is also very significant, then follows the product of the diameters, while the shrinkage of stature only comes fourth on the list.

If age be measured in years the following are the regression lines for Gladstone's data for stature, together with those for Swedes deduced from the reduction of Retzius' data by Pearl‡.

Shrinkage of Stature with Age.

Males: English: $\begin{cases} S = -\cdot0378\,A + 69\cdot02 \text{ (unit of stature} = \text{inch)} \\ S = -\cdot0960\,A + 175\cdot31 \text{ (unit of stature} = \text{cm.)} \end{cases}$

Swedes: $S = -\cdot0807\,A + 173\cdot42$ („ „ „)

Females: English: $\begin{cases} S = -\cdot0655\,A + 66\cdot05 \text{ (unit of stature} = \text{inch)} \\ S = -\cdot1662\,A + 167\cdot78 \text{ (unit of stature} = \text{cm.)} \end{cases}$

Swedes: $S = -\cdot1008\,A + 163\cdot51$ („ „ „).

From these we conclude that the English shrink in stature more than the Swedes with age, and that women of the General Hospital Population in both races shrink more than the men. The lines are represented in Fig. 1 below, which gives a fair idea of the effect of age on stature. We now inquire whether

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* See for example Macdonell's results for 3000 criminals, *Biometrika*, Vol. I. p. 202, and Fawcett and Pearson's results for 1000 Cambridge graduates, *R. S. Proc.*, Vol. 66, p. 23.

† *R. S. Proc.*, Vol. 61, pp. 365.

‡ *Biometrika*, Vol. IV. pp. 53, 54.

this shrinkage is peculiar to the General Hospital Population, or extends to the normal population of a district. The only comparative results that we know

Fig. 1.

———— Shrinkage in Stature with Age ————
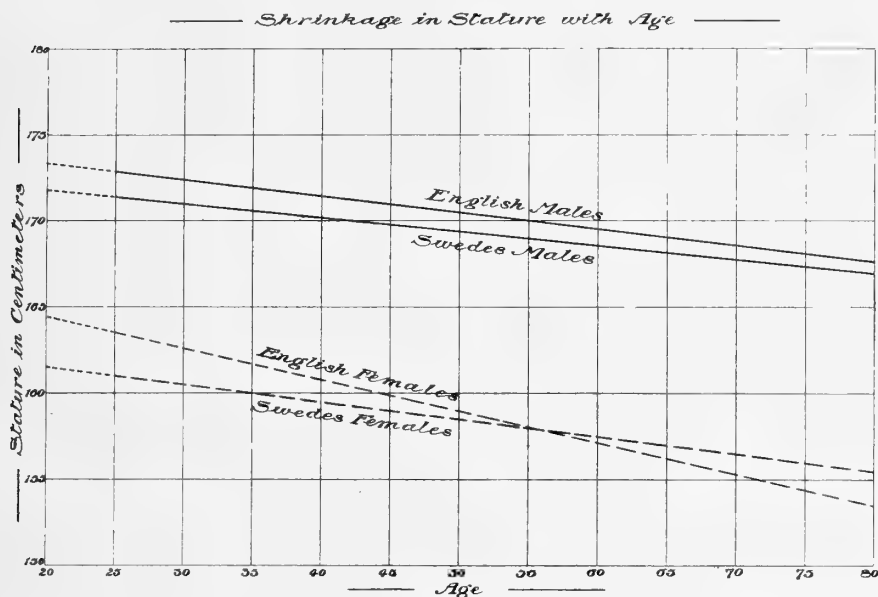
of are the regression lines calculated for A. O. Powys' experience of New South Wales criminals by K. Pearson*. We have:

English Males, Hospital Population : $S = -\cdot0378A + 69\cdot02$

New South Wales Males, Criminal Population : $S = -\cdot0337A + 68\cdot34$

English Females, Hospital Population : $S = -\cdot0655A + 66\cdot05$

New South Wales Females, Criminal Population : $S = -\cdot0361A + 63\cdot61$

In all cases the unit of stature is an inch.

Hence it would appear that the hospital population of males shrinks slightly more rapidly than the criminal population, and the hospital population of females very sensibly more. This is probably due to the larger proportion of male acute cases and of female chronic cases. We do not think the present results do more than confirm the conclusion of Powys' paper that there is for normally healthy individuals a shrinkage of a little more than $\frac{1}{3}''$ per ten years.

(c) If we take in the next place head diameters we are not able to compare our General Hospital Population with Matiegka's data, for his paper† gives no clue to the ages of the individuals whose head measurements he has taken, and further those measurements are scarcely comparable with external measurements

* Biometrika, Vol. i. p. 49.

† Sitzberichte des k. böhmischen Gesellschaft d. Wissenschaften, Math. Naturw. Classe, Jahrg. 1902, pp. 1—75.

on the head. Our results therefore must at present stand unsupported. They show a sensible shrinkage of the head diameters, greater in the woman and greatest in the auricular height for both sexes. The amount of it is represented graphically in Fig. 2. The equations to the regression lines are as follows, age being measured in years and the diameters in centimetres:

Maximum Head Length, Males: $L = -\cdot0348A + 192\cdot07,$

„ „ . Females: $L = -\cdot0623A + 186\cdot06,$

Maximum Head Breadth, Males: $B = -\cdot0406A + 151\cdot34,$

„ „ Females: $B = -\cdot0867A + 148\cdot53,$

Auricular Height, Males: $H = -\cdot0799A + 136\cdot84,$

„ „ Females: $H = -\cdot1059A + 134\cdot07.$

Fig. 2.

It will thus be seen that the woman shrinks with age about twice as fast as the man in head length and breadth, but only about $\frac{5}{4}$ as fast in auricular height. The very sensible shrinkage for both sexes in head height is peculiarly noteworthy, as it of course contributes to the shrinkage in stature. It seemed worth while investigating whether these results are peculiar to a hospital population. Accordingly 1306 criminals were taken, whose age, head length and head breadth but (unfortunately) not head height were recorded, and the correlations worked out for age and these two diameters. The correlation tables are given as

Tables XIV. and XV. below. The actual values obtained are, ages running from 21 to 70:

Mean Age: 33·32 ± ·19. Standard Deviation: 10·2219 yrs. ± ·1349.
Maximum Length: 191·44 ± ·11. Standard Deviation: 6·0025 cms. ± ·0792.
Maximum Breadth: 150·30 ± ·10. Standard Deviation: 5·1329 cms. ± ·0677.
Correlation of Age and Head Length + ·1138 ± ·0184.
Correlation of Age and Head Breadth: + ·0349 ± ·0186.

TABLE XIV.

1306 *Criminals. Correlation between Head Length and Age.*

Age.

Head Length.	21—25	26—30	31—35	36—40	41—45	46—50	51—55	56—60	61—65	66—70	Totals
171	—	1	—	—	—	—	—	—	—	—	1
172	—	—	—	1	—	—	—	—	—	—	1
173	1	—	1	—	—	—	—	—	—	—	2
174	—	—	—	—	—	—	—	—	—	—	—
175	1	—	—	1	—	1	—	—	—	—	3
176	—	2	—	1	—	—	—	—	—	—	3
177	1	4	—	—	—	—	—	—	—	—	5
178	3	3	—	1	—	—	—	—	—	—	7
179	5	2	3	—	—	1	—	1	—	—	12
180	4	3	2	3	—	—	—	1	—	—	13
181	7	5	3	1	—	1	—	—	—	—	17
182	7	7	6	3	3	1	—	—	1	—	28
183	6	6	2	6	2	2	—	—	—	—	24
184	15	15	2	4	1	1	4	—	—	1	43
185	18	17	6	5	4	1	1	—	1	—	53
186	19	15	7	5	7	3	1	—	—	—	57
187	17	15	3	7	5	3	4	—	1	—	55
188	26	10	14	7	6	2	2	1	—	—	68
189	25	16	13	15	4	6	—	1	3	—	83
190	24	19	14	13	5	4	3	2	—	1	85
191	34	27	12	3	9	3	2	1	2	3	96
192	30	14	21	15	7	7	3	3	1	1	102
193	22	20	15	12	3	3	2	—	2	—	79
194	28	12	14	10	9	3	2	1	4	—	83
195	18	13	10	5	7	8	2	—	3	—	66
196	14	16	11	10	4	6	3	1	—	1	66
197	18	6	10	5	6	8	2	—	1	—	56
198	15	6	10	6	2	1	2	—	—	1	43
199	6	10	4	6	5	1	1	1	1	—	35
200	5	9	5	4	2	1	1	2	—	1	30
201	6	4	1	2	—	4	1	—	2	—	20
202	6	9	4	—	1	—	1	1	2	—	24
203	3	4	3	2	1	—	1	—	—	—	14
204	1	7	2	2	—	—	1	—	—	—	13
205	2	2	—	2	—	1	—	1	—	—	8
206	—	1	1	1	—	—	—	—	—	—	3
207	—	1	—	1	—	2	1	—	1	—	6
208	—	—	—	—	—	—	—	—	—	—	—
209	—	—	—	—	1	—	—	—	—	—	1
210	—	—	—	—	1	—	—	—	—	—	1
Totals	387	301	199	159	95	74	40	17	25	9	1306

18—2

TABLE XV.

1306 *Criminals.　Correlation between Head Breadth and Age.*

Age.

	21—25	26—30	31—35	36—40	41—45	46—50	51—55	56—60	61—65	66—70	Totals
134	—	1	—	—	—	—	—	—	—	—	1
135	—	—	—	—	—	—	1	—	—	—	1
136	2	—	1	1	—	—	—	—	—	—	4
137	1	—	2	—	—	—	—	—	—	—	3
138	2	3	1	—	—	—	—	—	—	—	6
139	1	6	—	—	—	—	—	—	—	—	7
140	2	3	—	2	1	1	—	—	—	—	9
141	11	4	3	—	—	—	—	—	—	—	18
142	9	5	2	2	1	1	—	1	—	—	21
143	12	6	5	4	2	5	2	1	2	—	39
144	14	7	11	8	5	1	—	—	—	—	46
145	21	15	7	11	4	4	1	1	1	—	65
146	26	9	18	16	8	5	4	—	1	3	90
147	25	18	18	8	8	7	3	2	2	1	92
148	18	25	11	13	8	4	4	—	3	—	86
149	32	31	10	7	7	4	3	2	2	—	98
150	27	24	14	13	6	5	—	1	1	1	92
151	26	22	19	12	6	9	5	1	3	1	104
152	31	23	14	9	9	4	2	2	2	1	97
153	26	21	16	11	5	6	2	1	3	—	91
154	28	18	8	7	7	4	1	—	—	—	73
155	15	13	10	8	3	2	1	1	2	1	56
156	13	15	11	9	4	—	4	—	—	1	57
157	13	9	5	7	2	4	2	1	3	—	46
158	8	7	4	5	1	2	—	—	—	—	27
159	6	5	4	4	2	—	1	—	—	—	22
160	2	6	2	2	2	3	1	—	—	—	18
161	5	2	2	—	—	—	—	1	—	—	10
162	5	2	—	—	2	1	2	—	—	—	12
163	1	—	1	—	2	—	—	1	—	—	5
164	3	1	—	—	—	1	—	—	—	—	5
165	—	—	—	—	—	1	1	1	—	—	3
166	1	—	—	—	—	—	—	—	—	—	1
167	—	—	—	—	—	—	—	—	—	—	—
168	—	—	—	—	—	—	—	—	—	—	—
169	1	—	—	—	—	—	—	—	—	—	1
Totals	387	301	199	159	95	74	40	17	25	9	1306

(Head Breadth.)

A comparison of Table II. shows that the criminal is younger and less variable in age than the General Hospital Population. To test the effect of the more youthful population, we omitted the criminals less than 26 years of age. The results were as follows for head length of 916 cms.:

Mean Age:　　　　　37·46 ± ·21.　Standard Deviation: 9·5254 ± ·1498.
Maximum Length: 191·73 ± ·14.　Standard Deviation: 6·1534 ± ·0968.
Correlation of Age and Head Length: + ·1062 ± ·0220.

Clearly the correlation is not sensibly affected, while the age is sensibly and the mean head length slightly raised. We think we must conclude from this result

that in the criminal population there is no shrinkage of head breadth, perhaps a slight increase, with age, and a sensible increase, not a shrinkage, of head length with age. Unfortunately we could not test the auricular height change. It follows from this that the shrinkage of head size with age appears to differentiate the General Hospital from the criminal and very possibly from the normally healthy general population. Hence in determining the probable brain-weight from size of the head in a normal population probably little stress ought to be laid on any change of the diameters and therefore their product with age. It is possibly in our case a result of the increasing amount of chronic illness in the older members of the General Hospital Population. The possible shrinkage of the head with age in the general population would form an interesting subject for investigation.

Of course when the three diameters have negative correlation with age, we should expect to find, as we do find, that the product would shrink with age. We have no data to compare with the observed shrinkage of the horizontal circumference with age.

(d) The correlation of the horizontal circumference with stature, ·33, is we believe new, but is very much what we might have anticipated. So also is the correlation, ·31, of stature with the diametral product. Thus Macdonell found for 3000 criminals the correlation of stature and maximum head length, ·34, of stature and maximum head breadth, ·18, and of stature and face breadth, ·35*.

With regard to the correlation between the horizontal circumference U and the product P of the three diameters L, B, H, we note that if V be a coefficient of variation we must have:

$$r_{PU} = (V_L r_{UL} + V_B r_{UB} + V_H r_{UH})/V_P$$
$$= (2\cdot95 r_{UL} + 3\cdot62 r_{UB} + 3\cdot39 r_{UH})/7\cdot68$$

for English males from Table II. Now we are unacquainted at present with any determinations of r_{UL}, r_{UB} and r_{UH}, but in round numbers r_{UL} and r_{UB} cannot differ much from ·7 and r_{UH} from ·5. Substituting we find $r_{PU} = \cdot82$; so that the observed value ·83 for males appears quite probable. A similar investigation gives ·79 as against the observed ·80 for females.

(e) We turn lastly to the correlations between brain-weight and the other characters.

We see in the first place that cephalic index is the least important of these correlated characters, and further that its correlation with brain-weight varies markedly from male to female. It is within the limits of random sampling zero for men, and has the quite sensible value ·2 for women. We have no similar data to compare our results with. All we can do is to consider the correlation of skull capacity with cephalic index, which has been determined in a number of cases. The irregular and divergent values of this correlation were first pointed out by

* *Biometrika*, Vol. i. p. 202.

Dr A. Lee[*]. Thus she found it sensibly zero for ♀ Theban mummies and $-\cdot15$ for male Thebans. A list of values is given by Lewenz and Pearson[†] in which it is shown that ♂ English crania give a practically zero value for the correlation between cephalic index and capacity. That value was, however, obtained indirectly, i.e. from the L and B correlations with each other and the capacity, and accordingly it seemed worth while to investigate the matter directly on the Whitechapel crania. Accordingly we extracted from Macdonell's tables[‡] the capacities and cephalic indices of 70 ♂ and 80 ♀ crania and worked out directly the correlation between the two characters. We found:

Correlation of Cranial Capacity and Cephalic Index for Whitechapel Crania:

Males: $+ \cdot093 \pm \cdot080$.   Females: $- \cdot099 \pm \cdot075$.

Neither value is really significant having regard to the probable error, and we might therefore conclude that this correlation is sensibly zero for English crania. Such a result would be in good accordance with the sensibly zero correlation between brain-weight and cephalic index for males, but not in accordance with the result for English females. Assuming that there might be an error somewhere in the arithmetic the correlation between brain-weight (w) and cephalic index I was tested indirectly by the formula

$$r_{Iw} = (V_B\, r_{Bw} - V_L\, r_{Lw})/V_I \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots(2),$$

with the following results:

TABLE XVI.

Cephalic Index and Brain-weight.

Sex	Direct Calculation	By Formula (2)
Males ...	$+ \cdot0014$	$+ \cdot0373$
Females ...	$+ \cdot2110$	$+ \cdot2084$

The degree of closeness is as great as we might expect from the two methods, the square of $V/100$ being neglected in obtaining formula (ϵ). There is, therefore, we think, no doubt that, while the correlation between cephalic index and capacity is sensibly zero for English crania of both sexes, the females of a General Hospital Population have a sensible correlation between brain-weight and cephalic index. It is conceivable that this is due to some exaggeration of breadth in the measurement of the heads of the younger females, for whom the brain-weight would be larger and as a rule the hair interfere more than would be the case for older women or for the males[§]. On the whole we think we have very adequate grounds for omitting the cephalic index entirely from our prediction equations for the brain-weight.

[*] *Phil. Trans.* Vol. 196 A, p. 232.   [†] *Biometrika*, Vol. III. p. 374.
[‡] *Biometrika*, Vol. III. Tables after p. 244.
[§] The hair difficulty occurs *twice* in the case of measuring B and only *once* in measuring L. Hence B would tend to be more exaggerated than L.

Looking generally at the brain-weight correlations we see that predictions based on the diametral product and the circumference will give us the best results. Age and stature have formed such a large part of the anatomical discussions on the subject that we feel compelled to retain them, but clearly a knowledge of any diameter of the head is of more importance in predicting brain-weight than either stature or age. After careful consideration we determined to make our most complex prediction formula depend upon a knowledge of the diametral product, the horizontal circumference, the stature and the age of the subject.

Before considering prediction formulae, however, it is well to compare our correlations with any available material.

In the first place we take the English skull.

TABLE XVII.

Comparison of Skull and Head Correlations.

Character		General Hospital Population			Whitechapel Crania *	
		♂	♀		♂	♀
L with ...	w	·598 ± ·040	·545 ± ·049	C	·597 ± ·051	·691 ± ·040
B with ...	w	·489 ± ·048	·607 ± ·044	C	·631 ± ·048	·646 ± ·044
H with ...	w	·544 ± ·044	·673 ± ·038	C	[·501 ± ·059	·600 ± ·049]
U with ...	w	·623 ± ·032	·627 ± ·042	C	·820 ± ·026	·848 ± ·021
P with ...	w	·703 ± ·032	·749 ± ·031	C	[·750 ± ·035	·823 ± ·025]

We notice that the correlations of capacity and external measurements are all greater than the corresponding correlations for the brain-weight. The apparent exception (C and L, ♂ is within the limits of probable error), namely C and H for both sexes, arises, we think, from the fact that H is here the basio-bregmatic and not the auricular height of the skull, and no stress can thus be laid upon this exception[†]. The conclusion, which we might have anticipated, is therefore forced upon us that it is easier to predict capacity from external measurements on the skull than brain-weight from external measurements on the living head.

The values of the correlation between C and P, although obtained indirectly, are probably very close: thus A. Lee[‡] gives for the corresponding values in Bavarian crania: ♂, ·700 ± ·034 and ♀, ·814 ± ·023, the differences for the two races being well within the limits of random sampling.

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* Cited from Macdonell, *Biometrika*, Vol. iii. p. 232. Except those for P and C which are deduced from the formula : $r_{CP} = (V_L r_{CL} + V_B r_{CB} + V_H r_{CH})/V_P$. All these constants are known for the Whitechapel crania except r_{CH} the correlation of capacity and auricular height, we have used for this the known correlation of capacity and basio-bregmatic height, cited in brackets under C and H.

† If the auricular height be more highly correlated with capacity than the basio-bregmatic height, the result is very satisfactory, because the former is all that is obtainable on the living head. The point should be investigated as ample data exist.

‡ *Phil. Trans.* Vol. 196 A, p. 237.

If we pass from intra-racial to inter-racial comparisons, Pearl's memoir offers valuable material. Thus he has calculated the correlation of w with L and B for Bohemians, based, it is true, on Matiegka's rather doubtful skull measurements. Again, he provides us with the brain-weight and stature and brain-weight and age correlations for Swedes and Hessians. These are reproduced in the accompanying table. Considering how widely correlation results differ from race to race the general agreement is not without significance. It increases our confidence in the possibility of good results being obtained from careful post-mortem room measurements.

<div align="center">TABLE XVIII.</div>

Comparison of English and other European Brain-weight Correlations.

Correlation	Middlesex Hospital Data		Other European Data		
	♂	♀	Group	♂	♀
w and L	·598 ± ·040	·545 ± ·049	Bohemian	·548 ± ·027	·360 ± ·047
w and B	·489 ± ·048	·607 ± ·044	Bohemian	·466 ± ·031	·504 ± ·040
w and S	·289 ± ·057	·367 ± ·060	Swedes	·183 ± ·032	·349 ± ·039
,, ,,	,, ,,	,, ,,	Hessians	·182 ± ·030	·183 ± ·039
w and A	− ·256 ± ·058	− ·372 ± ·060	Swedes	− ·249 ± ·031	− ·234 ± ·042
,, ,,	,, ,,	,, ,,	Hessians	− ·167 ± ·030	− ·360 ± ·035

The general agreement is, we have said, not bad considering the differences in method of measuring and probably in heterogeneity of material. The Swedish are quite fairly concordant with the English. So are the Bohemians (considering the poor method used by Matiegka for determining the breadth and length of the skull), except for the correlation of weight and length for women, where there is a bad and inexplicable fall, although a slight fall might have been expected from the English results. The Hessian values appear to be irregular, and one is inclined to doubt the accuracy with which the corpse length has been determined, or the relative homogeneity of the series; individuals under 20 ought not to have been included in the age and stature correlations.

(7) *On the Regression Equations of Brain-weight on other Characters.*

(*a*) The following table will allow the reader to see at a glance the average changes in brain-weight w and product P involved in unit change from the mean in each of the observed characters.

As illustration, take the following case, which is closely that of Jeremy Bentham:

Stature: Mean (?), $U = 560$, $L = 192$, $B = 153$, $H^* = 127$, and Age = 85 yrs.

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* Measured from *top* of auricular orifice H is 121 (see *Biometrika*, Vol. III. p. 394), we must add, say, 6 mm. to compare with Gladstone's data from centre of orifice.

TABLE XIX.

Increase of Brain-weight in grs. and Diametral Product in cm.[3] above their means for each unit increase in the other observed Characters.

Regression Coefficient of Character with Probable Error.

		One mm. in L	One mm. in B	One mm. in H	One mm. in U	One Year in A	One inch in S
Mean of Character	♂	190·36	149·34	132·20	555·79	49·26	67·16
	♀	183·20	144·55	129·21	533·25	45·90	63·05
Brain-weight Increase in grs.	♂ *Mean* 1327·69 grs.	11·0728 ± ·9254	9·3936 ± 1·0463	12·5628 ± 1·2074	4·0895 ± ·3204	− 2·1974 ± ·5245	9·8277 ± 2·0338
	♀ 1224·90 grs.	9·7599 ± 1·0446	10·5593 ± ·9607	15·5037 ± 1·1841	3·7344 ± ·3224	− 2·7245 ± ·4721	13·1934 ± 2·3271
Diametral Product Increase in cms.	♂ 3782·91 cm.[3]	—	—	—	15·1711 ± ·6424	− 4·1097 ± 1·4730	29·6394 ± 5·6361
	♀ 3427·03 cm.[3]	—	—	—	13·5656 ± ·6913	− 5·9478 ± 1·3769	40·5999 ± 6·4775

A man of his age would have lost 78·5 grs. of his original mean age brain-weight, or have on this account only 1249 grs. instead of 1328 grs., the average at age 49·26. He has an excess of 1·64 mm. in head length, an excess of 3·66 mm. in head breadth, and a defect of 5·20 mm. in auricular height. Also an excess of 4·21 mm. in horizontal circumference. Thus on the first count his brain-weight would be 18·15 grs. above the average, on the second 34·38 grs. above the average, on the third 65·33 below the average, and on the fourth 17·22 grs. above the average. But it would not be proper to club all these together and say that on account of his head measurements we should expect Bentham to be 4·42 grs. above the average when at the mean age. For all these characters are correlated, and we have seen that they, especially the auricular height, are influenced by age. Hence it is absolutely needful to use multiple regression formulae. These will be applied to Bentham's measurements later, but the above will suffice to show how a single character may have its influence appreciated.

(*b*) We now pass to the multiple regression equations. These were found in the usual way, namely, by the calculation in this case of 25 constituent determinants and the minors of their first row of constituents. The work is straightforward, but laborious. For purposes of comparison the prediction formulae based on diametral

product and horizontal circumference together (without age and stature), and on each of these alone, are also added.

Prediction Formulae.

(i)[a]   ♂ : $w = \cdot 1987P + \cdot 8644U - 1 \cdot 1910A + 1 \cdot 7508S + 36 \cdot 8559.$
Probable error of prediction: 48·5 grs.   Mean error: 57·4 grs.

(i)[b]   ♀ : $w = \cdot 2195P + \cdot 5067U - 1 \cdot 2395A + 1 \cdot 0721S + 191 \cdot 8298.$
Probable error of prediction: 43·2 grs.   Mean error: 51·0 grs.

(ii)[a]   ♂ : $w = \cdot 2136P + \cdot 8486U + 47 \cdot 9443.$
Probable error of prediction: 49·6 grs.   Mean error: 58·7 grs.

(ii)[b]   ♀ : $w = \cdot 2466P + \cdot 3893U + 172 \cdot 2420.$
Probable error of prediction: 44·7 grs.   Mean error: 52·8 grs.

(iii)[a]   ♂ : $w = \cdot 2519P + 374 \cdot 7628.$
Probable error of prediction: 49·9 grs.   Mean error: 59·0 grs.

(iii)[b]   ♀ : $w = \cdot 2653P + 315 \cdot 8281.$
Probable error of prediction: 44·7 grs.   Mean error: 52·9 grs.

(iv)[a]   ♂ : $w = 4 \cdot 0895U - 945 \cdot 1785.$
Probable error of prediction: 54·9 grs.   Mean error: 64·9 grs.

(iv)[b]   ♀ : $w = 3 \cdot 7344U - 766 \cdot 4563.$
Probable error of prediction: 53·0 grs.   Mean error: 62·8 grs.

We see from these results: first, that owing to the higher correlations it is possible to predict female brain-weights somewhat more accurately than male brain-weights from external measurements of the head.   Secondly, that the fundamental character is the product of the three diameters.   A comparison of (iii) and (iv) shows that it is worth a good deal more than the circumference, and further comparing (iii) with (i) we see that if we predict from the diametral product *alone*, we shall only make 2 grs. worse mean error than if we predict from all four characters. From this standpoint it is clear that age and stature allowances are not nearly as important matters as head diameters if we wish to predict brain-weights from physical measurements.

The formulae were then applied to 24 ♂ and 24 ♀ cases selected at random, being every fourth or fifth case in the observation book.   It does not seem needful to reproduce the individual results, but the general conclusions are given in Table XX.

Our actual results are therefore somewhat in defect of the mean errors to be expected by theory, especially in the case of (i), (ii), and (iii) for the males.   In the case of the females the defect is very little, and the theoretical and observed mean errors are close.   We think it quite safe to say that the brain-weight of a living individual can be predicted with a mean error of not more than 50 grs.   It is

noteworthy that Dr Lee's diametral formula[*] gave a mean error in predicting skull capacity also of about 50 cm.[3] It seems to us that no linear formula based upon external measurements is likely to give any better result. For all such measurements are more or less closely correlated, and, as has been shown elsewhere, increasing the number of correlated variables does not much narrow the range of variation of the predicate[†].

TABLE XX.

Prediction of the Brain-weights of 48 *Individuals from External Measurements.*

Mean Error, in each instance of 24 cases.

Formula	♂ sex	♀ sex
(i)	47·50 grs.	50·65 grs.
(ii)	48·83 grs.	50·65 grs.
(iii)	52·63 grs.	49·10 grs.
(iv)	60·21 grs.	60·15 grs.

(c) It may be of interest to try these formulae on one or two actual heads.

In the *Société d'Anthropologie de Paris, Bulletins* 1866, II. Série, T. I. p. 209 are given the measurements of a skull reputed to be that of Dante. Unfortunately all the measurements we require are not given. We find for cranial lengths: $L = 178$ mm., $B = 140$ mm., $U = 525$ mm. The bi-auricular transverse arc, doubtless measured from centre to centre of auricular orifice, was 310 mm., and the total height of the skull, i.e. the basio-bregmatic height, 140 mm. By examining crania with a similar transverse arc and basio-bregmatic height, we consider that 118 mm. would be a close value of the auricular height of Dante's skull. The height of the corresponding skeleton was 155 cms. The mean stature of the Tuscans is 165 cms., and of the more northern Piedmontese 162 cms. We shall almost certainly exaggerate Dante's stature if we put it at 164 cms. We have next to convert skull measurements into head measurements, and in doing this we have rather magnified the values in the light of recent investigations on the thickness of the flesh. Dante died at 56 years of age. Alongside Dante we can take the measurements of Jeremy Bentham's head[‡], and for further illustration the head measurements of one of ourselves. We have then:

[*] *Phil. Trans.* Vol. 196 A, p. 238 *et seq.* *Biometrika*, Vol. III. p. 371.
[†] *Phil. Trans.* Vol. 190 A, p. 466.
[‡] *Biometrika*, Vol. III. p. 394.

TABLE XXI.

Special Individual Heads.

Characters	Dante	Bentham		P.
L	188	192		191
B	150	153		150
H	124	126*		135
U	525	560		555
Age	56	85		47
Stature ...	164 cms.	Mean (?)		71″
	At 56 years	At 85 years	At 49 years	At 47 years
Probable Brain-weight (i)	1244	1272	1351	1353
,, ,, (ii)	1240	1314		1328
,, ,, (iii)	1256	1307		1329
,, ,, (iv)	1202	1345		1324

The average English brain-weight for a "General Hospital Population" of the mean age of 49 years is 1328 grs. It is therefore clear that P.'s brain-weight is essentially mediocre. The great Bentham could have had a brain-weight which at the mean age was at most only a few grammes above the normal. This corresponds exactly to the general mediocrity which has been already noticed in Bentham's physical measurements[†]. As for Dante his probable brain-weight was 80 to 90 grs. below that of the mean of the English General Hospital Population, and this corresponds well with the observed skull capacity. If the reputed skull were really Dante's we are forced to conclude either (i) that there are most striking individual exceptions to any rule that extreme ability is associated with large brain-weight, or (ii) that it is impossible to apply the formulae deduced from one race to determine the brain-weight of a member of a second. As the present paper and Pearl's memoir both show that at least (ii) is true we may at once reject (*a*) any intra-racial reconstruction formula used inter-racially, and (*b*) any demonstration of the association of great intellectual power with large brain-weight, which is based on lists of the brain-weights of distinguished men of all nations, clubbed together without any regard to age, stature, or race, and compared with the mean brain-weight of a General Hospital Population.

We think, therefore, that the above formulae should be confined to English persons. That if they are to be applied to any other race the mean values of *w*, *P*, *U*, *A*, and *S*, say $\overline{w}$, $\overline{P}$, $\overline{U}$, $\overline{A}$, and $\overline{S}$, should be adopted for that race. The fundamental formulae (i) then become :

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* Formulae are calculated for *H* measured from centre of auricular orifices, and *OH* in *loc. cit.* was from the top.

† *Biometrika*, Vol. iii. p. 394.

(i)^a ♂: $w - \overline{w} = \cdot 1987\,(P - \overline{P}) + \cdot 8644\,(U - \overline{U}) + \cdot 6893\,(S - \overline{S}) - 1\cdot 1910\,(A - \overline{A})$,

(i)^b ♀: $w - \overline{w} = \cdot 2195\,(P - \overline{P}) + \cdot 5067\,(U - \overline{U}) + \cdot 4221\,(S - \overline{S}) - 1\cdot 2395\,(A - \overline{A})$,

where w and $\overline{w}$ are in grs., P and $\overline{P}$ in cm.^{3}, U and $\overline{U}$ in mm., S and $\overline{S}$ in centi-metres, no longer inches, and A and $\overline{A}$ in years. These formulae may enable the reader to obtain approximations for other groups, for which some of the mean characters are determinable. For example : a knowledge of $\overline{L}$, $\overline{B}$, and $\overline{H}$ for the Swedes would enable us to reach an approximate formula for Swedes from Retzius' data for $\overline{w}$, $\overline{S}$ and $\overline{A}$. The assumption made is that the multiple regression coefficients change far less from race to race than the "type *."

(8) *On the Graphic Representation of "Scatter" of Brain-weight about the Probable Value.*

In the course of the present investigation we have given the probable errors and mean errors of brain-weight prediction as deduced from the arithmetical deduction of our data. It seems desirable, with a view of impressing on anatomists the real character of variation in physical measurements, and how hopeless is the hunting for anything like an exact formula for brain-weight, to put the relationship between brain-weights and diametral products and brain-weights and horizontal circumferences in a graphical form for the present English data. In these diagrams each dot represents an actual observed case, and the best fitting straight line to the system of dots, calculated by the usual correlation method, is drawn slanting across the diagram. Figs. 3 and 4 give the results for males and females deducible from the diametral products. Figs. 5 and 6 give similar results for the horizontal circumferences. We see that the scatter is less for the diametral product than for the horizontal circumference, but we see further how hopeless it would be to attempt from these intra-racial data to achieve more by any curve than can be done by a straight line. When we pass, as in formula (ii), to double regression results we have merely such a system of dots in space of three dimensions, and multiple regression data, as in formula (i), only represent a more complicated system of dots in many dimensioned space. So soon as this conception of the actual conditions of variation is realized, we believe that anatomists will once for all abandon any attempt to represent by mathematical formulae, other than correlation results, this "scatter" which marks all the inter-relationships of anthropometric characters. We see in a general way also that increasing the number of dimensions of our space does not in the least necessitate any sensible reduction in the extent of the scatter—for if it did it would involve the principle that a plane must fit a system of points in space more closely than a line a system of points in a plane. A few variables highly correlated with the predicate and lowly correlated with each other is the ideal system to be hunted for, but it is not always to be found, even if it exists†.

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* *Phil. Trans.* Vol. 200 A, pp. 21 *et seq.*

† For example, two parents give better results for predicting the character of an individual than two of his brethren, because although the latter may be more highly correlated with him, these are more highly correlated together than the parents are.

Fig. 3. Male Brain-weights and Diametral Products.

*Diametral Product in cm.*³
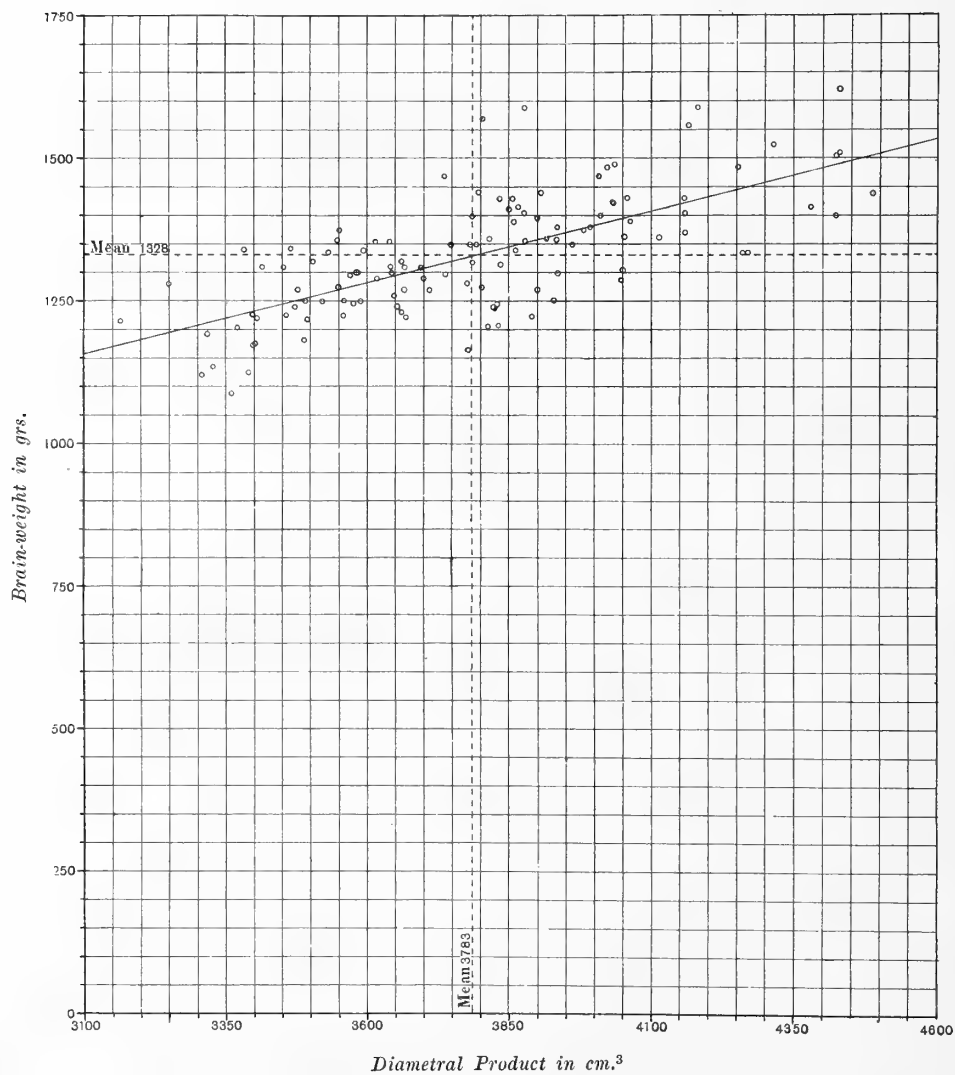
FIG. 4. Female Brain-weights and Diametral Products.

FIG. 5.   Male Brain-weights and Circumferences.

Horizontal Circumference in mm.

Fig. 6. Female Brain-weights and Circumferences.

Horizontal Circumference in mm.
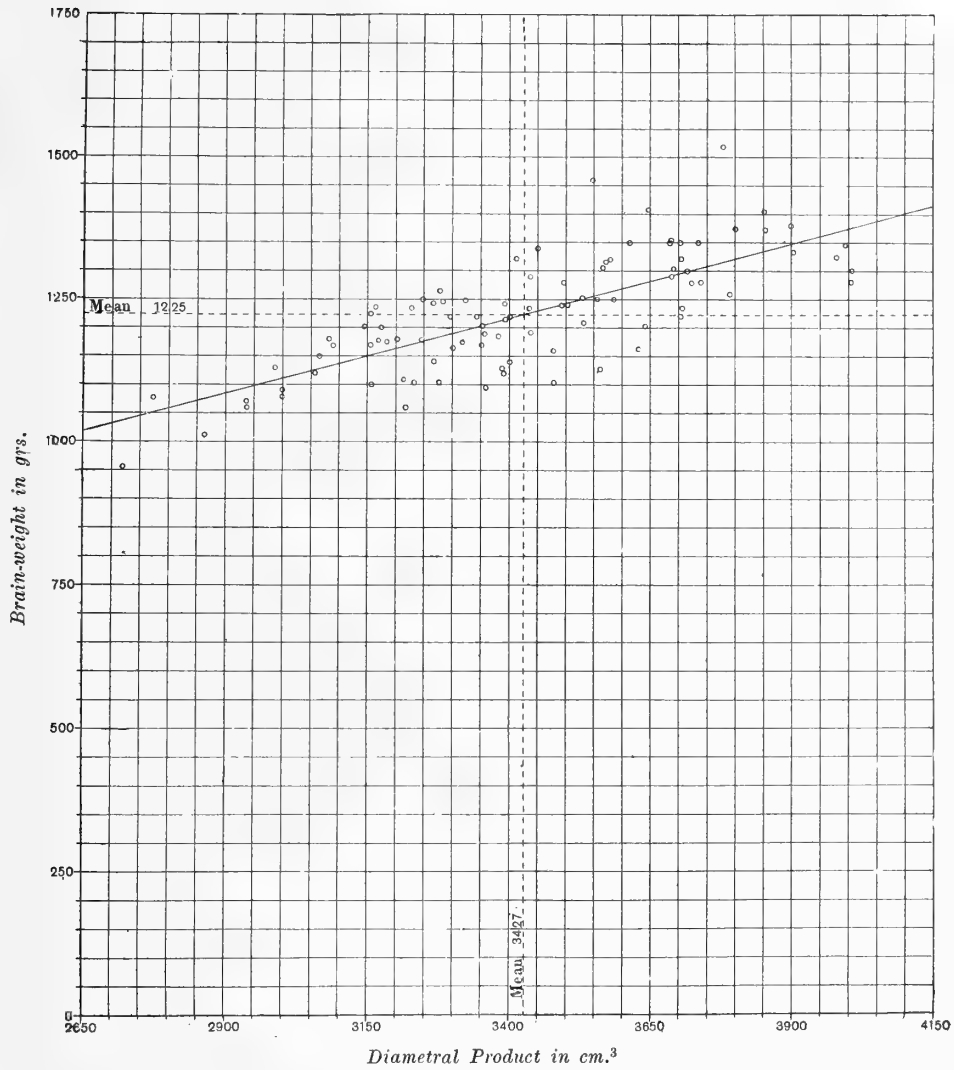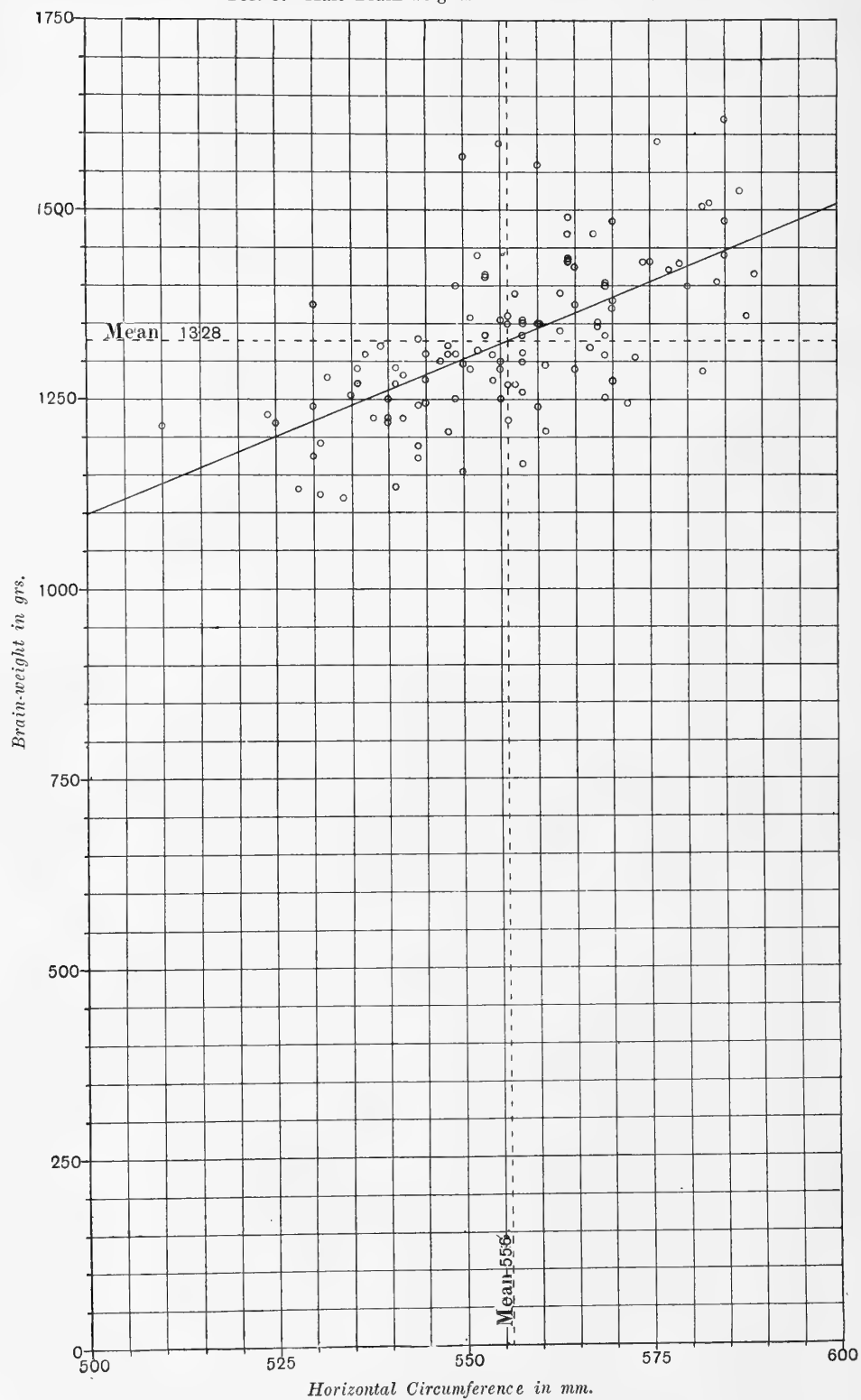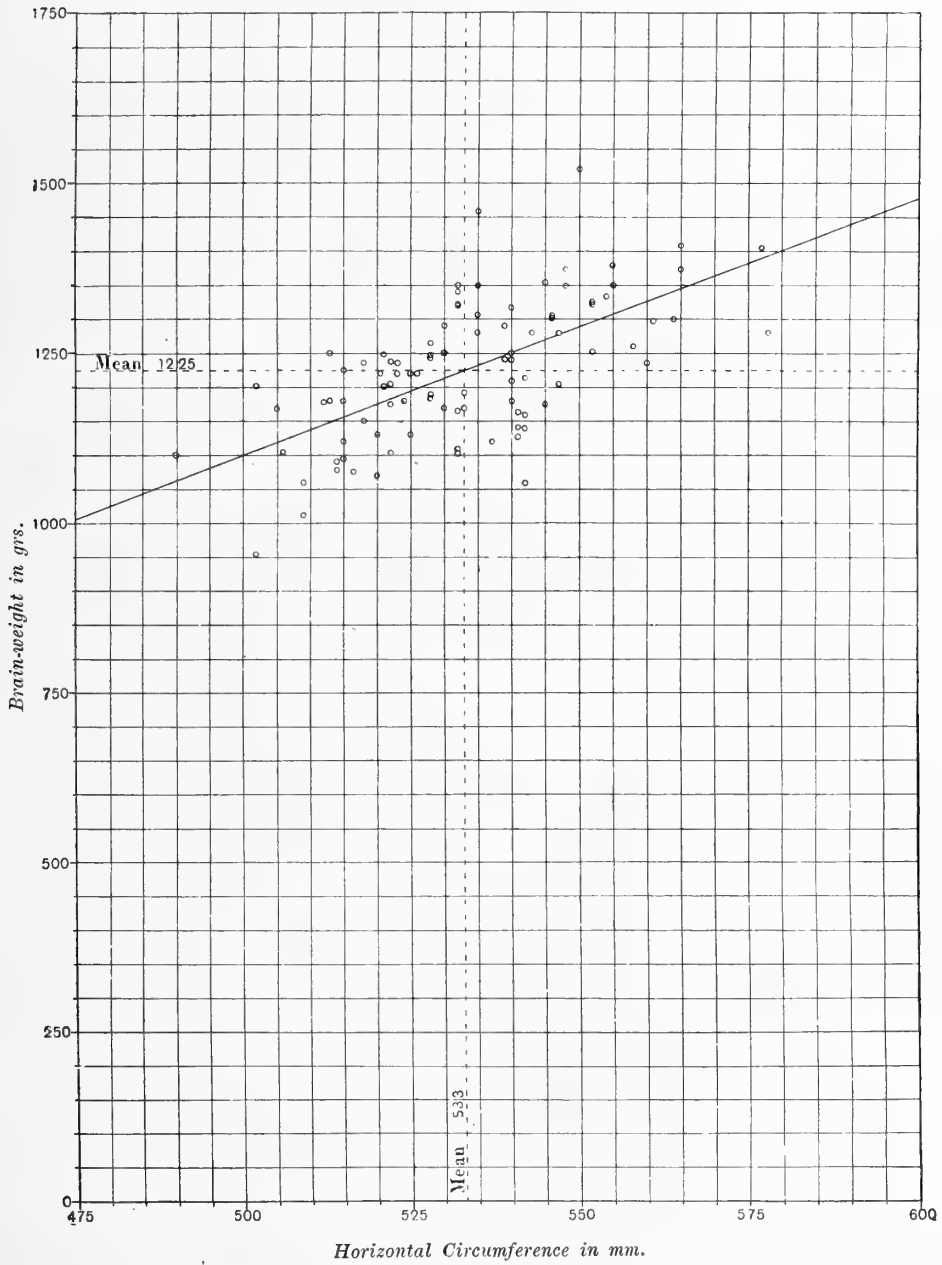
(9) *On the Human Prime in the Case of Brain-weight.*

Pearl cites[*] Marchand as stating that the human brain-weight reaches its maximum between 15 and 20 years, and remains practically constant until 50, when senile degeneration begins. Gladstone cites Boyd and Vierordt to the effect that the maximum brain-weights are to be found in youth. Gladstone's own data certainly show this maximum in almost extreme youth, but the degeneration begins immediately the prime is reached. It is true the data are very scanty, but such as they are, they are suggestive. We have:

Males	8—19,	Brain-weight	1407 grs.
„	20—25,	„	1394 grs.
„	25—46,	„	1380 grs.
„	over 46,	„	1320 grs.

Even among the 8 to 19 group the group 12 to 15 has the largest average brain-weight, i.e. 1443 grs. The numbers are very small, and of course the cases of chronic illness in the older groups are more numerous. But Table I, p. 125 shows that even among the acute cases the fall in brain-weight owing to age alone is very substantial. Accordingly the human prime in brain-weight seems to fall before 20 years, and after 20 years there is, on the whole, a continuous fall in brain-weight. This result must be steadily kept in view when any attempt is made to correlate brain-weight and intelligence. It can hardly be argued that the human intellectual prime corresponds to the human brain-weight prime. And while brain-weight steadily changes, first rising and then falling with age, intelligence will be shown in a forthcoming memoir by one of our number to be almost stationary during the rise of brain-weight. Further, it is clear that the correlations and regressions will be much modified if we include in our results the rising period of brain-weight with the falling period. For this reason we have limited the material of this paper to cases in which the individual was 20 years and upwards. In this way our regressions will be much more nearly linear. A glance, for example, at Powys's diagram for change of stature with age[†] would, we think, convince the reader that without the use of skew correlation curves it is not possible to mingle observations on a physical character for ages before and after the "prime." We think that the source of the irregularity in the Hessian correlations for weight, stature, and age may be due to including individuals between 15 and 20 years of age[‡].

The determination of the "prime" in every anthropometric measurement is a matter of very great importance and interest. Our own experience is that up to the prime we get a curved line of growth not widely divergent from the logarithmic in character. After the prime the degeneration is very gradual, and for practical purposes almost linear. It would be simple, therefore, to allow for this by age

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* *Biometrika*, Vol. iv. p. 20.   † *Biometrika*, Vol. i. p. 47.

‡ Stature has a later prime than brain-weight, and this might well account for the irregular Hessian results in Pearl's Table IX. See p. 44 of this number of the Journal.

correlations, were it not that the prime appears to vary sensibly from one character to another. In other words, the "human prime" is purely relative to the special organ considered. There appears to be also no *period* of prime, but rather on the average an *instant* of prime, the time-graph of the character not being horizontal for any very sensible period. In the determination of the prime we ought to proceed by repeated measurements of the *same* series of individuals, and then reducing the individual time-graphs to a mean time-graph. This, of course, is absolutely impossible in the case of brain-weight and other internal organs. We are in these cases thrown back on the measurements of populations at different ages, the population at one age not consisting of the same individuals in whole or even in part as the population of another age. This method is admittedly defective, for it tacitly dismisses the whole question of natural selection[*], and may lead us into strange pitfalls[†]; but it is often the only possible, and occasionally —since anthropometers will hardly work entirely for the future—the only practical method of approximating to the prime.

(10) We are able to a certain extent now to answer a problem which has much exercised anatomists. Has the woman *relatively* to the man less brain-weight, or is the absolute difference in brain-weight merely the result of the different sizes of the two sexes? There is, we think, only one proper method of answering this question. We must ask: What would be the brain-weight of a man having all his physical characters identical with that of the mean woman? Would he have more or less average brain-weight? Conversely, given a woman with all her physical characters identical with those of the mean man, would she have more or less brain-weight? Now unfortunately we do not know *all* the physical characters in either case, but we can judge from age, stature and the diametral product.

Using the multiple regression equation $(i)^a$ we find that:

The Englishman of the same age, stature and diametral product as the mean woman has 1235 grs. brain-weight, or only 10 grs. more than the average woman.

Further using $(1)^b$ we find that:

The Englishwoman of the same age, stature and diametral product as the mean man has 1315 grs. brain-weight, or only 13 grs. less than the average man.

But we are not certain that this even is the limit of difference because the woman reaches her prime rather sooner than the man and declines more rapidly. It would be really more reasonable to compare the man physically like the woman, when at his prime, and the woman physically like the man when at her prime. Brain-weight primes are so little determined at present that it seems impossible to do this. But we think that there is probably as in other physical

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[*] The selective death-rate in man is probably 60 to 80 p. c. See *Biometrika*, Vol. I. p. 74.

[†] Pfitzner's conclusions drawn from the dead are probably to be largely discounted for this reason: see *Biometrika*, Vol. III. p. 465.

characters several years' interval between the two primes. If we change the problem and ask, what is the brain-weight of a man of *male mean age* equal to the mean woman in physical characters, we find it is 1230 grs. or only 5 grs. more than the brain-weight of the mean woman. On the other hand the brain-weight of a woman of female mean age equal to the mean man in physical characters is 1319 grs. or only 9 grs. less than the brain-weight of the mean man.

These differences are extremely slight, and would probably disappear entirely if we took another important cranial character like frontal breadth into consideration and gave the woman equal frontal breadth with the mean man and *vice versâ*. On the whole as far as present evidence goes, we can safely conclude that there is no sensible *relative* difference in the brain-weights of man and woman, the absolute differences observed are quite compatible with the differences which result from the relative sizes of the two sexes. In other words any argument as to the relative intellectual capacity of men and women based upon their absolute brain-weights applies equally to the thesis that the average big man has more intellectual capacity than the average small man, because he has on the average greater absolute brain-weight.

While our results thus apparently contradict those of Pearl on p. 51 of this Journal, the contradiction is only on the surface, for we have been able to use a far more complete system of physical measurements; and it is clear that the more measurements taken the closer is the approach to relative equality in the brain-weights of man and woman.

(11) *On the Capitulo-Statural Index.*

Gladstone has introduced a quantity equal to $100 \times \sqrt[3]{\text{Diametral Product}}/\text{Stature}$ or to $100 P^{\frac{1}{3}}/S$ in our notation and termed it the capitulo-statural index[*]. He shows that it decreases with stature—an almost necessary result of its algebraic form. As he has not discussed it with special relation to the Middlesex Hospital measurements, it appears worth while to discuss some of its properties and correlations. We will represent it by the letter χ and take

$$\chi = P^{\frac{1}{3}}/S.$$

Clearly, since it involves no new measurement, all its properties can be deduced from the biometric constants already tabled.

We have at once $\delta\chi = \frac{1}{3}\bar{P}^{-\frac{2}{3}}\delta P/\bar{S} - \bar{P}^{\frac{1}{3}}\delta S/\bar{S}^2,$

whence squaring, summing and dividing by the number of individuals, we find:

$$\sigma_\chi^2 = \{\tfrac{1}{9}\bar{S}^2\sigma_P^2 + \bar{P}^2\sigma_S^2 - \tfrac{2}{3}\bar{P}\bar{S}\sigma_P\sigma_S r_{PS}\}/\bar{S}^4\bar{P}^{\frac{4}{3}},$$

or $$V_\chi^2 = (100\sigma_\chi)^2/\chi^2 = \tfrac{1}{9}V_P^2 + V_S^2 - \tfrac{2}{3}r_{PS}V_S V_P \quad\dots\dots\dots\dots\dots(i).$$

Again, multiply the value of $\delta\chi$ above by δS, sum and divide by number of individuals, and we have:

$$r_{\chi S} = (\tfrac{1}{3}r_{PS}V_P - V_S)/V_\chi\dots\dots\dots\dots\dots\dots\dots\dots(ii).$$

[*] *Biometrika*, Vol. IV. p. 115.

Here V_J denotes the coefficient of variation of any character J, or $V_J = 100\sigma_J/\bar{J}$, and $\bar{J}$ is a mean value.

For the practical purposes of the present inquiry we may take $\bar{\chi} = \bar{P}^{\frac{1}{3}}/\bar{S}$ although this is not absolutely exact*. As $\bar{P}$ is in cm.³ we must reduce $\bar{S}$ to cm. We then find:

TABLE XXII.

Capitulo-Statural Index.

Sex	Mean $= 100\,\chi$	Standard Deviation $= 100\,\sigma_\chi$	Coefficient of Variation $= \dfrac{100\,\sigma_\chi}{\bar{\chi}}$	Correlation with Stature	Regression with Stature
♂	9·1340 ± ·0255	·4095 ± ·0180	4·4828 ± ·1977	− ·8366 ± ·0187	− ·0442 ± ·0018
♀	9·4136 ± ·0273	·3924 ± ·0193	4·1686 ± ·2051	− ·7957 ± ·0255	− ·0441 ± ·0024

From the regression coefficients we at once find:

For ♂s:
$$\text{Capitulo-Statural Index} = 16\cdot6674 - \cdot04416 S,$$

For ♀s:
$$\text{Capitulo-Statural Index} = 16\cdot4812 - \cdot04413 S,$$

S being measured in centimetres.

Comparing this with Gladstone's Table on p. 116 of this volume of the Journal we have:

TABLE XXIII.

Capitulo-Statural Index for Different Statures.

Stature	Gladstone's Observed Values on Males	Values deduced from Post-mortem data
185·3 cms. ...	8·66	8·48
177·7 cms. ...	8·98	8·82
170·0 cms. ...	9·00	9·16
162·4 cms. ...	9·69	9·50
154·8 cms. ...	10·05	9·83

Probably the fair homogeneity of the post-mortem material gives sensibly better results, especially as it is smoothed by the correlation process, than the

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* See *R. S. Proc.* Vol. 60, pp. 491, 492.

averages of Gladstone's groups, which were composed of post-mortem, work-house, and professional class measurements *.

Generally we see that the woman has a rather higher capitulo-statural index than the man; absolutely it is about equally variable, but its coefficient of variation is less. These coefficients, however, have very much the values which have been found for other bone measurements on man. The drop of this index per centimetre of stature is almost identical in male and female.

For anthropometric purposes the index appears to be of little value, for the change in the diametral product with stature is given at once by the regression coefficient tabled in Table XIX., where the reader will also find the influence of age and horizontal circumference on this product. In fact, the theory of linear correlation shows that we should expect $P^{\frac{1}{3}}$ for a given S to be closely of the form $P^{\frac{1}{3}} = C_1 S + C_2$, and accordingly if C_2 is positive we might not unnaturally anticipate that $P^{\frac{1}{3}}/S$ would decrease with S. Such an index would only be of value if C_2 were zero, and up to the present a wide experience shows that even a sensible approach to zero in C_2, if it ever does occur, is an occurrence of the most marked variety †.

(12)　*On the Index = Ratio of Diametral Product to Brain-weight.*

What we have said at the conclusion of the previous article applies with a good deal of force to the index : Diametral Product/Brain-weight, which we will represent by the letter z. All the properties of it are involved in the correlations already tabled between the diametral product, the brain-weight, and the other characters. Still there is a physical side to this index which may be of some interest. It measures in a rough sort of way the ratio of volume to weight of enclosed brain-matter. We will investigate how far this index changes with age. All the requisite constants are given in Table XXIV.

The results were deduced, as in the case of the capitulo-statural index, from the formulae :

$$V_{\bar{z}}^2 = (100\sigma_{\bar{z}}/\bar{z})^2 = V_P^2 + V_w^2 - 2V_P V_w r_{Pw} \dots\dots\dots\dots\dots(iii),$$

$$r_{\bar{z}A} = (V_P r_{PA} - V_w r_{wA}) V_{\bar{z}} \dots\dots\dots\dots\dots\dots\dots\dots\dots(iv),$$

* If we apply the above formula to male giants and dwarfs (see Gladstone, p. 117), we find :

Individual and Stature	Gladstone's Value	Formula
Irish Giant, O'Brian, 231·1 cms.　...	7·12	6·46
American Giant, Freeman, 205·7 cms.	7·77	7·59
Dwarf from Kiel, 121·9 cms.　...	11·15	11·29
Dwarf from Holstein, 97·8 cms.　...	13·78	12·35

but quite different values have been given by other authorities for these statures and accordingly no great stress can be laid on such divergences as occur between the formula and "observed" values.

† Cf. Lewenz and Pearson, *Biometrika*, Vol. iii. pp. 373 and 374.

by aid of the known values of the coefficients of variation V_P, V_w (see Table II.), and the known correlations r_{Pw}, r_{PA}, and r_{wA} (see Table XIII.)*.

TABLE XXIV.

Index: Diametral Product/Brain-weight.

Sex	Mean	Standard Deviation	Coefficient of Variation	Correlation with Age	Regression Coefficient on Age
♂	2·8492 ± ·0106	·1703 ± ·0075	5·9771 ± ·2636	+ ·1153 ± ·0615	+ ·0016 ± ·0008
♀	2·7978 ± ·0113	·1628 ± ·0080	5·8189 ± ·2863	+ ·1150 ± ·0696	+ ·0014 ± ·0008

The mean value of the index is closely the same, 2·8 for both sexes. The difference of the variations is also within the difference due to random sampling. The influence of age is, within the limits of the probable errors, the same for both sexes, and for the sixty years covered by the data would not alter the index by as much as ·1 or by sensibly less than four per cent. We have thus the interesting result that while age tends to shrink diametral product and brain-weight its effect on their ratio is comparatively insignificant. It tends slightly to increase it, the shrinkage of the brain-weight being somewhat more than the shrinkage of the diameters. Turning to the regression coefficients in Table XIX., we see that the shrinkage of brain-weight in 60 years is 132 grs. on 1328 grs. average, or roughly 10 per cent., and the shrinkage in diametral product 247 cm.³ on 3783 cm.³ average, or roughly 6·5 per cent. We have thus the source of the 4 per cent. change in the index. It is, therefore, not surprising that the index increases and does not shrink like the simple characters with increasing age.

Supposing the index were absolutely constant, then equations (iii) p. 158 show us that it would only be a very inexact method of determining brain-weight from diametral product; for the constant terms are here 25 per cent. of the total probable brain-weights.

(13) *General Conclusions.*

While the writers of the present paper would admit that many of its conclusions are only tentative as the data are too sparse, they yet consider that they will be useful as suggesting lines for further research and as confirming and supplementing some of the results of Pearl's more elaborate study. They hold that:

(i) The biometric constants determined fit in reasonably well with the biometric results hitherto obtained for other brain-weight material.

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* See footnote, however, to Table II.

(ii)   They are, allowing for the differentiation of a general hospital population, consonant with what we know of other English anthropometric and craniometric material.

(iii)   They show that a general hospital population differs in mean from other groups of the general population, but that the difference in variability and correlation is not very marked.

(iv)   Important results would be reached if the change in physical characters with age were more completely studied; especially if the human prime for various physical measurements were more adequately determined.

(v)   Allowance can be made for the effect of age and stature on brain-weight, but neither of these factors is nearly as important for brain-weight prediction as the knowledge of diametral product or horizontal circumference.

(vi)   The probable brain-weight of an individual can be determined from stature, age, diametral product, and horizontal circumference with a mean error of about 50 grs.   The mean deviation in brain-weight from the mean of the group is about 80 grs.   In other words, we improve our prediction in the ratio of 8 to 5 by taking into account these characters.   Nothing better can probably be achieved (see Figures 3 to 6) by introducing further external characters, or by considering regression as curved instead of plane.

(vii)   Generally Pearl's conclusion that brain-weights for homogeneous material, uniformly measured, can be dealt with as satisfactorily as any other anthropometric characters is confirmed.   We would add that the material ought to be restricted so that the highest prime of any character dealt with is the minimum age included in the data.

(viii)   The apparent shrinkage of the diameters of the head with age is probably peculiar to general hospital material.   Its absence in criminal data is significant.   The subject, however, is at present being directly investigated by one of the present writers.

(ix)   There is no sensible *relative* difference between the brain-weights of man and woman, when proper allowance is made for the relative difference in size of man and woman.

A QUANTITATIVE STUDY OF THE RESEMBLANCE BETWEEN MAN AND WOMAN.

By E. TSCHEPOURKOWSKY, Moscow.

In my studies on the cephalic index, which are partly published in Russian and partly in German journals, I affirm that sexual selection plays a very important *rôle* in the development of its varieties. Since then I have collected from different sources a certain amount of data for the study of interracial and sexual correlation, which can possibly give an answer to the following question: "If different characters of the human frame are inherited and if we admit that some of them are the result of natural selection, then the environment—in a wider sense—not being the same for both sexes we may expect that some characters are characteristic for one sex only. How far are they inherited in comparison with others by the opposite sex?" We may suppose *à priori* that the cephalic or nasal index has not the same importance in the struggle for life as for instance the arm length. If all kinds of characters are inherited equally by both sexes, the coefficients of correlation of their male and female values in different races must be equally high for them all.

In order to investigate this point I have used the data of various Russian authorities for living subjects; firstly, because they are collected in a very uniform manner, and secondly, because the diversity of races within the large territory of Russia is itself sufficient. The work of collecting the data was very much facilitated by the new compilation of Iwanowsky: "Anthropological Constituents of the Population of Russia." The raw material is represented in the tables given below in which the number of cases and the authors' names are also indicated. The numerical values for crania which I have used are partly extracted from different authors but mostly deduced from my own measurements made in the various anthropological museums of Europe. The cross-correlations can also be studied from these tables by anyone who wishes to do so*.

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* See the remark on the next page.

The number of races is not very high, but the persistently high value of the coefficients of correlation and the fact that the measurement of men and women of the same race was made by the same investigator make the results very valuable.

TABLE I.

Constants for Interracial Variation and Correlation between Man and Woman*.

Source	No.	Character	Standard Deviation		Coefficient of Variation		Mean		Correlation	Sexual Ratio
			♂	♀	♂	♀	♂	♀		♀ to ♂ °/₀
Iwanowsky	31	Stature	3·59	3·11	2·20	2·04	163·16	152·42	+ ·842	93·4
,,	33	Head Length ...	3·61	3·17	1·92	1·76	187·09	179·76	+ ·844	96·0
,,	45	Cephalic Index ...	2·59	2·54	3·16	3·10	81·71	81·93	+ ·875	100·2
,,	22	Face Index †	1·81	1·89	2·39	2·43	77·91	77·73	+ ·726	99·7
,,	25	Nasal Index	4·43	4·60	6·51	6·75	68·00	68·08	+ ·836	100·1
,,	22	Relative Arm Length‡	1·30	1·50	2·86	3·32	45·41	45·18	+ ·731	99·4
Crania ...	49	Cephalic Index ...	4·53	4·00	5·92	5·18	76·51	77·16	+ ·929	100·8
Means ...	—	—	2·73	2·60	3·12	3·07	—	—	+ ·826	—

TABLE I a.§

Character	Mean	Mean	Standard Deviation		Correlation	No.	Coefficient of Variation	
	♂	♀	♂	♀			♂	♀
Stature	164·00 ± ·42	152·28 ± ·39	4·10 ± ·21	3·76 ± ·19	·836 ± ·023	49	2·50	2·47
Head Length ...	187·11 ± ·37	178·36 ± ·33	3·76 ± ·26	3·36 ± ·23	·909 ± ·009	49	2·01	1·88
Relative Arm Length	45·82 ± ·29	45·41 ± ·27	2·00 ± ·20	1·85 ± ·19	·832 ± ·033	22	4·36	4·09
Cephalic Index ...	81·94 ± ·24	82·37 ± ·24	2·53 ± ·12	2·50 ± ·12	·869 ± ·017	43	3·08	3·03
Facial Index ...	77·35 ± ·29	77·94 ± ·29	2·04 ± ·29	2·06 ± ·29	·687 ± ·057	23	2·67	2·61
Nasal Index ...	68·00 ± ·64	68·01 ± ·60	4·96 ± ·36	4·66 ± ·40	·688 ± ·054	27	7·28	6·84

* In forming the correlation tables of this memoir as well as those of a second to be shortly published in *Biometrika*, I have sometimes used not all the given data but have chosen from the means of the same nationality only a few for groups which are geographically isolated.

† The relation of Bizygomatic Breadth to the total Face Length (from the top of the forehead to the chin).

‡ Ratio of the absolute arm length to stature.

§ I owe this table to the kindness of my friend M. Çekanowsky (Zürich-Warshawa), who formed (using my material quite independently from me) the correlation tables and calculated the coefficients with their probable errors. The number of cases is different, because he had not seen my tables—(and I excluded the means deduced from a small number), but the constants calculated by him are not very different from mine. This table can be used as control table. The mean resemblance (interracial) between man and woman is close to ·8.

We see firstly, that the coefficients lie between ·726 and ·929, and in the mean they are equal to ·826 (Table I.). Secondly, that in sexual resemblance there does not exist a large difference between characters which are apparently indifferent and those which can be presumed to play an important *rôle* in the struggle for life. This is shown also by the means of indices and relative lengths, which are nearly the same for both sexes. Two explanations of this fact are I think possible: 1. The racial proportions of the human frame have no special value in the struggle with environment, or they have the same value for both sexes. 2. The characters acquired in this struggle under the influence of natural selection by one sex are inherited by another.

If we admit that the racial characters of the human frame are evolved under the influence of an environment which is different for each sex, then why are they not characteristic for one sex only, i.e. why in the species *Homo* is the male not different from the female, as in many other species of animals and plants? It shows I think that in the formation of the different races of man many characters are probably the result of homogamic and endogamic instincts. From this point of view a race is very clearly defined by Karl Pearson as a population which by geographical, religious, or other causes has been isolated during a time sufficient for thorough intercrossing and for the work of selection. If that be so, the degree of resemblance between male and female of the same race is the product of this selection. Within the limits of one race the degree of resemblance between the closest relatives of the opposite sex is nearly ·4. If a part of the members of this race with a special deviation of any one character become isolated and form a new race, then the resemblance between their male and female elements increases to ·8. The difference ·4 is perhaps the measure of the work of selection, and the sexual correlation coefficient is the criterion of the formation of a new race.

From the Table I. it follows, that judged by coefficients of interracial variation woman is in the mean less variable than man; but the difference is very small. In three characters out of seven she is more variable than man. But in at least five of the seven cases the difference is not sensible, having regard to the probable error of the difference.

APPENDIX I.

TABLE II.

Cephalic Index-Skulls.

Nationality	♂	♀	Authority	Nationality	♂	♀	Authority
Whitechapel ...	74·7	75·0	Pearson *	Fijians	69·6	68·9	Flower, etc.
French Paris, City	79·2	78·0	,,	Duke of York Islanders	70·9	71·8	Godefroi Collection
Italian	80·8	80·0	,,	Australians	71·0	72·7	Different authors
Ancient Romans ...	78·1	78·7	,,	Micronesians ...	74·4	75·0	,, ,,
Dutch	80·0	79·4	,,	Lenftemberg	84·2	85·1	Niederle
Bavarians	83·4	83·1	,,	Australians	68·8	71·4	Krause
Waichenfeld Graves	83·9	84·6	,,	Czechs	83·2	83·6	Matiegka
Badeners	83·8	83·4	,,	Australians	69·0	72·0	Challenger Expedition
Swiss	78·9	80·5	,,	Wurtembergers ...	82·6	82·7	German Catalogue
Row-graves ...	73·7	74·1	,,	Kalmuek Astrackan ...	81·6	81·0	Iwanowsky
Ancient Gaulish ...	78·4	75·4	,,	Minousinsk	79·6	75·9	Goroshenko
Bretons	77·3	76·8	,,	Thebans	75·1	76·5	Pearson
Romans	77·3	79·0	,,	Bashkirs	81·0	81·0	Nikolsky
Friesians	77·7	79·0	,,	Hottentots	71·4	74·9	Broca (registers)
Anglo-Saxons ...	75·0	75·0	,,	Eskimo	71·4	71·8	,, ,,
Swedes	77·9	78·1	,,	New Caledonians ...	71·3	74·4	Crania Ethnica
Etruscans	78·5	78·2	,,	Hawaiians	75·5	78·4	,, ,,
Ancient Egyptians	75·1	76·5	,,	Loyalty I.	69·8	71·8	,, ,,
Naqadas	73·2	74·6	,,	Dzungarians	78·2	80·7	,, ,,
Ainos	76·5	77·7	,,	Taithi	75·1	77·4	,, ,,
Negroes	74·3	74·8	,,	Tasmanians	76·1	74·8	Broca
Panjabis	70·7	72·3	,,	Chinese	77·9	76·5	,,
Kanakas	79·3	80·3	,,	Javanese	81·5	80·9	,,
Andamanese ...	80·6	82·7	,,	Marquesans	75·0	78·2	,,
Torres Straits ...	68·3	70·1	Thomas				

* Extracted from "Variation in Man and Woman." *Chances of Death*, Vol. I. pp. 349—372.

APPENDIX II. TABLE III. Living Races of Russia (Mean Values).

Nationality	Stature ♀	Stature ♂	Head Index ♀	Head Index ♂	Nasal Index ♀	Nasal Index ♂	Head Length ♀	Head Length ♂	Facial Index ♀	Facial Index ♂	Relative Arm Length ♀	Relative Arm Length ♂	Authority
Aino	1471(69)	1567(91)	784(71)	773(95)	679(54)	687(87)	184(71)	194(85)	780(55)	781(73)	451(66)	460(90)	Koganei
Armenians	1529(19)	1685(231)	841(49)	858(292)	649(48)	646(291)	177(49)	183(292)	—	—	—	—	Chantre
Bielorussians*, Smolensk	1544(275)	1652(100)	825(275)	802(445)	711(275)	694(445)	179(275)	190(445)	789(275)	763(445)	444(10)	447(57)	Shadrowizky
„ Minsk	1539(17)	1648(57)	823(17)	815(57)	633(17)	680(57)	182(40)	188(99)	754(17)	759(57)	442(40)	445(100)	Rosdestwensky
Bouriats	—	1644(47)	807(40)	824(100)	686(40)	725(100)	—	—	—	—	—	—	Porotow
Great Russians, Moskwa	1528(411)	1651(325)	840(100)	815(325)	—	—	177(100)	188(325)	775(100)	771(323)	464(100)	464(200)	Rosdestwensky
„ „ Kazan	1530(100)	1660(200)	833(110)	830(110)	674(110)	697(200)	172(100)	179(200)	750(100)	746(200)	454(50)	462(235)	Worobiew
„ „ Twer	1528(100)	1640(235)	810(50)	817(235)	—	—	178(50)	185(235)	784(50)	775(235)	—	—	Galay
„ „ Kursk	1525(50)	1671(51)	822(36)	813(51)	—	—	179(36)	187(51)	757(100)	737(147)	—	—	Lokine
„ „ Semiriechensk (Siberia)	1569(37)	1610(200)	829(125)	819(200)	607(100)	620(147)	176(25)	184(200)	748(48)	762(148)	449(43)	448(27)	Zeanland
Jews, Warchau	1506(125)	1617(139)	807(100)	817(36)	683(86)	600(70)	178(80)	183(139)	—	—	—	—	Elkina
„ Mohilew	1507(100)	—	830(206)	829(438)	—	—	—	—	756(13)	764(98)	—	—	Jakowenko
„ Kiew	—	—	824(50)	824(100)	—	—	176(50)	183(100)	800(30)	793(85)	—	—	Talko-Grinzewitch
„ South of Russia	—	—	825(23)	808(51)	648(13)	647(98)	—	—	—	—	—	—	Weissenberg
„ Different Goubernians	—	—	836(16)	822(20)	746(30)	734(47)	181(30)	189(90)	—	—	442(93)	431(98)	Jkow
Zirians	1537(21)	1628(97)	826(30)	811(96)	—	—	—	—	—	—	456(9)	460(93)	Sommier
Kalmueks, Astrachan	1504(19)	1635(30)	833(13)	821(36)	775(10)	762(40)	—	—	—	804(99)	430(10)	471(20)	Korolew
Karagass	—	—	897(10)	873(40)	590(10)	622(20)	176(30)	181(99)	830(30)	—	446(10)	453(39)	Sommier
Karakirgiz	—	—	856(10)	848(20)	722(30)	718(99)	—	—	—	—	—	—	Korolew
Karaims	1505(30)	1651(99)	863(30)	894(99)	626(60)	623(30)	182(60)	189(30)	760(60)	752(300)	451(30)	445(99)	Zeanland
Kirgiz Sredni Orda	—	—	770(15)	776(300)	648(15)	658(97)	183(34)	193(140)	—	—	423(10)	419(31)	Weissenberg
Courds from Transkaukasian	—	—	764(15)	762(67)	702(34)	677(140)	—	—	—	—	451(60)	453(30)	Iwanowsky
„ Turkmens „	—	—	776(34)	785(140)	718(11)	728(35)	—	—	—	—	—	—	Tronow
„ from Syria	—	—	825(11)	825(35)	—	—	—	—	—	—	—	—	Iwanowsky
Courtins (Transkaukasia)	1524(60)	1686(30)	796(40)	805(60)	636(40)	638(60)	185(40)	190(60)	—	—	—	—	Chantre
Latish	1563(40)	1705(60)	835(40)	818(66)	711(40)	729(60)	184(40)	191(60)	777(30)	752(70)	414(40)	449(60)	Waeber
Lituanians	1548(70)	1659(130)	825(30)	827(70)	—	716(120)	178(30)	184(70)	781(40)	778(120)	467(40)	469(60)	Brennsohn
Little Russians, Chernigor	—	—	836(40)	856(120)	716(40)	—	175(40)	182(120)	786(48)	772(102)	468(30)	472(70)	Bielodied
„ „ Woronief	1529(48)	1632(102)	805(48)	811(102)	686(75)	667(162)	179(48)	187(102)	—	—	—	—	Koshuchow
Morawa	1546(75)	1643(162)	819(60)	815(300)	—	—	181(75)	187(162)	777(60)	813(120)	489(75)	472(162)	Mainow
Ossetins	1573(60)	1690(30)	790(30)	792(95)	609(149)	633(226)	182(60)	190(300)	—	—	443(66)	449(120)	Riskine
Ostiaks	1441(27)	1563(95)	813(149)	808(226)	734(85)	729(100)	185(30)	193(95)	783(145)	—	—	—	Sommier
Poles, Warchau	1533(144)	1610(191)	836(85)	827(116)	—	—	177(49)	185(223)	772(85)	768(207)	458(50)	459(100)	Tezinsky
„ Lublin	1537(83)	1655(116)	827(59)	830(131)	—	—	178(85)	186(116)	—	753(100)	—	—	Olechnowiz
„ Radom	1531(59)	1617(131)	845(36)	837(60)	628(13)	642(114)	179(59)	186(131)	—	—	—	—	Olechnowiz
„ Lublin bis	—	—	839(38)	844(66)	667(100)	812(150)	181(36)	190(60)	—	—	—	—	Sommier
Samoieds	1430(46)	1550(84)	826(20)	830(72)	—	—	179(39)	189(68)	774(36)	786(205)	449(10)	453(21)	Goroshanko
Soiots	—	—	818(37)	821(206)	—	—	—	—	—	—	—	—	Warushkine
Tatars, Kazan	1521(37)	1645(206)	767(18)	769(114)	—	—	179(37)	186(207)	771(100)	768(150)	—	—	Chantre
„ Azerbeïsan	—	—	819(60)	804(150)	—	—	181(100)	191(150)	793(30)	794(24)	—	—	Kolmogorow
Finns, Wiborg	1545(105)	1666(150)	833(31)	823(24)	—	—	181(31)	186(24)	—	—	457(50)	461(100)	Retzius
„ Tawastgouse	1552(31)	1679(22)	767(29)	790(48)	—	—	182(29)	184(48)	—	—	—	—	Malieff
Cheremiss	1517(45)	1615(48)	802(6)	794(38)	—	—	—	—	—	—	—	—	Sommier
Jakuts	1509(105)	1617(324)	809(62)	827(207)	—	—	186(62)	191(207)	789(62)	792(207)	—	—	Mainow
Tasiks	1541(20)	1693(392)	829(15)	823(46)	—	—	—	—	—	—	471(16)	475(45)	Maslowsky
Jakuts bis	—	—	—	—	—	—	—	—	—	—	—	—	Witashewsky

* White-Russians.

APPENDIX III.

CORRELATION TABLES.

I. *Stature.*

Female.

Male	143	144	—	147	148	149	150	151	152	153	154	155	156	157	158	Totals
155	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
156	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
157	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
162	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	3
163	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	3
164	—	—	—	—	—	—	—	—	2	2	—	1	—	—	—	5
165	—	—	—	—	—	—	1	—	—	1	3	—	—	—	—	5
166	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	2
167	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	2
168	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	2
169	—	—	—	—	—	—	—	—	1	—	1	—	—	1	—	3
170	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	1	1	—	1	—	—	2	3	4	7	6	3	1	2	—	31

II. *Cephalic Index-Skulls.*

Male.

Female	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	Totals
69	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
70	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
71	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
72	—	1	1	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
73	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
75	—	—	—	1	—	1	2	2	1	—	1	—	—	—	—	—	—	—	—	8
76	—	—	—	—	—	—	—	2	—	1	1	—	1	—	—	—	—	—	—	5
77	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
78	—	—	—	—	—	—	—	2	1	—	1	1	1	—	—	—	—	—	—	6
79	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	3
80	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	3
81	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	—	—	—	—	4
82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
83	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	3
84	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
85	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	2
Totals	1	3	1	6	—	1	3	7	2	2	6	3	3	4	1	3	3	—	—	49

III. *Head Index, Living Subjects.*

Female.

Male.	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	Totals
75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
77	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
78	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
79	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	3
80	—	—	—	—	—	1	—	2	—	—	—	—	—	—	—	—	3
81	—	—	—	—	—	1	1	4	1	1	—	—	—	—	—	—	8
82	—	—	—	—	—	—	3	3	5	1	—	—	—	—	—	—	12
83	—	—	—	—	—	—	1	1	4	1	—	—	—	—	—	—	7
84	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
85	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
86	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
87	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
88	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
89	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	—	1	3	2	1	3	5	10	10	7	—	2	—	—	—	1	45

IV. *Nasal Index.*

Female.

Male.	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	Totals
60	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
61	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
63	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
64	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
65	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
66	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
67	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
68	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
69	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
70	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
71	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
72	—	—	—	—	—	—	—	—	—	—	1	—	—	2	—	—	—	—	—	3
73	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	3
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	2	—	3	1	2	—	1	2	2	2	2	3	1	1	1	—	1	25

V.   Head Length.
Female.

Male.

	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	Totals
179	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
180	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
181	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
182	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
183	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	3
184	—	—	—	—	1	—	1	—	—	—	1	—	—	—	—	—	3
185	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	2
186	—	—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	4
187	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	3
188	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	2
189	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	3
190	—	—	—	—	—	—	—	1	—	1	1	—	—	1	—	—	4
191	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	—	3
192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
193	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	2
194	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
195	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
196	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	—	1	3	3	4	6	—	5	4	1	2	2	1	—	33

VI.   Facial Index.
Female.

Male.

	74	75	76	77	78	79	80	—	83	Totals
74	—	—	1	—	—	—	—	—	—	1
75	—	1	1	1	1	—	—	—	—	4
76	—	2	1	—	—	1	—	—	—	4
77	—	—	—	2	2	1	—	—	—	5
78	—	—	—	—	1	—	1	—	—	2
79	—	—	—	1	—	2	1	—	—	4
80	—	—	—	—	—	—	—	—	1	1
81	—	—	—	—	1	—	—	—	—	1
82	—	—	—	—	—	—	—	—	—	—
83	—	—	—	—	—	—	—	—	—	—
Totals	—	3	3	4	5	4	2	—	1	22

VII.   Relative Arm Length.
Female.

Male.

	42	43	44	45	46	47	48	49	Totals
42	1	—	—	—	—	—	—	—	1
43	—	—	1	—	—	—	—	—	1
44	—	—	1	1	—	—	—	—	2
45	—	—	3	4	—	—	—	—	7
46	—	—	—	2	4	—	—	—	6
47	—	1	—	—	—	3	—	1	5
48	—	—	—	—	—	—	—	—	—
49	—	—	—	—	—	—	—	—	—
Totals	1	1	5	7	4	3	—	1	22

"DAS FEHLERGESETZ UND SEINE VERALLGEMEINER-UNGEN DURCH FECHNER UND PEARSON*." A REJOINDER.

By KARL PEARSON, F.R.S.

THERE is not much profit as a rule in complaining of the treatment one receives at the hands of critics, but still I think the needlessly hostile tone of Dr K. E. Ranke and Dr Greiner's review of my memoir on skew variation requires some protest on my part. As an illustration of the want of courtesy of which I complain I would cite for example the following statement (p. 323):

Er (Pearson) weist aber einmal darauf hin, dass zwar seine eingeschriebenen Kurven teilweise unbegrenzt werden können, das Gesetz selbst, die hypergeometrische Reihe aber nicht, und vertröstet auf eine spätere Arbeit, in der an Stelle der eingeschriebenen Kurven die Anpassung der Reihen selbst gegeben werden solle. Diese spätere Arbeit ist nie geschrieben worden.

This is not an isolated instance of the manner in which the authors criticise my work. It was quite open to them to have examined the customary sources of bibliographical information, or even to have written to me and asked if the memoir in question had been published. But here as elsewhere they assumed, without making proper investigation, that I could say nothing further and therefore had said nothing further. The memoir in question appeared so long ago as 1899 in a well known British scientific journal† from which the authors actually cite another paper of mine. Although my memoir is *nie geschrieben*

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* The criticism of my work appeared in the *Archiv für Anthropologie*, Bd. II. pp. 295—331, 1904. The proper place to reply to an attack of the kind would be in the *Archiv* itself. Professor J. Ranke accepted a rejoinder and asked that it should be in German and not exceed 40 pp. I have heartily to thank Miss M. Lewenz for the labour of a translation, which I much regret I cannot make use of, because the Editors of the *Archiv* do not now see fit to publish this reply to K. E. Ranke's attack. As the reply was an endeavour to give an historico-critical account of the theory of skew variation it may interest readers of *Biometrika*, and will possibly reach in the course of time some readers of the *Archiv für Anthropologie*.

† "On certain Properties of the Hypergeometrical Series, and on the fitting of such series to Observation Polygons in the Theory of Chance," *Phil. Mag.* 1899.

worden according to my critics, it was actually written in 1895 and its substance given in academic lectures of the same or the following year; it was not published till some years afterwards owing to that want of leisure for preparing matter for press which every teacher who has to lecture four hours a day will appreciate. Dr Ranke says that had the paper been written, it would not have influenced his judgment. That is quite possible, and I only cite the matter here to indicate the tone adopted by my critics.

A very similar instance occurs in Dr K. E. Ranke's treatment of my fitting of Professor J. Ranke's data for 900 Altbaierisch crania. I used this example purposely because it had already been used by Stieda. I should not myself have mixed, even for the cephalic index, ♂ and ♀ data as Stieda did, but I wished to compare the results reached by the generalised curve with those reached by the Gaussian curve. I actually spoke of the resulting curves in my memoir, whether the generalised curve or the Gaussian, as being "quite good for this type of statistics*." My object of course was to show that the generalised method did not fail where the Gaussian succeeded, but surpassed it. Now how does Dr Ranke treat this instance? He cites an example (actually inserted† by me!) in a memoir by Palin Elderton giving the Ranke'sche Messungen as an illustration of my method of testing goodness of fit in the case of the normal curve. Undoubtedly as I said in 1894 the Gaussian curve is quite good for J. Ranke's data, but it does not follow that the Type IV. frequency curve does not give a *better fit*, and is not significant for constants which the Gaussian process cannot deal with. Now it was open to Dr Ranke to test the values given for the distance from mode to mean, the skewness and the other constants in the case of the Altbaierisch crania. I have given the probable errors of these constants in my memoir: *On the Mathematical Theory of Errors of Judgment*, etc., *Philosophical Transactions*, Vol. 198 A, see p. 278. Had Dr Ranke fairly tested my results he would have found that the asymmetry was not significant, and that the mode sensibly coincided with the mean, but that the constant β_2, which should equal 3 for the normal curve, has a value 3·65 with a probable error of only about ·11. Now this constant and its probable error have no relation at all to any particular theory of variation. They follow quite easily from the general Gaussian theory. It is accordingly extremely improbable that Ranke's measurements are truly given by a Gaussian distribution in all their features. The fact that β_2 is > 3 points to an emphasis

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* p. 389.

† The same remark applies to the illustrations of goodness of fit given by Fawcett, cited in footnote Ranke u. Greiner, p. 326. The reference to Powys is inexact; his paper shows that in at least three cases the Gaussian curve is quite impossible. Dr Macdonell's work on the English skull shows that at least in 4 out of 13 cases the asymmetry is significant. "Die englische Schule," by which Dr Ranke refers to workers in my Biometric Laboratory has not discovered a truth which had escaped me; they have shown that the Gaussian curve is of wide applicability, but not of universal truth in anthropometric measurements. This result was reached with a view to testing whether the theory of inheritance, so far as it is based on the Gaussian theory, might be safely applied to human characters. In testing this validity of the Gaussian theory, it was of course needful to have a more general theory from which to determine the chief physical constants involved in non-Gaussian distributions.

of the modal frequency and to a reduction of the extreme frequencies which are inconsistent with the Gaussian curve. This is actually shown in my Plate 11 and referred to in my text, and corresponds to a sensible deviation from the Gaussian law. It was open to Dr Ranke to attribute this exaggeration of the modal at the expense of the extreme variates to heterogeneity in the material. But he had no right when the material for a judgment was before him in my memoirs to conclude that, because the *general* distribution of frequency was not on the average incompatible with the Gaussian law, the deviation of a particular constant of the distribution from its Gaussian value might not be most significant. This case of the deviation of the Ranke's measurements from the Gaussian form is of special interest, for it is not one of asymmetry, but of a non-Gaussian type of symmetry. Dr Ranke suggests that because my test of goodness of fit shows that the Gaussian curve is "quite a good fit," my generalised method of dealing with frequency is idle. Assisted by a mathematician he ought to have recognised that the expression $\beta_2 - 3$ (which measures whether the frequency towards the mean is emphasised more or less than that required by the Gaussian law) had a sensible value, and that my method not only led to the discovery of this deviation but provided a method of allowing for it in the description of the frequency. Had Dr Ranke read my memoir on errors of observation (*Phil. Trans.*, Vol. 198 A, pp. 274—286), he would have recognised that the two tests (a) whether special physical constants of the distribution satisfy the Gaussian law, and (b) whether the general distribution of frequency satisfies within reasonable limits the Gaussian law, are not necessarily identical. Finally had he concluded that (b) for the Altbaierisch crania was satisfied, but not (a), and that there was thus no necessary discrepancy between my memoir of 1894 and the statement in Palin Elderton's paper of 1900, he might indeed have fallen back on his customary assumption that when frequency is not Gaussian it is heterogeneous. But at any rate had he adopted this course he would have avoided the appearance of criticising his author without endeavouring to understand what the meaning of his investigations was, or striving to elucidate them by a study of his other memoirs on the same subject.

I do not wish to say anything further on this point. I want merely to indicate by these two out of several cases that the reader must not look for a really impartial statement of my position from Drs Ranke and Greiner. I am quite unable to account for the peculiar tone they have at times given to their criticism.

(2)   With this preamble I should like to divide my reply under these headings:

(A)   The need for generalised frequency curves, even in anthropological science.

(B)   The nature of the assumptions made in the Gaussian theory and their insufficiency.

(C)   The hypotheses made to generalise the Gaussian law.

  (i)   Poisson, Laplace and the early writers.

  (ii)   Francis Galton and D. McAlister.

  (iii) Fechner.

  (iv)   Edgeworth and Kapteyn.

  (v)   General Results for Asymmetry*.

(D)   The criticisms of my theory by Dr Ranke and Dr Greiner and the reply to be made to them.

A. *The need for Generalised Frequency Curves, even in Anthropological Science.*

I have already pointed out that even Prof. Ranke's measurements are not fully in accordance with the Gaussian theory—for the odds are great against a quantity exceeding its probable error more than five times. It is perfectly true that the English School have found that many characters, especially craniological characters, are for *practical* purposes sufficiently described by the Gaussian curve. But it is equally true that they have found other cases in which the deviation from the Gaussian curve is significant, and that they have only been able to measure this significance because they had a wider theory to base their researches upon. Dr Ranke entirely disregards the statements of Miss Fawcett and Dr Macdonell on this point. Both find a definite number of cases, the one in Egyptian skulls, the other in English skulls, in which the deviation from the Gaussian law is definitely significant†. Both conclude as I have done that in the case of many characters for a variety of practical purposes the Gaussian curve is sufficient; this is, however, not a *theoretical* justification of the Gaussian curve, but an argument in favour of its empirical use in a certain definite number of cases. Dr Ranke may of course say that the exceptions that we have found are due to heterogeneity of our material. If so he must face the difficulty that the *same* set of crania can be homogeneous and give the Gaussian curve for their length and be heterogeneous for their breadth, deviating therein largely from the Gaussian curve. If he asserts that this is quite possible then he must meet the further difficulty that they can be homogeneous for their cephalic indices, which are based upon the ratio of the supposed heterogeneous to the homogeneous material! The fact is that no unprejudiced observer can examine the constants by which we have defined the deviations from the Gaussian law without seeing that they present every variety of value, starting from the values to be expected on the Gaussian theory and rising to values which are absolutely incompatible with any Gaussian theory at all. In fact he must come to the conclusion that some theory is absolutely needful, which will provide a curve or series of curves capable of representing the

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* I have left out of consideration the general method of Thiele, followed in Germany by Lipps, because I have dealt with these authors in a recent memoir.

† *Biometrika*, Vol. i. p. 443, and *Biometrika*, Vol. iii. p. 227.

fundamental deviations of any distribution from the Gaussian curve and determining whether these deviations are significant or not. Looked at solely from this standpoint—which I am very far from accepting—my curves provide an empirical series which accurately measures the deviations from the Gaussian law and enables the enquirer to determine how far that law is applicable. Each one of them passes into the Gaussian curve if that curve is the better fit to the observations. This is not true of many of the other remedies which have been proposed to supplement what I venture to call the universally recognised inadequacy of the Gaussian law. They cannot as we shall see in the sequel effectively describe the chief deviations from the Gaussian distribution.

The chief physical differences between actual frequency distributions and the Gaussian theoretical distribution are:

(i) The significant separation between the mode or position of maximum frequency and the average or mean character.

(ii) The ratio of this separation between mean and mode to the variability of the character—a quantity I have termed the *skewness.*

(iii) A degree of flat-toppedness which is greater or less than that of the normal curve. Given two frequency distributions which have the same variability as measured by the standard deviation, they may be relatively more or less flat-topped than the normal curve. If more flat-topped I term them *platykurtic,* if less flat-topped *leptokurtic,* and if equally flat-topped *mesokurtic.* A frequency distribution may be symmetrical, satisfying both the first two conditions for normality, but it may fail to be *mesokurtic,* and thus the Gaussian curve cannot describe it.

The Gaussian curve is usually fitted from the mean square deviation, but it may also be fitted from the probable error, or the mean error, or again from the mean fourth power of the deviations—μ_4 in my notation. Whichever method is adopted we ought to get the same result within the errors of random sampling. When I first began to describe frequency data by the normal curve, I was startled to find the very large number of cases in which these different processes led to Gaussian curves, differing widely from one another, i.e. beyond all the limits of probable error. I was soon led to see that in actual statistics two distributions might have equal total frequency, be sensibly symmetrical, and have the same standard deviation and yet differ largely in their flat-toppedness. The mesokurtosis of the Gaussian curve is not a universal characteristic of frequency distributions.

When we test a theoretical distribution of frequency against observation, we may find an excellent fit for the *total* distribution and yet the distinction between mode and mean, the skewness, and the deviation from mesokurtosis may be most significant. The reason for this is that the test for goodness leaves a margin of variation which may be due to random sampling, or to the non-normal character of an important constant of the distribution. For example, 10 coins are tossed a hundred times, and the proportion of cases with five and more heads is somewhat

in excess of the theoretical distribution $100 \left(\frac{1}{2} + \frac{1}{2}\right)^{10}$, but within the limits of a random sample. It is quite conceivable that if the returns for each individual coin were analysed it would be found that those of one exceeded in proportion of heads the limits of random sampling, and that the coin proved to be loaded when delicately tested. Thus as I have shown in my memoir on errors of observation, we have not only to test for general goodness of fit, but also to consider the probable errors of the fundamental constants of the distribution. Because the general distribution of frequency is given within the limits of random sampling by a normal curve it does not follow that the system will be mesokurtic.

Consider for example the two curves:

$$y = y_1 \left\{ 1 - \frac{x^2}{2\sigma^2 \left(m_1 + \frac{1}{2}\right)} \right\}^{m_1 - 1}, \qquad \text{where:} \quad y_1 = \frac{N}{\sqrt{2\pi}\sigma} \frac{\Gamma\left(m_1 + \frac{1}{2}\right)}{\sqrt{\left(m_1 + \frac{1}{2}\right)} \Gamma\left(m_1\right)},$$

and

$$y = y_2 \left\{ 1 + \frac{x^2}{2\sigma^2 \left(m_2 - \frac{1}{2}\right)} \right\}^{-(m_2 + 1)}, \quad \text{where:} \quad y_2 = \frac{N}{\sqrt{2\pi}\sigma} \frac{\Gamma\left(m_2 + 1\right)}{\sqrt{\left(m_2 - \frac{1}{2}\right)} \Gamma\left(m_2 + \frac{1}{2}\right)}.$$

They are both symmetrical, they both for any value of m_1 or m_2 which is moderately large are indistinguishable in appearance from the Gaussian curve. If they represented actual observations, we should try to fit them (i) by finding the area, (ii) by finding the standard deviation. The former for both curves is N and the latter for both curves is σ. Hence we should fit them with

$$y = \frac{N}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(i)}.$$

But this in both cases would be incorrect. Both cases would only pass into the Gaussian curve when m_1 and m_2 are theoretically infinite, practically large. No Gaussian fitting could distinguish one of these curves from the other. Why?— Because it does not proceed further than the standard deviation. To measure the difference between either of the above distributions and the Gaussian curve we must proceed to higher moments. Let $N\mu_n$ be the nth moment about the mean, i.e. if $\bar{x}$ be the mean value of x,

$$N\mu_n = \int y \, (x - \bar{x})^n \, dx,$$

where the limits of the integral are those of the range. Then if $\beta_2 = \mu_4/\mu_2^2$, we easily find:

$$m_1 = \frac{3\left(\beta_2 - 1\right)}{2\left(\beta_2 - 3\right)} \quad \text{and} \quad m_2 = \frac{3\left(\beta_2 - 1\right)}{3 - \beta_2},$$

or

$$\beta_2 - 3 = \frac{6}{2m_1 - 3}, \qquad 3 - \beta_2 = \frac{6}{2m_2 + 3}.$$

Thus we reach one of the conditions for the Gaussian curve, i.e. $\beta_2 = 3$, in either case when m_1 and m_2 are considerable, but if β_2 be > 3, m_1 will be positive and if $\beta_2 < 3$, m_2 will be positive. Now since $\dfrac{m_1 - 1}{m_1 + \frac{1}{2}}$ is always less than $\dfrac{m_2 + 1}{m_2 - \frac{1}{2}}$, it is easy

to show that in the neighbourhood of the origin y/y_1 is always greater than y/y_2 for the same value of x. In other words, the first curve is flatter topped than the second, and both lie on different sides of the corresponding Gaussian curve. The first curve type is platykurtic and the second leptokurtic.

Now there is nothing to prevent us fitting curves of the above types to any series of frequency observations. Supposing those observations are truly normal, then m_1 or m_2 will be so large that $\beta_2 = 3$ within the error of random sampling. Now the probable error of β_2 for a Gaussian distribution of total frequency N^*:

$$= \cdot 67449 \sqrt{\frac{24}{N}},$$

and if β_2 differs from 3 by several times this probable error, it is absolutely impossible to treat the system as mesokurtic. In any such case one or other of the above curves *must* give a truer representation than the Gaussian curve. It is easy to show that for leptokurtic distributions the maximum frequency is greater than that given by the normal curve and for platykurtic distributions it is less. The Gaussian curve compels us to assert that the product of the maximum frequency into the standard deviation is a constant (i.e. $y_0 \sigma = N/\sqrt{2\pi}$). This condition of mesokurtosis is unfulfilled—within the limits of random sampling— for a great variety of frequency distributions.

Further it is absolutely certain that divergencies from the Gaussian or normal curve are not exclusively in the direction of either platykurtic or leptokurtic distributions. Thus the *symmetrical* binomial is essentially leptokurtic, i.e. $\beta_2 < 3$, and therefore cannot be used for a great variety of distributions. In general all skew binomials with $p > \cdot 2113$ and $< \cdot 7887$ are leptokurtic; outside these limits they are platykurtic.

The test whether a curve satisfies the mesokurtic condition has nothing to do with my particular views on frequency, it is merely deduced from the general principles of probability and is a test of normal distribution. Of course there are many other conditions to be satisfied, e.g. μ_{2n} should equal $(2n-1)\mu_{2n-2}$. But as I have shown elsewhere the probable errors of the high moments increase so rapidly, that it becomes easier and easier to satisfy such conditions within the errors of random sampling, and without very large numbers they are of little practical value.

The following are significantly platykurtic distributions:

The Maximum Breadth in English ♂ skulls,
The Nasal Breadth in English ♂ skulls,
The Cephalic Index of Altbaierisch skulls,
The Auricular Height in ♀ Naqada skulls.

* Pearson: *Phil. Trans.* Vol. 198 A, p. 278.

As a rule the data available in craniological investigations are too sparse to give any real test of mesokurtosis, and this is the true reason why we must content ourselves with the Gaussian curve.

Again Mr Powys found out of twelve frequency distributions for the stature of men and women that 11 were leptokurtic and the twelfth essentially mesokurtic. This tendency to leptokurtic distributions—which can hardly be due to chance—is actually given by Ranke and Greiner as a case in favour of the Gaussian curve! (*Anmerkung* S. 326). They further cite Fawcett and Lee in the following manner :

In der an letzter Stelle zitierten Arbeit ist der Nachweis einer bestimmt gerichteten Asymmetrie für die Mehrzahl der Masse und zwar in der nach Fechner zu erwartenden Richtung besonders beachtenswerth.

They do not say that of the 24 curves given by Fawcett and Lee 14 are leptokurtic and that Fechner's curve can only represent platykurtic distributions. They do not draw attention to the fact that the Fechner curve would be impossible for the whole of Powys' stature data, and for 12 out of Macdonell's 26 curves for the English skull! In other words, if the *Abweichungen* of Fawcett and Macdonell and Powys' data are to be used as an argument at all, 38 out of these 62 distributions diverge from the normal curve in a manner which cannot possibly be represented by Fechner's theory!

If we turn from the condition for mesokurtosis to those for differentiation of mode and mean and for skewness we meet other considerations. So far we are not dependent for anything we have said on any theory of frequency other than the Gaussian. On that theory $\beta_2 = 3$. and if the difference $\beta_2 - 3$ be significant the distribution cannot be Gaussian. If we want to distinguish between the mode and the mean, we cannot start from the Gaussian theory, because that theory supposes the two values absolutely the same. On the other hand if we consider asymmetry, we ought to have, within the limits of random sampling, all the odd moments zero, i.e.

$$\mu_3 = \mu_5 = \mu_7 = \ldots = \mu_{2n-1} = 0.$$

Now it is of very little practical value testing the high moments because their probable errors are excessive. The probable error of μ_3 for the normal curve $= \cdot 67449 \sqrt{\dfrac{6}{N}} \sigma^3$ and of $\mu_5 = \cdot 67449 \sqrt{\dfrac{945}{N}} \sigma^5$, or in terms of σ as our unit is thirteen times as large. These are the *gross* errors; the percentage probable errors are of course infinite. As a rule it is hardly worth testing these conditions beyond μ_3. We determine whether the third moment is zero within the limits of random sampling. If we wish a relative magnitude we can take $\beta_1 = \mu_3^2/\mu_2^3$, a quantity which occurs over and over again in frequency discussions. The probable error of β_1 is obviously zero for the normal curve, because β_1 is of the square of the order of small quantities. The probable error of $\sqrt{\beta_1} = \cdot 67449 \sqrt{\dfrac{6}{N}}$, and

$\sqrt{\beta_1}$ for all truly normal distributions ought not to differ by more than two or three times the above expression from zero.

We can form other expressions involving β_1 and β_2 and ask what their value is for the Gaussian curve. We can calculate their probable errors, and determine whether the given distribution satisfies the Gaussian value within the limits of random sampling.

Thus I take the expression:

$$\chi = \frac{1}{2} \frac{\sqrt{\beta_1}(\beta_2 + 3)}{5\beta_2 - 6\beta_1 - 9} = \frac{1}{2} \frac{\sqrt{\beta_1}(6 + \beta_2 - 3)}{6 + 5(\beta_2 - 3) - 6\beta_1} \quad \text{......(ii)}.$$

Clearly this expression vanishes for the normal curve, and $= \frac{1}{2}\sqrt{\beta_1}$ nearly when $\sqrt{\beta_1}$ and $\beta_2 - 3$ are not very large, i.e. when we have not a very wide deviation from normality. The probable error of this expression, if the distribution be really normal, is $\cdot 67449 \sqrt{\dfrac{3}{2N}}$.

Again, consider the expression:

$$d = \tfrac{1}{2}\sigma \frac{\sqrt{\beta_1}(\beta_2 + 3)}{5\beta_2 - 6\beta_1 - 9} \quad \text{......(iii)}.$$

This is a length which vanishes, if the distribution be truly normal. Its probable error is $\cdot 67449 \sqrt{\dfrac{3}{2N}} \, \sigma$ in the case of the Gaussian curve, and accordingly d should not differ from σ by more than two or three times the above probable error.

Now let us write $\eta = \beta_2 - 3$. Then it is absolutely impossible for any distribution to be looked upon as Gaussian unless χ, d and η are zero within the limits of random sampling. These limits being deduced from their known probable errors.

Now it will be asked why choose such an expression as χ instead of the simpler $\sqrt{\beta_1}$? The answer is quite simple. We want to determine whether the mode coincides with the mean or not, and we cannot do this on the basis of the Gaussian curve where no distinction is made between the two. We must take some curve which is not Gaussian to determine this important quantity from. Now the equation to the Gaussian curve is

$$y = y_0 \, e^{-\frac{(x-m)^2}{2\sigma_0^2}},$$

where m is the mean value of x and σ_0 the standard deviation, and we have for its differential equation:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x - m}{\sigma_0^2}.$$

Now if we assume that the actually observed character is not x but X, and that X is some function of x, we shall not in plotting the frequencies to X obtain a normal curve, but we ought if Y be the ordinate of this curve to have

$$Y dX = y dx \quad \text{or} \quad Y = y \frac{dx}{dX}.$$

Taking logarithmic differentials

$$\frac{1}{Y}\frac{dY}{dX} = \frac{1}{y}\frac{dy}{dx}\frac{dx}{dX} + \frac{d^2x}{dX^2}\bigg/\frac{dx}{dX}$$

$$= -\frac{1}{\sigma_0^2}(x-m)\frac{dx}{dX} + \frac{d^2x}{dX^2}\bigg/\frac{dx}{dX}.$$

Assume $x - m = f(X)$ and we find:

$$\frac{1}{Y}\frac{dY}{dX} = -\frac{X}{\sigma_0^2 F(X)} \quad\dotfill(\text{iv}),$$

where $\qquad F(X) = Xf'(X)/\{f(X)(f'(X))^2 - \sigma_0^2 f''(X)\}.$

The form has been so chosen that the origin is the mode, i.e. dY/dX vanishes with X. The proposal to thus generally transform the Gaussian curve is due in a quite different form to Edgeworth[*]. Kapteyn following Edgeworth and without any acknowledgment takes:

$$f(X) = \beta(X + \kappa)^q$$

where β, κ and q are constants to be determined.

He therefore puts:

$$F(X) = \frac{X(X + \kappa)}{\sigma_0^2(q-1) - mq\beta(X+\kappa)^q - q\beta^2(X+\kappa)^{2q}}.$$

This is a somewhat complex expression. The resulting frequency curve is

$$Y = Y_0(X + \kappa)^{q-1} e^{-\frac{1}{2\sigma_0^2}\{(X+\kappa)^q - m\}^2} \quad\dotfill(\text{v}),$$

and has been suggested by Kapteyn as a general form of the skew frequency curve. We shall consider it later.

Galton and McAlister as early as 1879 took

$$f(X) = b \log \frac{X}{a} - m,$$

where b and a are constants. Ranke and Greiner, without apparently knowing the history of research in this field, take the same value and attribute to Fechner the well-known Galton-McAlister curve of the geometric mean which results. We find

$$F(X) = \frac{X^2}{\sigma_0^2 + b^2 \log \dfrac{X}{a}},$$

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[*] He has developed it in a long series of papers published in the *R. Statistical Society's Journal*,

whence
$$\frac{1}{Y}\frac{dY}{dX} = -\frac{1}{X}\left(1 + \frac{b^2}{\sigma^2}\log\frac{X}{a}\right),$$

and
$$Y = Y_0\frac{1}{X}e^{-\frac{b^2}{2\sigma_0^2}\left(\log\frac{X}{a}\right)^2}\dots\dots\dots\dots\dots\dots(vi).$$

Edgeworth himself has made other suggestions as to suitable values for $f(X)$ and accordingly of $F(X)$. Now it is quite clear that assuming the character to be a definite function of another character which really obeys the normal law, there is no more reason for assuming one form of $f(X)$ than another, because we are in absolute ignorance of the nature of this function. Kapteyn's, or Edgeworth's, or Galton's are equally valid, and the only test of their relative suitability lies in the extent to which the resulting curves fit actual data. Clearly to assume $x = f(X)$ is to assume the actual frequency distribution to follow any law whatever. It is only screening the generality of the assumption

$$y = \phi(x),$$

where ϕ is unknown, by an appeal to the supposed universality of the Gaussian curve and by a perfectly arbitrary selection of the subsidiary function f.

But there is another manner of looking at this proposal. Returning to the equation

$$\frac{1}{Y}\frac{dY}{dX} = -\frac{X}{\sigma_0^2 F(X)},$$

and writing $\sigma^2 = \sigma_0^2 F(X)$, we see that it becomes identical in form with the normal equation, i.e.

$$\frac{1}{Y}\frac{dY}{dX} = -\frac{X}{\sigma^2}.$$

In other words the distribution of any frequency may be looked upon as given in the neighbourhood of any point by a normal curve of standard deviation $\sigma_0\sqrt{F(x)}$. Hence the conception arises that if the causes which produced variation in the immediate neighbourhood of any value x_0 of the character, were constants for the whole range of variation, we should have a normal curve of standard deviation $\sigma_0\sqrt{F(x_0)}$*. In reality there is a continuous and gradual change of the tendency to variability as we pass from one value of the character to a second†. Analytically

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* This method of looking at the matter throws light on another point. If a curve be of limited range, it signifies that $\sigma = 0$ at certain points, or the curve stops because we have reached the limits of local variation. In a curve of unlimited range it is not the capacity for local variation but the absence of individuals to vary, which is the special feature.

† The matter is of such importance relative to some of Ranke's criticisms that I give another proof of equation (iv) here, based on the conception of an infinite number of infinitely small cause groups which Ranke considers can only lead to the normal curve. Let y_{r+1} be the $(r+1)$th term of a binomial, skew or symmetrical, say for simplicity the latter, i.e. $(\frac{1}{2}+\frac{1}{2})^n$. Then

$$\frac{y_{r+1} - y_r}{\frac{1}{2}(y_{r+1}+y_r)} = \frac{n+1-2r}{\frac{1}{2}(n+1)}.$$

Now let c_r be the distance between y_r and y_{r+1} used in plotting these ordinates to obtain a curve, and let it be related to some small value c_0 by the relation $c_r = c_0 \times$ function of $r = c_0 \times \phi(r)$. Let X_r be measured

we may look at it in this way: If δx_0 be the variation in the neighbourhood of x_0, then δx_0 is not independent of x_0, *but correlated with it.* We may have a perfectly continuous population from dwarfs to giants, but it does not follow that the actual tendency to vary of dwarfs and of giants is identical. All proofs that I have seen of the normal curve fail in this respect. They assume that the character x is due to a number of increments, which are due to an indefinitely large number of *independent* cause groups. They assume that δx_0 is not correlated with the already accrued value x_0. All processes like those of Edgeworth, Galton, Kapteyn and Fechner are really devices for getting over this gradual change in the tendency to vary from point to point of the range. It appears to me best to directly acknowledge and face this difficulty by selecting a fitting function for $F(X)$.

If we drop now the distinction between X and x as unnecessary we reach as our frequency equation:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{\sigma_0^2 F'(x)},$$

or if we use Maclaurin's theorem for $F(x)$:

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2(1 + \overline{a_1}x + \overline{a_2}x^2 + \overline{a_3}x^3 + \ldots)} \quad\ldots\ldots\ldots\ldots\ldots(\text{vii}).$$

Now I have shown* how to determine the successive constants σ_0^2, $\sigma_0^2 a_1$, $\sigma_0^2 a_2$, etc. Further all these constants but σ_0^2 are zero, when the distribution is normal, and the series will be found to converge rapidly, when the distribution is

from the largest term of the binomial, then $X_r = c_0\sqrt{(n+1)} \times$ function of $r = c_0\sqrt{(n+1)}\,f(r)$, say, and conversely $r = $ a function of $X_r/\{c_0\sqrt{(n+1)}\}$. Divide both sides of the above equation by c_r, which may be written on the left ΔX, and we obtain:

$$\frac{\Delta Y}{X\Delta X} = \frac{-X_r}{\frac{1}{4}(n+1)c_0^2 f(r)\,\phi(r)/\{r/(n+1) - \frac{1}{2}\}}.$$

Put $\sigma_0 = \frac{1}{2}\sqrt{n+1}\,c_0$ and $F(r)$ for the expression $f(r)\,\phi(r)/\{r/(n+1) - \frac{1}{2}\}$ which does not become infinite with $2r = n+1$, because X_r and therefore $f(r)$ vanishes for this value of r, and accordingly $f(r)$ contains $2r - (n+1)$ as a factor. We then have:

$$\frac{\Delta Y}{y\Delta X} = -\frac{X_r}{\sigma_0^2 F(X_r/\sigma_0)}.$$

Now make n infinite and c_0 vanishingly small, then we have if $\sigma_0 = \frac{1}{2}\sqrt{(n+1)}\,c_0$ be still finite

$$\frac{dY}{Y dX} = -\frac{X}{\sigma_0^2 F(X/\sigma_0)},$$

a result in agreement with the above investigation. In other words this, and not the Gaussian curve, is the generalised frequency curve we reach if we directly abrogate the third Gaussian principle, that contributory increments of the variate are independent. Of course the first two Gaussian principles simultaneously disappear. This view of the matter occurred to me many years ago, when considering Hagen and Crofton's proofs of the Gaussian law. It was expressed in my memoir of 1894 by the statement that we require curves produced by conditions in which the contributory cause groups are not independent, i.e. in which an increment δx to the variate x depends upon the value of x, or is correlated with it. My method of reaching such curves, however, was a direct appeal to discrete series in which such a condition was fulfilled.

* "Mathematical Contributions to the Theory of Evolution, XIV." p. 6. Dulau and Co., London.

in the least approximately normal. Accordingly if we wish to get a good interpolation curve to determine the distance between the mode and the mean, we may assume

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^{2}(1 + a_1 x + a_2 x^2)} \quad \dots\dots\dots\dots\dots\dots \text{(vii)} \, bis.$$

In this case we discover with the previous notation, that

$$d = \frac{1}{2}\frac{\sqrt{\beta_1}\,(\beta_2 + 3)}{5\beta_2 - 6\beta_1 - 9}\,\sigma$$

is the distance between mean and mode, and that $\chi = \dfrac{\frac{1}{2}\sqrt{\beta_1}\,(\beta_2 + 3)}{5\beta_2 - 6\beta_1 - 9}$ is the ratio of this distance to the variability, or what I term the skewness, or the asymmetry relative to the variability.

If we leave out a_2 we find the skewness given by $\chi = \frac{1}{2}\sqrt{\beta_1}$ and the distance between mean and mode $d = \frac{1}{2}\sqrt{\beta_1}\sigma$. In practice these give fairly closely the same values as the fuller expressions above, and the fuller expressions are not numerically much modified if we include a_3. Shortly we have got a very fair mathematical process of determining the position of the mode and the degree of asymmetry.

Now the constants of such a curve as (vii) *bis* are absolutely determined by a knowledge of σ, β_1, and β_2; or looked at inversely they suffice to fix σ, β_1, and β_2. In other words the degree of kurtosis $(\beta_2 - 3)$, the skewness χ and the distance between mean and mode—all most definite physical constants—are at once fixed by a knowledge of the constants of the curve, or on the other hand, being known, they fix those constants. It is of course allowable to replace any one of the three by the variability of the system. The actual position of the mode and the total magnitude of frequency suffice to fix the position and size of the curve. I have already called $\beta_2 - 3 = \eta$ the degree of kurtosis; I call d the modal divergence. Then unless

η, the degree of kurtosis be zero, subject to probable error, $\cdot 67449\sqrt{24/N}$,

χ, the skewness be zero, subject to probable error, $\cdot 67449\sqrt{1\cdot 5/N}$,

d, the modal divergence be zero, subject to probable error, $\cdot 67449\sigma\sqrt{1\cdot 5/N}$,

no distribution can be legitimately described as normal or Gaussian.

It would be of interest to know how far Ranke and Greiner have applied such tests to any series containing a large number N of individuals. I think if they had done so, they must have come to the same conclusion as the majority of statisticians that the normal curve has only a limited range of application.

Of course if N be small, as in most craniological series, we find our probable errors so large, that it is not possible to say more than that for *short* series the Gaussian curve may roughly describe the result. But for long series in economics, sociology, zoology, botany and anthropometry the Gaussian curve over and over again fails. If in all these cases Ranke and Greiner assert that the material is

heterogeneous they are merely arguing in a circle. The distributions are as continuous and smooth as those which occur in the case of the Gaussian curve, and *they occur for characters in the same group of individuals which present for other characters the normal distribution.*

Thus the length of meropodite of right claw in *Gelasimus pugilator** is quite sensibly normal, but the length of the carpodite of the right claw is almost as certainly platykurtic and skew. If two characters are normal, a third character which is their difference, whether they be correlated or not, should have a normal distribution, yet in the case of *Gelasimus pugilator* for the whole series of measurements the difference distributions are essentially platykurtic†.

The size of the disc in *Ophicoma nigra*‡ has a modal difference in the distribution of 1000 cases of ·271 mm. and the probable error on the basis of a normal curve is ·056. The deviation is thus five times its probable error and the asymmetry undoubtedly significant.

The outer diameter of *Arcella vulgaris* § in 504 cases gives a modal difference of 3·226 mikrons, and the probable error of this difference is only ·211. The asymmetry is therefore undoubtedly significant.

The distance between the mean and mode in the case of the length of shell of *Nassa obsoleta* ‖ from Lloyd Point, Long Island, U.S. was ·68 mm. for 368 individuals, the probable error of this modal difference was ·08. The asymmetry is therefore significant. Other characters of *N. obsoleta* were as definitely asymmetrical, while some from exactly the same individuals were sensibly normal.

The transverse arc in ♂ Naqada skulls has for 115 individuals a modal difference of 2·34 mm. and the probable error of this difference is ·78 mm., or it is probably significant. Yet the breadth of the male Naqada skulls is significantly symmetrical.

The height/length index of 117 English ♀ skulls has a modal difference of ·85 and a probable error of only ·22, the skewness is therefore significant.

The same is true of the distribution of many internal organs in man. For example, if we exclude recognised diseased hearts, we obtain a markedly skew distribution such as is given in the broken line of Fig. 1. This is for 1382 heart-weights. If this be supposed to be due to the great variety of ages, we have only to look at the continuous curve for hearts of 358 young adults, 25 to 35, to see the same asymmetry. This is drawn for four times the scale.

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* G. Duncker : *Biometrika*, Vol. II. p. 313.

† There is another point to which I will only refer briefly here. If characters were always distributed according to the Gaussian law the regression curves must be *straight* lines. The generalised Mendelian theory of determinants I have developed makes them, however, hyperbolas, and I have given instances in a recent memoir of a variety of curved regression lines

‡ McIntosh : *Biometrika*, Vol. II. p. 470.

§ R. Pearl and F. J. Dunbar : *Biometrika*, Vol. II. p. 327.

‖ A. C. Dimon : *Biometrika*, Vol. II. p. 29.

Lastly if we consider that the question of health determines the skewness, we have in the dotted curve the weight-distribution of 699 hearts stated to be "healthy."   We see that there is still the same essential skewness; the pathologist has merely cut off a small portion of the tail on the left and far too much of the tail on the right, i.e. unusually big hearts were discarded as necessarily "unhealthy." The form of the curve undoubtedly indicates that many of these large hearts are abnormal, but any continuous curve fitted to the remainder, the "healthy hearts," would not only be significantly skew, but would project a long way into the portion of the tail discarded as "unhealthy."   The list of asymmetrical distributions might be indefinitely extended, but these must suffice to indicate that asymmetry cannot be lightly put on one side in the manner adopted by Ranke and Greiner.

Weight of Heart in English ozs.
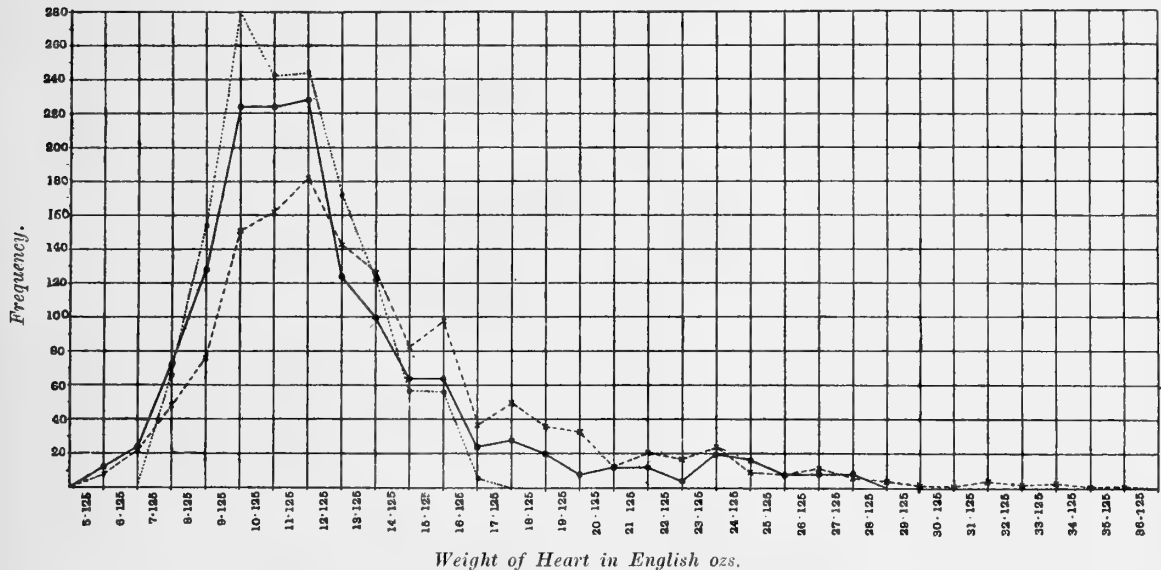
Fig. 1.   Frequency Polygons of Weight of Heart in Males (Greenwood*).

I.   *------*   25 to 55 years.   Without specific disease of heart.   Number of cases 1382.
II.   O——O   25 to 35 years.   Without specific disease of heart.   Number of cases 358.
III.   •········•   25 to 55 years.   Definitely sound hearts.   Number of cases 669.

The scale is four times as great for II. and twice as great for III. as for I.

If we pass to discrete variates, we find as large a number, if not a larger number of distributions in which skewness is well marked, for example, fertility in the Aphis *Hyalopterus Trirhodus*†, fertility in man‡, fecundity in race-horses§, and fertility and fecundity in mammals generally.   I illustrate this with an example of fertility in English mothers in Fig. 2.   It will be seen at once that no normal curve could be used to describe this distribution.   It is equally

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* *Biometrika*, Vol. III. p. 45 *et seq.*
† Warren : *Biometrika*, Vol. I. p. 127.
‡ Pearson : *Phil. Trans.* Vol. 192 A, p. 257, and *The Chances of Death*, Vol. I. p. 63.
§ Pearson : *Biometrika*, Vol. I. p. 292.

FIG. 2.   Number of Children in English Families.
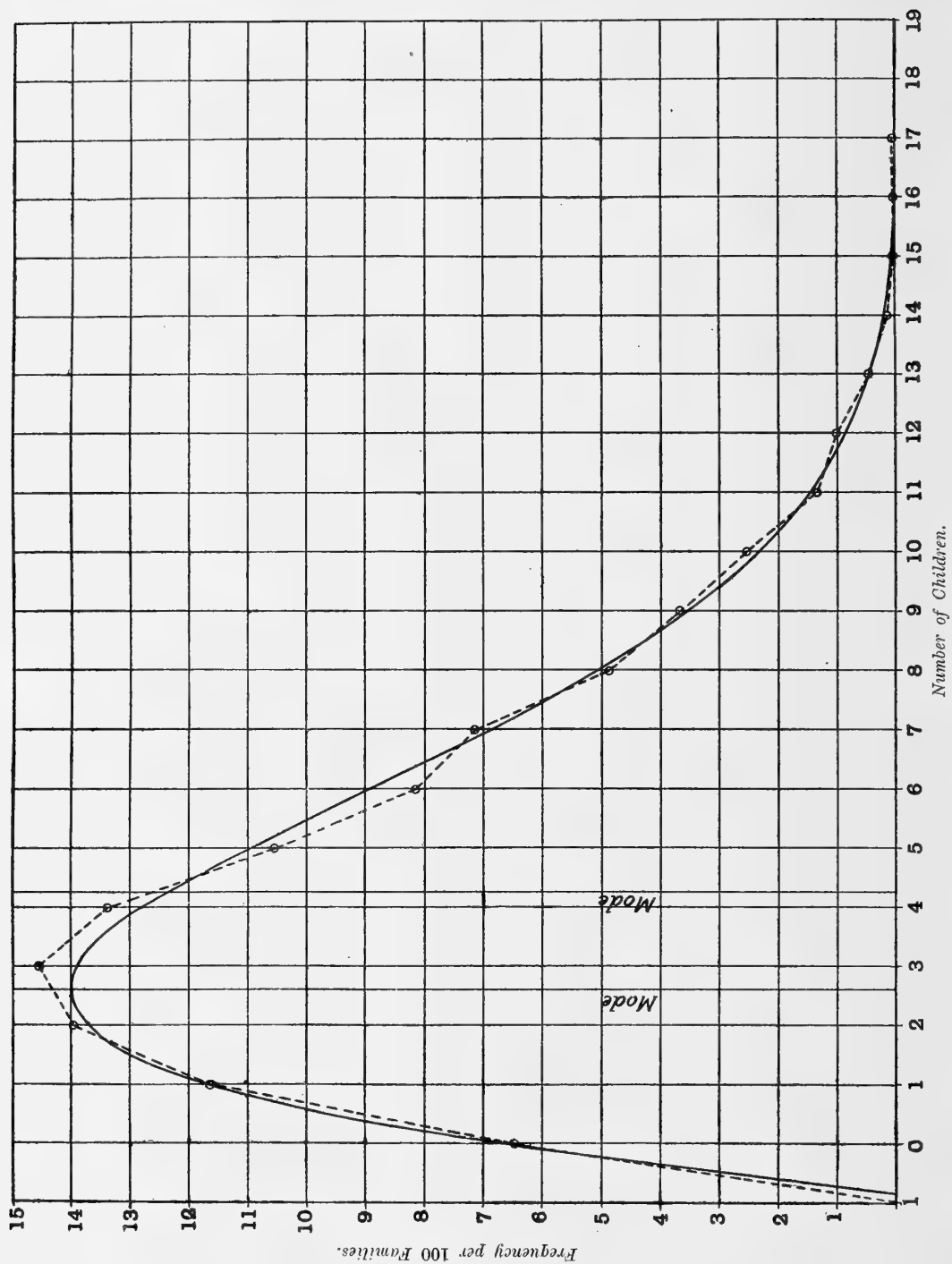
impossible also for a curve of the McAlister-Galton type, for the simple reason that that curve has high contact at both ends of the range. Now I contend that the anthropologist who either neglects such matters as human fertility, or confesses that he has no means of succinctly describing their distributions—and as long as he sticks to the Gaussian curve he certainly will not have—is simply putting on one side a fundamental factor in the science of man. Ranke lays great stress on homogeneity. He does not, however, clearly define what he means by the term. Apparently any series which follows a Gaussian curve is to him homogeneous, any other series is not. I should be glad if he would then consider any craniological series, say, of adult crania. This will involve crania of adults from perhaps 25 or 30 years of age to 50, and these are rather narrow limits considering our paucity of material. Now our data show that the correlation between head measurements and age may be of the order of about $-\cdot1$ to $-\cdot2$. After about 25 to 28 years of age in man, there is a continual shrinkage not only of stature but of skull capacity, brain-weight, circumferences and diameters of the skull. Under the circumstances, what right have we because the Gaussian curve is obtained to call this material "homogeneous"? I will go further; suppose we could, and we can, obtain the measurements on one or two thousand individuals of the same age; are these to be considered as a homogeneous distribution? My reply will be, in man the order of birth is an essential feature in determining the dimensions of the physical characters. My investigations show that physique and health are sensibly correlated with the position of a member in his own family. In mammals others have shown that the physique of an individual is sensibly correlated with the number of members born in that individual's litter. Now with these facts before us what stress can be laid on Ranke's conceptions of homogeneity? The practical anthropologist requires curves which will successfully *graduate* his data. Only on the basis of such graduations can he allow for the influence of disturbing factors like age, order of birth, season or special position of production in the organism. I take this very case: How is it possible to allow for the influence of order of birth, unless you know the size of families or the distribution of births within the community? It can only be achieved provided the distribution can be represented by a few simple constants which allow of definite mathematical handling.

Ranke and Greiner say with considerable asperity that my method of determining the range from given data can be of no service. Yet take this very case of size of families. In the English middle classes for 4390 instances, I find that the *observed* limit is 17, but fitting a skew curve the range is determined as 22 children running practically from just before 0 to over 21 births (Fig. 2). For Denmark by the same process in 34,000 cases the theoretical range is 26 and the observed range 22*. I then proceeded to take statistics for the Argentine Republic, and found for the town of Buenos Ayres, 27,510 births, that the range of the curve was from ·25 to 36·61 births, or 37 possible births. The maximum observed in these 27,510 births was 23. But among the South- and Mid-American

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* *The Chances of Death*, Vol. I. *Reproduction Selection.*

populations cases of 24 (Trinidad), 26 (Cuba), 27 (Nicaragua) and 34 (Colombia), the women beginning to bear at 13 and continuing to 50, have been recorded! It will, I think, be realised by the impartial reader that Ranke's statements:

Für den Anthropologen ist also nur das Gaussche Gesetz von Wichtigkeit. Für seine Probleme beansprucht es aber auch völlige Gültigkeit (S. 327)

and

So haben anderseits auch die Werte von Variationsumfängen die bislang aufgefunden sind, keinerlei weitere Erkenntniss gebracht (S. 324)

fall wide of the mark. Ranke has either very much circumscribed the field of the anthropologist, or he has not in this, as in other cases noted in his paper, studied the literature of the subject, or finally he has disregarded the results reached.

It is quite true that the range cannot always be determined and being determined does not always give a very good result. The reasons for this are not far to seek. For example, If a discrete quantity has for its minimum 0 units the start of the curve must naturally fall on the negative side of the origin—since its area measures frequency between $-\cdot5$ and $+\cdot5$. Ranke would probably find something mysterious in this "reichen oft ins Negative." Actually it is to be expected, especially if due allowance be given for the probable error of the range. In most biometric statistics, we cannot as in the case of births deal with 20,000 to 30,000 cases and get small errors for our constants. We have only perhaps 500 to 2000 cases and even less than this in craniology. This may denote an error of 14 to 17 p.c. in the calculated range, and it is quite possible that the range may "reichen ins Negative." Take the case worked out by me* of the number of Müllerian glands in the forelegs of ♀ swine. The range theoretically calculated is 18 glands with a probable error of $\pm 2\cdot54$; the start of this range ought not to have exceeded $-\cdot5$. It is actually $-\cdot82$ with a probable error of $\cdot16$. The actual skewness of this distribution is $\cdot31$ with a probable error of $\cdot02$. The distribution is accordingly significantly asymmetrical.

I have cited these cases as sufficient for our present purpose, but there are many other cases in which the discovery of the range has been of biological or special anthropological interest, e.g. the earliest appearance of certain diseases in childhood, the range of cancer attacks, the first occurrence of signs of puberty, etc. It has been applied also effectively to a number of zoological and botanical data. A more striking case, perhaps, of usefulness is the limit to high barometric pressure obtained by dealing with the frequency statistics of barometric height at series of stations†. Throughout the whole of the stations of the British Isles dealt in, there is sensible skewness of distribution, and with one Irish exception, which is sensibly mesokurtic, the whole series of curves are platykurtic, and this deviation from normality cannot be chance, but is a significant character of the frequency distributions. In these cases the limit to high pressure has been found, and appears to be a constant of considerable physical importance for the local climate.

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* Pearson and Filon: *Phil. Trans.* Vol. 191 A, p. 289.

† Pearson and Lee: *Phil. Trans.* Vol. 190 A, p. 423 *et seq.*

I again am forced to consider that Ranke has not been aware of what has been published, still less what has been done in this matter. He appears to base his conclusions chiefly on my first paper on skew-variation, and he has not noticed the fact that being the *first* paper much has to be corrected in the light of more recent work in the past ten years. Ranke speaks of the writer's:

Andwendung von allerlei grösseren und kleineren Änderungen in seiner Methode *ad hoc* in eine für den vorliegenden Zweck nicht zu unwahrscheinliche Form bringen (S. 324).

Now I contend that this gives a grossly unjust description of the paper in question. Had Ranke read recent literature, he would have been aware that the great difficulty with frequency distributions is to obtain the true values of the "moments" from records which merely give data for arbitrary "Spielräume," often far too large and usually selected by the observer without any regard to the needs of the computator. My method is one based on the method of moments, but to deduce the moments from given data is the real difficulty which Ranke never for an instant seems to grasp or at any rate refer to. The standard deviation (which he appears to consider sufficient for anthropologists) will vary, and often very sensibly, with the nature of the grouping of the data. This difficulty was very present in my mind in 1894, and is constantly referred to in my memoir, the "allerlei grössere und kleinere Änderungen in seiner Methode" are no changes in method at all but attempts to obtain some approximation to the true moments of the data. It was not till 1898 that Sheppard showed the correct manner of calculating the moments from the raw data in his important memoir on frequency constants*, for *one* type and one type only of frequency distribution. The curves calculated by Sheppard's method, now in general use, would give better results undoubtedly than are to be found in my memoir of 1894. Further, however, Sheppard's method applies only to curves with high contact with the horizontal axis at both ends. It leaves us still in doubt as to how to find the moments of curves, which cut the axis at the end of the range or are asymptotic at one or both ends to the vertical axis. At such ends of the range, the real solution lies in recording the frequency for very small elements, but this was not provided in any of the statistics which were then before me. It is just these cases of limited range at one or both ends which present difficulty in the determination of the moments. The difficulty will be familiar to all statisticians, if it has escaped Ranke. To some extent it is met in my memoir on the systematic fitting of curves issued in April, 1902†. Yet granting all these difficulties what do we find in my memoir of 1894? An analysis of the cases in which range is dealt with seems justified by the charges made:

Example I. Range determined of Cambridge Barometric Heights. There is nothing physically improbable in the result.

Example VI. Range found for enteric fever runs from -1.35 years to about 385 years. The probable error of the range is not given, but the whole difficulty

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* *Proc. London Math. Society*, Vol. xxix. p. 353 *et seq.*   † *Biometrika*, Vol. i. p. 265.

turns upon the *great* changes introduced into the range by different methods of calculating the moments. More recent investigations, in which the sexes are separated, the moments more accurately determined, and larger numbers dealt with, give far better results for zymotic diseases. I presume that one character being age, however, Ranke and Greiner would dismiss these data from consideration under any circumstances.

Example VII. Guesses at 9 tints. Possible range 1 to 9, i.e. curve to run from ·5 to 9·5. Observed guesses run from 1 to 8. Theoretical range of 11 instead of 9. The paucity of the observations gives a probable error of at least 20 to 30 per cent. in the determination of the range, and the result is rather better than might have been anticipated.

Example VIII. Ratio of forehead to body length in *Carcinus moenas*, observed range 30, calculated range 51. This range is probably not very close but it is not in any way that I can see impossible. The material is probably dimorphic.

Example XI. H. de Vries's data for *Ranunculus bulbosus*. Actually observed range 5 to 10 petals. Calculated range 5 to 11 petals.

Example XII. H. de Vries's data for a race of *Trifolium repens*. Actually observed range 0 to 10 high blossoms. Theoretical range in complete agreement.

Example XIV. Pauperism percentages for 632 cases. Observed range 18 for the year 1891 dealt with. Calculated range 31. This range gives 2 units of negative pauperism. Its probable error is, perhaps, 14 per cent.

It will be seen that out of the seven examples in which range is calculated only three *reichen ins Negative*, and that this *reichen* is well within the limits of the errors arising on the one hand from random sampling and on the other from the defective methods of determining the moments, which were alone available in 1894. While quite appreciating the honour done me when other workers use my methods, I must decline to be responsible in any way for their application of my formulae. I have so often found that their failure to fit my curves is due to a misapprehension of my methods or to actual errors in arithmetic, that I have long given up any attempt to set such matters right. The frequent assumption made that statistical methods can be applied without adequate mathematical training is the source of most of the slips in this matter*.

So far then I think we may conclude that Ranke is completely unjustified both in his statement that the Gaussian curve fully describes all the frequency that is of importance to the biologist, and in his attempt to discredit any result of scientific value which flows from endeavouring to measure such differences from the Gaussian law as we find in the distance between mode and mean, the skewness, the kurtosis and range of many actual frequency distributions.

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* A good illustration, by no means unique, of this is F. Reinöhl: *Die Variation im Andröceum der Stellaria Media*, 1903. He finds it impossible to fit certain distributions with my curves, owing to ignorance of the full literature and to faulty determination of the moments. He then argues from this want of fit to biological conclusions.

All the leading statisticians, from Poisson to Quetelet, Galton, Edgeworth, and Fechner, with botanists like de Vries, zoologists like Weldon have realised that asymmetry must be in some way described before we can advance in our theory of variation. In innumerable cases the important quantities measured by η, χ and d actually exist; these have each their physical significance and they must be found. It is perfectly open to Ranke and Greiner to criticise my method of determining these quantities, but that they should shut their eyes to their existence appears to me only compatible with a very small acquaintance with the data of variation.

Let us see now how various authorities have met this difficulty of skewness.

B. *The Gaussian Curve*.*

Gauss proceeds from the axiom that: *The arithmetical mean of a series of observations gives their most probable value,* i.e. the mean is the value of maximum frequency. This result is not axiomatic. It can only be a result of experience, and if it were true it would make the normal curve as much a result of experiment, i.e. an empirical result, as any other proposed curve of frequency. Gauss's proof demands, however, something more than this first statement. It involves (i) the equal probability of errors in excess of the mean and of errors in defect, (ii) the continuity of magnitude in the errors, and (iii) the independence of all the small contributions to the total error.

Experience shows that Gauss's fundamental axiom as to the mode and mean coinciding is not universally true. It is not true of errors of observations, it is not true of variations in living forms. Gauss reaches a differential equation which leads to the normal curve. His proof seems to me, as it has done to many others, quite invalid, because the equal probability of errors in defect and excess of the mean is not demonstrated, the possible dependence of contributory elements is not discussed, and the question of continuity of errors is not considered.

C. (i) *Laplace and Poisson.*

Laplace and after him Poisson took, I venture to think, much firmer ground. They did not assume (i) and (ii), but they did not realise the importance of (iii). They proceeded by evaluating the terms of the binomial:

$$(p + q)^N.$$

* In writing for Germans I naturally spoke of the Gaussian curve. But I am not clear that precedence is to be given to Gauss. Gauss first gave a proof of the well-known equation $y = y_0 e^{-\frac{1}{2}x^2/\sigma^2}$ in his *Theoria Motus Corporum Coelestium* of 1809. This was three years before the publication of Laplace's *Théorie Analytique des Probabilités* of 1812. But to give absolute priority to Gauss is to disregard Laplace's earlier memoirs, particularly those of 1782, "Sur les approximations des Formules qui sont fonctions des très-grands nombres," and its *Suite du Mémoire* of 1783. On p. 433 of the latter memoir Laplace actually suggests the importance of forming a table of the probability integral $\int e^{-t^2} dt$. The *Théorie des Probabilités* reproduces the substance of this memoir, and on this account some writers have post-dated Laplace's work. Gauss stated that he had used the method of least squares in 1795, but this does not necessarily involve a knowledge of the probability integral, and if it did, it is ten years after Laplace. On the whole my custom of terming the curve the Gauss-Laplacian or *normal* curve saves us from proportioning the merit of discovery between the two great astronomer mathematicians.

Using Stirling's theorem they showed that if the mth be the largest term in the binomial, then the sum p_r of all the terms from $m - r$ to $m + r$ is very nearly given by:

$$p_r = \frac{2}{\sqrt{2\pi}\sigma} \int_0^r e^{-\frac{x^2}{2\sigma^2}} dx + \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{r^2}{2\sigma^2}} \dots\dots\dots\dots\dots\dots(viii),$$

where $\sigma = \sqrt{Npq}$.

Here we have the first appearance of the probability integral as representing a series of *discontinuous* binomial terms. In fact when N is fairly large Laplace and Poisson show that sums of terms of the binomial are closely given by the areas of the probability curve. It is an approximate result based upon Stirling's theorem, and it does not for a moment involve making N infinitely large, or the spacing apart of the binomial terms very small. This representation of a number of finite terms by the probability integral seems to be unfamiliar to Ranke and Greiner, but no practical statistician would calculate the sum of r terms in the binomial $(p + q)^N$ for even moderate values of N. He would simply calculate the standard deviation $\sigma = \sqrt{Npq}$ of the binomial and turn up tables of the probability integral. This fundamental property of the normal curve, i.e. that it closely represents a *discontinuous* series, is passed over in silence by my critics. It is the very purpose for which the probability integral was originally introduced by Laplace. In other words it arises without any consideration of (i) continuity of variation, or (ii) equal probability of negative and positive deviations.

It will be observed that the above approximation to the binomial, i.e. to $(p + q)^N$ is symmetrical, but we can easily allow for some degree of asymmetry. Still writing $\sigma = \sqrt{Npq}$, and for the binomial

$$\beta_1 = (1 - 4pq)/(Npq), \quad \eta = \beta_2 - 3 = (1 - 6pq)/(Npq).$$

I have shown[*], y_0 being the maximum term in the binomial, that the rth term from the maximum is given by:

$$y_r = y_0 e^{-\frac{r^2}{2\sigma^2}(1 - \beta_1 + \frac{1}{2}\eta) - \frac{1}{2}\sqrt{\beta_1}\frac{r}{\sigma} + \frac{1}{6}\sqrt{\beta_1}\frac{r^3}{\sigma^3}(1 - \frac{5}{2}\beta_1 + \frac{9}{2}\eta) - \text{etc.}} \dots\dots\dots\dots(ix).$$

The term in r/σ was, I believe, first added by Poisson, and expresses his attempt to allow for asymmetrical variation. Edgeworth expanding the exponential has adopted for his asymmetrical curve, a form easily deduced from (ix),

$$y_r = y_0 e^{-\frac{r^2}{2\sigma^2}} \left\{ 1 - \frac{1}{2}\sqrt{\beta_1} \left(\frac{r}{\sigma} - \frac{1}{3}\frac{r^3}{\sigma^3} \right) \right\} \dots\dots\dots\dots\dots(x).$$

It will thus be seen that the normal function and the probability integral arise naturally from the expression for a single term or a series of terms of the binomial polygon. This is their historical origin and the historical origin of the conception of asymmetrical variation. Instead of the complex form given above resulting from Stirling's theorem, I approached the subject by looking at the relation of the

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[*] *Phil. Trans.* Vol. 186 A, p. 348, footnote.

normal curve to a symmetrical binomial in a totally different manner. I succeeded in showing that the ordinates and areas of the normal curve gave exceedingly closely the terms and sums of terms of the symmetrical binomial even for relatively small values of n. This had already been done by Laplace. The reader will realise this if he looks at the closeness of the normal curve with

$$\sigma = \sqrt{Npq} = \tfrac{1}{2}\sqrt{N},$$

and the binomial $(\tfrac{1}{2} + \tfrac{1}{2})^{10}$ with $N = 10$ in the accompanying Figure 3. But my method enabled me to give a simpler expression to the asymmetrical binomial
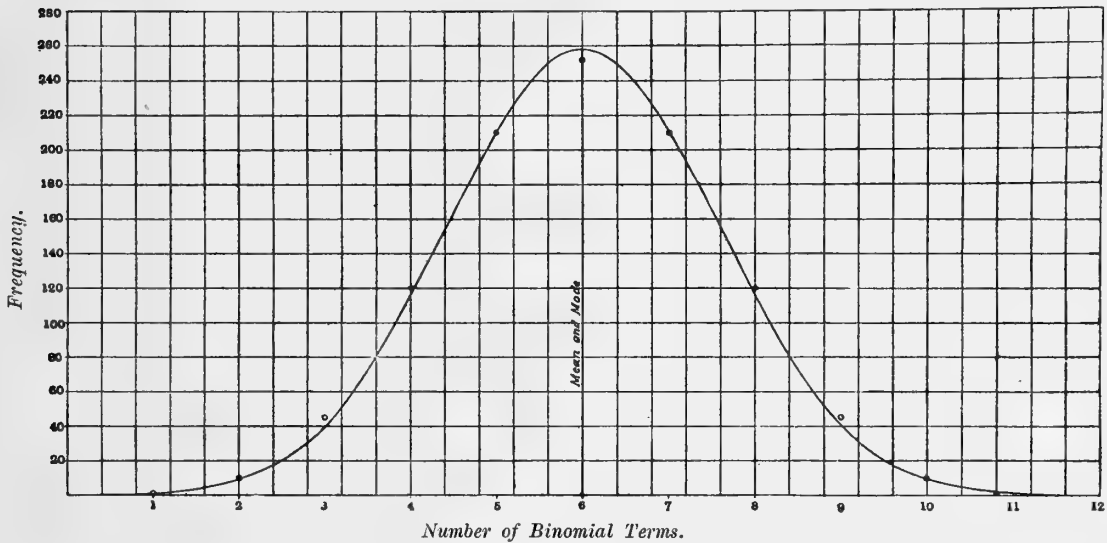
Fig. 3. Comparison of Point Binomial $1024 (\tfrac{1}{2} + \tfrac{1}{2})^{10}$ with the Gaussian Curve.

S.D. $= \sqrt{Npq} = 1\cdot5811$.   Maximum Ordinate $258\cdot35$.

than had been obtained by Poisson or Edgeworth using Stirling's theorem. Figure 4 shows how closely the terms of the asymmetrical binomial $5000 \left(\tfrac{1}{6} + \tfrac{5}{6}\right)^{12}$ and the sums of terms are reproduced by my curve of Type III., i.e.

$$y = 1536\cdot54 \left(1 + \frac{x}{5}\right)^{14} e^{-3x}.$$

I had no higher ambition—nor could I have had one higher—than Laplace had when he discovered the normal curve. I wanted to find a close mathematical expression for the terms of the asymmetrical binomial for relatively small values of N.

Now Laplace and Poisson had both retained the last of Gauss's limiting conditions, i.e. they had by adopting the binomial supposed each increment of the deviation to be independent of previous increments. It seemed needful to me to get rid of this condition, and I therefore introduced instead of the binomial the hypergeometrical series. Here the successive increments are correlated. In order

to place this new representation on the same footing as the symmetrical binomial
to which Laplace approximated with the normal curve, I deduced as I had done for
the symmetrical and the asymmetrical binomials, curves which gave the hyper-
geometrical series and the sum of its terms as closely as Laplace's normal curve
gave the symmetrical binomial. This is the complete history of the development
of my skew curves. Before I proceed to discuss Ranke and Greiner's criticisms,
I must remark that their attack on this point does not concern me only. Every
practical statistician uses Laplace's representation of the point binomial by the
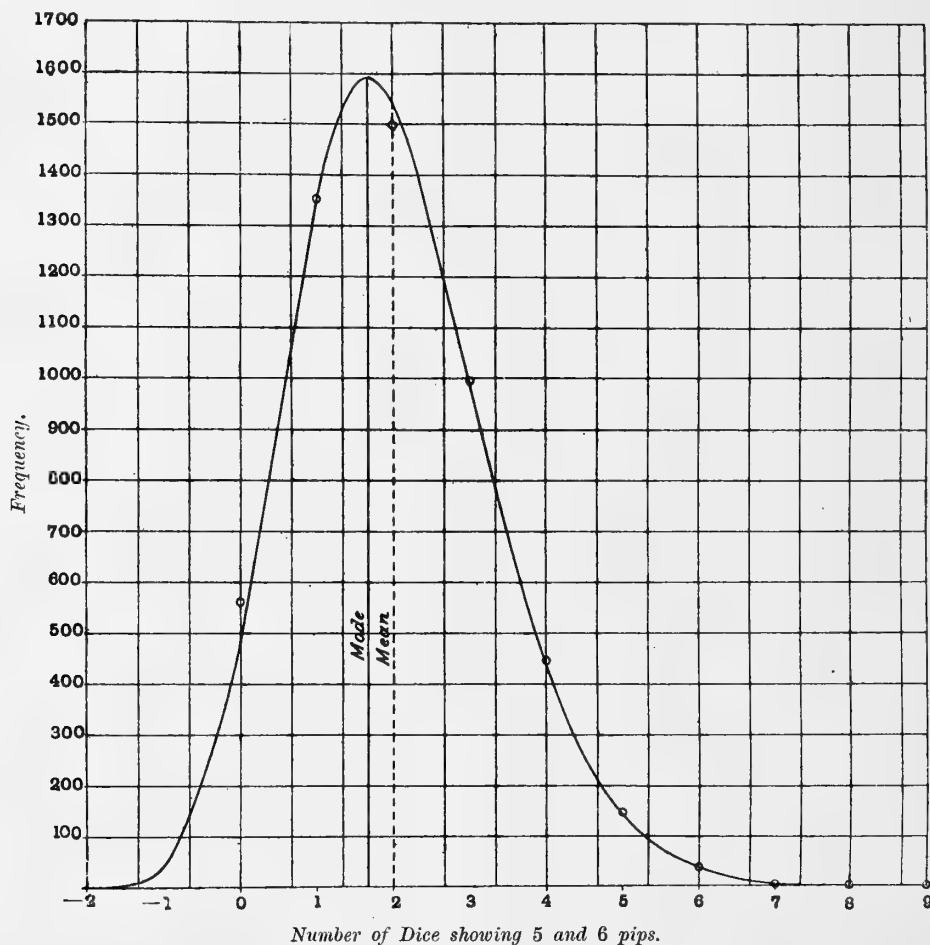
Fig. 4. 5000 Throws with 12 Dice.

○   ○   Points of the Binomial $(\frac{1}{6} + \frac{5}{6})^{12} \times 5000$.

————   Curve $y = 1536 \cdot 54 \left(1 + \dfrac{x}{5}\right)^{14} e^{-3x}$.

Origin at the mean 2. Mode of curve at $x = -\frac{1}{3}$.

Frequency of each number of 5 and 6 pips	−1	0	1	2	3	4	5	6	7	8	9
Ordinates of Binomial		0	561	1346	1480	987	444	142	33	6	1	0
Areas of Curve		557		1340	1493	982	443	139	37	8	1	0

probability integral, when he is discussing the probable errors of death rates, class indices and a multitude of other problems, and this when the binomial is skew and N relatively small! It is the whole theory of current statistics which Ranke and Greiner are tilting at when they object to the use of what is equivalent to the Euler-Maclaurin theorem, i.e. the mathematical representation of a finite sum of terms by a definite integral*.

There is another point also which may be noted here before we leave the binomial. The quantity β_1 is a measure of the asymmetry. Now consider the ratio β_1/η, or the ratio of this measure of asymmetry to the kurtosis. For the asymmetrical binomial we have $\beta_1/\eta = (1 - 4pq)/(1 - 6pq)$. Since p and q if positive must give a product lying between 0 and $\frac{1}{4}$, this ratio cannot take any value between 0 and $+1$. Hence any curve which gives for β_1/η a value less than unity cannot possibly diverge from the normal curve in the direction of a binomial series. We shall see the application of this later.

C. (ii) *The Galton-McAlister Curve.*

I have already referred to the attempt of Poisson to give the skew binomial by an extra term applied to the Gauss-Laplacian probability integral. Quetelet endeavoured to meet the asymmetry of frequency distributions by placing graphically skew binomials on top of the frequency polygon—a very rough and somewhat deceptive process. The next step in the advance was taken by Francis Galton, who in 1879 suggested that the geometrical mean and not the arithmetical mean is likely to give the most probable result in many vital phenomena[†]. Galton refers to Gauss's assumption that errors in excess or in defect are equally probable, and says "this assumption cannot be justified in vital phenomena." He cites especially the cases of errors in human judgment, guessing at temperatures, tints, pitch, etc. He appeals to Fechner's law in its simplest form as evidence to the contrary, and placing the matter in the hands of D. McAlister[‡], the law of frequency

$$y = y_0 \frac{h}{\sqrt{\pi} x} e^{-h^2 \left(\log \frac{x}{a}\right)^2}$$

was deduced, and methods for fitting this curve were discussed. The curve is well-known in England and also on the continent[§]. It is therefore curious to find Ranke and Greiner attributing this curve to Fechner's work which was not published till 18 years later. It was not till I had made a fairly complete set of experimental determinations of the kind supposed to give this curve, that I finally discarded it. Thus I asked audiences of 100 to 300 persons to match tints in several ways, I asked them to guess heights, to determine mid-lengths, to state which figures in randomly distributed series were most closely circles,

* See Lacroix : *Traité du Calcul différentiel et intégral*, Tom. iii. p. 136.
† *R. S. Proc.* Vol. 29, p. 365 *et seq.* "The Geometric Mean in Vital and Social Statistics."
‡ *R. S. Proc.* Vol. 29, p. 367 *et seq.* "The Law of the Geometric Mean."
§ It is cited by Kapteyn, for example.

squares, equilateral triangles, etc. etc. In all these results I found the distribution asymmetrical, but the most probable value was *not* the geometrical mean, nor the distribution the Galton-McAlister curve. One of the striking defects of the curve was its high contact at both ends. The distributions clearly often corresponded to curves in which the contour cut the axis at a finite angle. Another point was that the skewness was *in the opposite direction* to that presupposed by the reasoning from which the curve is deduced. It was precisely this experience which showed me that putting $x = \beta \log (\xi/a)$ in the Gaussian curve is not a sufficient generalisation.

Ranke and Greiner, to say nothing of Fechner himself, are remarkably vague as to the accurate determination of the position and constants of the Galton-McAlister curve. McAlister gives no clear description of how the curve is to be placed if neither the mode nor start of the range is known. I think it desirable therefore, having regard to the inferences I wish to draw, to give the fitting by my method of moments. I write the curve:

$$y = \frac{y_0}{x} e^{-\frac{1}{2c^2}\left(\log \frac{x}{a}\right)^2} \quad\dots\dots\dots\dots\dots\dots\dots\dots\text{(xi)}.$$

Differentiation shows us at once that the distance x_{mo} of the mode from the origin is given by:

$$x_{mo} = ae^{-c^2} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xii)}.$$

Integrating the expression $N\mu_n' = \int_0^\infty yx^n dx$ we find if N = total frequency:

$$y_0 = N/(\sqrt{2\pi}c) \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xiii)},$$

and generally:

$$\mu_n' = a^n e^{\frac{n^2}{2}c^2} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xiv)}.$$

Thus the distance from the origin to the mean, x_{me}, is given by

$$x_{me} = ae^{\frac{1}{2}c^2} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xv)}.$$

Now write $e^{c^2} = \lambda$ and we have if μ_n be a moment coefficient about mean:

$$\begin{aligned}
\mu_2 &= \mu_2' - \mu_1'^2 = a^2\lambda\,(\lambda - 1) \\
\mu_3 &= \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = a^3\sqrt{\lambda}\,(\lambda^4 - 3\lambda^2 + 2\lambda) \\
\mu_4 &= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = a^4\lambda^2\,(\lambda^6 - 4\lambda^3 + 6\lambda - 3)
\end{aligned} \right\} \quad\dots\text{(xvi)}.$$

Forming the usual constants of frequency we have:

$$\beta_1 = \mu_3^2/\mu_2^3 = (\lambda - 1)\,(\lambda + 2)^2 \quad\dots\dots\dots\dots\dots\text{(xvii)},$$

$$\eta = \beta_2 - 3 = (\lambda - 1)\,(\lambda^3 + 3\lambda^2 + 6\lambda + 6) \quad\dots\dots\dots\dots\text{(xviii)},$$

where $\beta_2 = \mu_4/\mu_2^2$;

$$d = x_{me} - x_{mo} = a\,(\sqrt{\lambda} - \lambda^{-1}) \quad\dots\dots\dots\dots\dots\text{(xix)},$$

$$\chi = \frac{1 - \lambda^{-\frac{3}{2}}}{\sqrt{\lambda - 1}} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xx)}.$$

We see then that the mean and second and third moments must be found to determine this curve. From the second and third moments we have β_1, whence by equation (xvii) λ is determined. The first equation of (xvi) then gives

$$a = \sigma/\sqrt{\lambda\,(\lambda - 1)}.$$

Then equation (xv) gives the distance of the start of the curve from the known mean. Further since $\lambda = e^{c^2}$, we determine c and finally y_0 is determined by equation (xiii). There is no obscurity or difficulty with the fitting, if we use the method of moments. The cubic equation (xvii) is solvable at once either by Lill's, Mehmke's or Reuschle's mechanisms.

But what are the objections?

(i) The curve touches the axis at the end of the range. Skew curves extremely often cut it at a finite angle.

(ii) The skewness has a definite direction, which to be logically consistent we ought not to neglect, i.e. since λ is always > 1, d remains always of one sign.

(iii) Since $\lambda > 1$, η, the kurtosis, is always positive and the curve can only represent platykurtic distributions. It can never give a curve which deviates from the Gaussian curve in the direction of the Laplace-Poisson skew binomial for $p > \cdot2113 < \cdot7887$, because this is essentially leptokurtic.

(iv) The range of skewness given by χ is very limited. Differentiating χ we find it is a maximum for $\lambda = 1\cdot7200$ and this gives $\chi = \cdot2075$. The Galton-McAlister curve cannot therefore describe any curve whose skewness does not lie between 0 and $\cdot2$. A cursory examination of the observational results reached, shows that the skewness in all kinds of data over and over again exceeds $\cdot2$.

(v) β_1 and η are both functions of λ only*. Hence there is a relation between them or between η and χ. That is to say the kurtosis is determined by the skewness. The kurtosis must vanish with the skewness. But experience shows that many distributions are sensibly symmetrical and yet have far from zero kurtosis, e.g. nasal breadth in English women, etc. etc.

Finally consider the ratio β_1/η. If we approach the normal curve as the limit to a point binomial $(p + q)^N$ we have seen that

$$\beta_1/\eta = (1 - 4pq)/(1 - 6pq) \quad\quad\dots\dots\dots\dots\dots\dots(xxi),$$

and this equals nothing if we take the symmetrical binomial. Otherwise it has a finite value depending upon the particular binomial along which we reach the Gaussian curve. The Galton-McAlister curve, if we make $a\sqrt{\lambda}$ infinite, but $a\sqrt{\lambda}\sqrt{\lambda - 1}$ finite, approaches the Gaussian curve.

* Actually it is

$$\beta_1^4 - 12\beta_1^3 + 156\beta_1^2 + 64\beta_1 - \eta^3 + 12\eta^2 - 36\eta + 18\beta_1^2\eta - 6\beta_1\eta^2 - 117\beta_1\eta = 0.$$

Now for the Galton-McAlister curve

$$\beta_1/\eta = \frac{(\lambda+2)^2}{\lambda^3 + 3\lambda^2 + 6\lambda + 6},$$

and this approaches the limit 9/16, its maximum value, when λ approaches unity. If we take $(1 - 4pq)/(1 - 6pq) = 9/16$, we get imaginary values for p and q. Thus while the normal curve itself gives an indeterminate value for $\beta_1/\eta = 0/0$, and as Laplace has shown describes with fair accuracy any slightly skew binomial with large power, the Galton-McAlister curve cannot describe even approximately any skew binomial, however near to a normal distribution.

On all these grounds we see that the "law of the geometric mean" fails to supply the fundamental need of describing the modal difference, the kurtosis and the skewness of actual frequency distributions. It cannot describe these physical characteristics of the frequency.

C. (iii) *Fechner's Double Gaussian Curve**.

We have noted that the Gaussian curve was first deduced by Laplace to represent a finite number of the terms of a binomial expression, and that Gauss deduced it on hypotheses which amount to the following:

(i) The arithmetic mean is the most probable value.

(ii) Deviations in excess and defect of the mean are equally probable if of the same magnitude.

(iii) The facility of an increment is the same for all values of the character.

Now every one of these assumptions is negatived when the double Gaussian curve is used, and yet the Gaussian curve *which is only deduced by aid of them* is adopted to describe what conflicts with its fundamental axioms. This proceeding is the reverse of logical. However, if the double Gaussian curve be adopted, there is absolutely no reason why we should adopt the rough process by which Fechner determines the mode and obtains the constants of the distribution. The fitting by my method of moments is perfectly straightforward, and as it leads to the points we have to consider it will be indicated here. Let the two half curves be:

$$\left. \begin{aligned} y_1 &= \frac{n_1}{\sqrt{2\pi}\,\sigma_1}\, e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}}, \quad x > 0 \\[2mm] y_2 &= \frac{n_2}{\sqrt{2\pi}\,\sigma_2}\, e^{-\frac{1}{2}\frac{x^2}{\sigma_2^2}}, \quad x < 0 \end{aligned} \right\} \quad \dots\dots\dots\dots\dots\text{(xxii)}.$$

Then, since the modal value is common, $\sigma_1/n_1 = \sigma_2/n_2$. Further, the total frequency $N = \frac{1}{2}(n_1 + n_2)$. Now write $\kappa = \sqrt{2/\pi}$ and $u = \sigma_1 - \sigma_2$, $v = \sigma_1\sigma_2$. Then taking moments round the mode we easily find:

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* Here again it is historically incorrect to attribute these curves to Fechner. They had been proposed by De Vries in 1894, and termed "half-Galton curves," and Galton was certainly using them in 1897. See the discussion in Yule's memoir, *R. Statist. Soc. Jour.* Vol. LX. p. 45 *et seq.*

$$\mu_1' = \kappa u, \quad \mu_2' = u^2 + v, \quad \mu_3' = 2\kappa u (u^2 + 2v), \quad \mu_4' = 3\{u^2(u^2 + 3v) + v^2\}.$$

Transferring to the mean we deduce:

$$\mu_2 = u^2 + v - \kappa^2 u^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xxiii)},$$

$$\mu_3 = \kappa u (v - u^2 + 2\kappa^2 u^2) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xxiv)},$$

$$\mu_4 = 3\{u^2(u^2 + 3v) + v^2\} - \kappa^2 u^2\{2u^2 + 10v + 3\kappa^2 u^2\} \quad \dots\dots\text{(xxv)}.$$

μ_2, μ_3, μ_4 will be known quantities as soon as the frequency distribution is known.

Now determine $\beta_1 = \mu_3^2/\mu_2^2$ and write $\lambda' = u/\sqrt{\mu_2}$, we easily find:

$$\left(\frac{6}{\pi} - 2\right)\lambda'^3 + \lambda' - \sqrt{\frac{\pi}{2}}\,\beta_1 = 0 \quad \dots\dots\dots\dots\dots\text{(xxvi)}.$$

This cubic* gives by its real root the value of $u = \sigma_1 - \sigma_2$. We then easily deduce

$$\left.\begin{array}{l} \sigma_1 = \tfrac{1}{2}\sqrt{\mu_2}\left(\{4 + (4\kappa^2 - 3)\lambda'^2\}^{\frac{1}{2}} + \lambda'\right) \\ \sigma_2 = \tfrac{1}{2}\sqrt{\mu_2}\left(\{4 + (4\kappa^2 - 3)\lambda'^2\}^{\frac{1}{2}} - \lambda'\right) \end{array}\right\} \quad \dots\dots\dots\text{(xxvii)}.$$

These determine the different variabilities of the two halves. Then

$$n_1 = \frac{2N\sigma_1}{\sigma_1 + \sigma_2}, \qquad n_2 = \frac{2N\sigma_2}{\sigma_1 + \sigma_2} \quad \dots\dots\dots\dots\dots\text{(xxviii)}$$

give the frequencies in each Gaussian curve, while

$$\mu_1' = \kappa\sqrt{\mu_2}\,\lambda' \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xxix)}$$

fixes the position of the origin relative to the known mean value of the system. Thus the complete solution depends on a knowledge of the mean, and the second and third moment coefficients. As before the cubic is readily solved by Lill, Reuschle or Mehmke's mechanisms.

The analysis is now a little more complex than in the case of the Galton-McAlister curve. Write $\epsilon = v/u^2 = \sigma_1\sigma_2/(\sigma_1 - \sigma_2)^2$. Then we have:

$$\left.\begin{array}{l} \beta_1 = \kappa^2(\epsilon - 1 + 2\kappa^2)^2/(1 - \kappa^2 + \epsilon)^3 \\ \beta_2 = \{3(1 + 3\epsilon + \epsilon^2) - \kappa^2(2 + 3\kappa^2 + 10\epsilon)\}/(1 - \kappa^2 + \epsilon)^3 \end{array}\right\} \quad \dots\dots\text{(xxx)}.$$

Thus again we see that β_1 and β_2 are both functions of ϵ only, or the skewness is not independent of the kurtosis†. Whenever the skewness is zero, the kurtosis must also be zero or the curve be normal.

Now consider the expression $1 - \kappa^2 + \epsilon$ which we will write γ, or,

$$\gamma = \cdot36338 + \sigma_1\sigma_2/(\sigma_1 - \sigma_2)^2.$$

The last term is positive or γ must be $> \cdot36338$.

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* This cubic was, I believe, first given by Edgeworth.

† The actual relation is:

$$29521\eta^2 + 62500\beta_1^2 - 110506\beta_1\eta + 13468\eta^3 - 11345\beta_1 + 15925\eta = 0,$$

which, as in the Galton-McAlister case, has no obvious physical significance.

Now :

$$\eta = \beta_2 - 3 = \frac{\cdot 45352}{\gamma} - \frac{\cdot 05003}{\gamma^2},$$

and this with the above limitation to the value of γ can never become negative. Hence the double Gaussian curve is, like the Galton-McAlister curve, invariably platykurtic. Now consider the value of

$$\beta_1 = \frac{\cdot 63662}{\gamma} - \frac{\cdot 11477}{\gamma^2} + \frac{\cdot 00518}{\gamma^3}.$$

Hence the ratio $\dfrac{\beta_1}{\eta}$ tends as γ increases to take the value $1\cdot40374$. Equating this to $(1 - 4pq)/(1 - 6pq)$ we see that the double Gaussian curve approaches the normal curve along the particular platykurtic binomial $p = \cdot 8985$, $q = \cdot 1015$, or it cannot in the neighbourhood of the normal curve represent any skew binomial but this.

Lastly it may be shown that β_1 has its maximum value when $\gamma = \cdot 36338$ or its minimum value. Thus we find that the maximum possible value of β_1 is about $\cdot 99$. In the same way the maximum skewness is $1\cdot3236$. These values are sufficiently high to cover the great bulk of cases, but I have found $\beta_1 = 4\cdot071$ for scarlet fever incidence, $= 1\cdot9396$ for age of brides who marry men in their 24th year and $= 4\cdot1683$ for the distribution of lips in the medusa *P. pentata*. These exceptions suffice to show that the curve is not general enough.

Summing up we conclude that the double Gaussian curve is not satisfactory because theoretically

(i) It starts by denying the very axioms from which alone we can reach the Gaussian curve ;

and empirically because

(ii) It can describe no frequency distribution which cuts the axis at a finite angle, and such distributions constantly occur.

(iii) It is essentially platykurtic. Therefore it is not available for leptokurtic curves, nor even for any but very special skew binomials, i.e. those in which p does not lie between $\cdot 2113$ and $\cdot 7887$. As we approach close to the normal curve we get nearer and nearer to one definite point binomial, i.e. that in which $p = \cdot 8985$.

(iv) There is always a relation between the skewness and the kurtosis, or these important physical constants are not independent. In particular we cannot have any form of symmetry but the mesokurtic.

(v) The range of β_1 and of the skewness is fairly large, but frequency distributions actually occur markedly outside this range.

(vi) Lastly, and of much import, the kurtosis can never exceed $\cdot 8692$, or the maximum value of $\beta_2 = 3\cdot8692$. This degree of kurtosis is exceeded in a great number of distributions. Thus in the lips of *P. pentata*, in tint guessing, in the breadth of male English skulls, in the nasal breadth of female English skulls, in

no less than eight of Duncker's series in the case of *Gelasimus pugilator* and in various other distributions. In all these cases the platykurtosis is significant, and the double Gaussian curve fails us hopelessly.

C. (iv) *The Edgeworth-Kapteyn Curves.*

Kapteyn[*], without recognizing Edgeworth's priority, has proceeded in the manner indicated on p. 178 above. He assumes that some quantity x obeys the normal distribution

$$y = \frac{N}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}x^2/\sigma^2}.$$

He then takes $x = F(X) - M$ and reaches the frequency distribution:

$$Y = \frac{N}{\sqrt{2\pi}\sigma} F'(X) e^{-\frac{1}{2}\{F(X)-M\}^2/\sigma^2} \dots\dots\dots\dots\dots\dots\dots(xxxi).$$

Thus far (as we have already shown) nothing has been achieved, because this equation may by a proper choice of $F(X)$ represent any curve whatever. As Kapteyn himself says, following Edgeworth, "as $F(x)$ may represent any function, we see that the equation may be made to represent any curve whatever. Therefore it must be the most general form of frequency curve possible" (p. 17). There is, however, one point to be raised here. What is x of which the observed character X is a function? Is it, as in the explanatory illustrations cited by Kapteyn, another characteristic of the organism? If so we ought in some cases to be able to determine it. What is the character which obeys the normal law? For example, sagittal arc in English women is almost exactly normal in its distribution, and nasal breadth is very asymmetrical. Shall we take $x =$ sagittal arc and $X =$ nasal breadth and make

$$x = F(X) - M?$$

Now every biologist knows that such a relation is not in the least true. No two characters in an organism are in any way connected by a mathematical function, such that when one is given the other is determined. The relation is always of the loose kind that we term association or correlation. X does not fix x, and a multitude of x's with varying degree of probability are associated with a given x. This correlation is often of a very low order. Between any two characters of a given organism, no such relation of perfect correlation as that involved in Edgeworth or Kapteyn's relation has ever been discovered. Very imperfect correlation or at any rate all degrees of correlation have been invariably demonstrated to exist. The function x has no real existence as a biological entity. It is only a mechanism for introducing the normal curve, and is not a true character of the organism at all. Supposing, as in English female crania, nasal breadth is asymmetrical, what is the quantity which is symmetrically distributed of which nasal breadth is a function? It has no reality in the organism at all, and Kapteyn proceeds to make it still more impossible in the following manner. If x has

[*] *Skew Frequency Curves in Biology and Statistics*, Groningen, 1903.

existence at all, its limits lie between $-\infty$ and $+\infty$, i.e. the whole range of the normal curve. But in order to get a range limited at one end, not the whole series of values of X corresponding to x are taken. A value is selected for X, such that X becomes impossible after a certain value of x. In other words, x is a character which although following the normal curve is abruptly terminated as far as X is concerned at a value with a finite frequency! Kapteyn takes[*]

$$x = (X + \kappa)^q - M,$$

where κ, q and M are to be determined from the data.

For example, in Professor Weldon's data for the measurement of foreheads of *Carcinus moenas*, Kapteyn (p. 39) finds that with our notation

$$\sigma = \quad \cdot002,204,$$
$$M = \quad \cdot002,561,$$
$$\kappa = - 0\cdot5781,$$
$$q = \quad 2\cdot21.$$

Thus when $X = -\kappa$, we have $x = -M = -\sigma$ roughly, or the Gaussian frequency curve for x is to be abruptly cut off, and about 15 per cent. of its tail discarded. If it be said that this could be achieved by natural selection of foreheads, the reply is the simple question: Please show what physical character in a crab is given by an abruptly truncated normal curve! The fact is no such character has ever been met with, and it must be recognised that x represents a wholly fictitious variable having no physiological relation to the character X at all, but introduced solely to reduce the frequency by hook or by crook to that fetish distribution the Gaussian curve.

We can now sum up the objections to Kapteyn's method.

Theoretical :

(i) There is no justification whatever for assuming that some character x actually exists which obeys the normal law of distribution, and that the observed character is a function of this. Some characters are found as a rule in any organism which obey the normal law, but no two characters in an organism have ever been found to be the one a mathematical function of a second, they are always imperfectly correlated.

(ii) Kapteyn's hypothesis involves if his normal character were a physiological entity, that distributions of organic characters should occur which would be represented by fragments of Gaussian curves, or such curves abruptly curtailed. We have no experience of such distributions in actual vital statistics. If they did exist they would contradict the first two axioms on which the Gaussian law itself is based, and would thus deprive that law of the sole justification for its application. As it cannot be supposed that all skewly distributed characters X in an organism are functions of one and the same x, for in this case they would be

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[*] This becomes the Galton-McAlister curve for the limit $q = 0$.

perfectly correlated with each other, which is contrary to experience, it must follow that if Kapteyn's hypothesis were correct large quantities of characters distributed in truncated Gaussian curves ought to appear when we deal with variation. The total absence of such characters is evidence that the x-characters are shadow variables and of no biological import.

(iii) The previous statements reduce Kapteyn's special choice of

$$F(X) = (X + \kappa)^q$$

to a mere artifice adopted to get an empirical curve of variation by Edgeworth's hypothesis. Many other functions are *à priori* equally valuable, and might be adopted to get curves of limited range, e.g.

$$F(X) = (X + \kappa)^q (X - \kappa')^{q'}.$$

The hypothesis gains nothing in logical consistency by its appeal to the Gaussian curve; that appeal is one adopted for convenience of fitting, and the sole test of Kapteyn's curve is empirical goodness of fit.

Practical:

(iv) *Every frequency curve should be a graduation formula.* Kapteyn's method of fitting is by equating certain *total* frequencies in order to determine his four constants. They thus fail to successfully smooth any special causes tending to exaggerate any particular frequency group. Such screening of special causes of frequency deviation is far less likely to occur when we use the method of moments, which is a true method of graduation*.

(v) *We ought in every law of frequency distribution to be able to judge of the effect of the unit of grouping on the values of the constants.* This has been satisfactorily achieved for, perhaps, the bulk of cases, when the method of moments† is used by Sheppard's corrections†. In Kapteyn's process we have no means of ascertaining the extent to which the size of the unit of grouping influences the constants of his distribution.

In his Example II., for instance, he takes his curve to accurately reproduce the total area of the group of houses under £10 annual value. What difference would

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* Thus Kapteyn deals with some statistics of the values of house property in England fitted by me (*Phil. Trans.* Vol. 186 A, p. 396). I specially state that £20 was the limit to taxable value, and that accordingly the frequency of houses immediately below this value will be exaggerated. Kapteyn's method fails to indicate such a source of *à priori* recognised irregularity. For example, one of his conditions is that the houses of value less than £10, i.e. more than half the total frequency, shall be *identical* in his result with the observed frequency. He thus cuts away at once any possibility of smoothing this group or allowing for the large probable error in it due to random sampling even. His method leads to a limiting house value of £2. 2s., while mine leads to £4. 4s. Mine corresponds to a weekly rent of about 2s.; his to a weekly rent of 1s. The latter rent hardly occurs in England unless the house is given in part payment of wages, or in charity. Kapteyn says that his distribution starts with a zero frequency, and mine with an infinite ordinate. " It seems hardly admissible that the latter solution can be in accordance with nature (*sic*) in this particular." Why not? An infinite ordinate may and does in my case give a finite frequency.

† See reference, p. 187.

be made if the first group had included only houses under £5 ? We are unable to answer this question.

(vi) *Every frequency curve should be determined by constants of which the probable errors are easily deducible.* The method of moments admits of the probable errors of the moments being easily determined (see *Biometrika*, Vol. II. pp. 273 *et seq.*). My system of skew curves gives all the constants in terms of moments whose probable errors are known.

The moments in Kapteyn's theory depend on the integration of :

$$\int_{-M}^{\infty} (x + M)^{\frac{n}{q}} e^{-\frac{1}{2}x^2/\sigma^2} \, dx,$$

and there is no means of readily evaluating this integral. *In fact the arithmetical mean*, the standard deviation, the skewness and the kurtosis, and the modal divergence are unobtainable from the constants of Kapteyn's theory.* This seems to me sufficient to deprive the method of any practical significance even as an empirical representation.

It has further been shown by Sheppard that the probable errors of constants determined by class frequencies (partial areas) are higher than when these constants are determined by the method of moments. We may give the above statement a separate paragraph as :

(vii) The fundamental physical constants of the frequency distribution are not determinable from Kapteyn's empirical curve.

To illustrate the results of this want of a knowledge of the probable errors, I turn to the three illustrations given by Kapteyn.

Example (i). Observations on the *Threshold of Sensation*. Kapteyn himself shows that his solution is hardly less satisfactory if he uses the Galton-McAlister curve (our equation (xi) p. 194). He does not therefore know whether $q = \cdot 00$ and $q = - \cdot 04$ differ within the probable error of q.

Example (ii). *Valuation of House Property.* Kapteyn fits this with a Galton-McAlister curve for his q comes out $\cdot 00$. Owing to the difficulty in calculating moments, we cannot do more than approximate to the value of χ the skewness in these data. I make it $1 \cdot 8$. It is certainly well over unity. We have already seen that it is impossible for a Galton-McAlister curve to give a skewness above $\cdot 21$. The apparent agreement Kapteyn finds for the frequencies is not therefore sufficient evidence that the fundamental constants of the distribution will be really given by reasonable values.

Example (iii). Foreheads of *Carcinus moenas*. Kapteyn fits these first with $q = 2 \cdot 21$. "The agreement seems satisfactory." Then with $q = 0$, or a Galton-McAlister curve, "The representation is hardly less satisfactory." Then with

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* "The arithmetic mean of all the X's cannot be *generally* found in a simple and rigorous way," Kapteyn, p. 44.

$q = \infty$, "The representation, though sensibly less satisfactory than that by the [previous] solutions, is still pretty close."

A method by which a fundamental constant of the distribution may take any value between 0 and ∞ and still give a "pretty close" representation, must I think condemn itself. Such a statement demonstrates effectively that the author has not yet determined numerically or even approximately in his own mind the probable errors of the constants he uses.

It will be seen that as far as the three illustrations Kapteyn himself gives go he has not advanced the matter beyond the Galton-McAlister curve. That curve fits reasonably (according to Kapteyn) all his three series. But the skewnesses of the three series are respectively ·72, 1·8 and ·32. I have calculated these roughly, but I think there can be no doubt of their approximate correctness. In every one of these cases the skewness sensibly exceeds the maximum limit of skewness, i.e. ·21, possible for the Galton-McAlister curve which Kapteyn applies to them *.

C. (v) *The General Results which flow without the Third Gaussian Axiom.*

It seems to me accordingly that very grave objections can be raised not only from the theoretical but from the practical standpoint to the methods I have discussed which attempt to allow for asymmetry, i.e.

   (i)   The Galton-McAlister Geometrical Mean Law

   (ii) The Galton-Fechner use of Half Gaussian Curves,

   (iii) The Edgeworth-Kapteyn use of transformed Gaussian Curves.

All these experienced statisticians differ *in toto* from the opinion of Ranke and Greiner—that we need not trouble about descriptive curves for asymmetrical distributions—but their methods seem to me unsatisfactory theoretically and insufficient practically, because they still make a fetish of the Gaussian axioms. They do not return to the Laplace-Poisson method of replacing those fundamental axioms by more general conceptions. If a Gaussian curve does not fit, they will consent to deduce their own curves from a truncated Gaussian curve, which some shadow variable of the mathematician is supposed to follow, and of which we have no experience in any organic characters hitherto measured. Indeed if we had such experience, it would at once negative the very axioms on which the Gaussian curve is based.

Now it seems to me that all these attempts, whether embodied in the general method of Edgeworth or in the special hypotheses of Galton-McAlister or Kapteyn, amount to abolishing the third of the Gaussian assumptions, namely that small increments of the variable or the character are independent of the total already reached. That is to say that they amount to saying that increments of the

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* I am unable to say how far the general form of Kapteyn allows for the requisite range of skewness and kurtosis, because neither the modal difference, nor the standard deviation, to say nothing of the higher moments, can in general be evaluated.

variate are *correlated* with the value of the variate already reached[*]. Galton and Fechner made the increment proportional to the variate. But in our ignorance of the actual nature of variation in organisms, we have no reason at all for making such a narrow assumption. We can to please our critics put the matter as I have already indicated in the Gaussian form. We simply assume that if the causes of variation in the immediate neighbourhood of the character x_0 remained the same for the whole range they would give a normal curve, hence we should have a relation of the form:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{\sigma^2}.$$

They do not, however, remain the same; the tendency to vary at x is a function of x, in other words $\sigma = \sigma_0 \sqrt{F(x)}$, where $F(x)$ is an arbitrary function. We have then

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{\sigma_0^2 F(x)}.$$

This is a result as general as Edgeworth's and more so than Kapteyn's or Fechner's. We now take the simplest possible functional series for $F(x)$, i.e.

$$= a_0 + a_1 x + a_2 x^2 + \dots.$$

The coefficients a_0, a_1, $a_2 \dots a_n$ can be found at once in terms of the moments[†], and my special curves result if we stop at a_2. Against going to higher powers are the objections I have raised in my memoir on skew correlation[‡], namely (i) that the higher powers involve moments of the 5th and higher orders and their probable errors are very large, (ii) that it has not yet been shown that going to a_2 does not suffice to describe all the types of frequency which occur in common practice.

The above is the simplest and most general form into which I would put my theory of asymmetrical frequency for those who feel compelled to approach all frequency from the Gaussian standpoint.

D. *Specific Criticisms of Ranke and Greiner on my Theory.*

I think these may be summed up as follows:

(*a*) That all distributions of variates are continuous, and that accordingly no curves, however closely they may approximate to finite discontinuous series like the binomial and the hypergeometrical series, can be applicable to variation in nature.

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[*] Suppose we draw r cards from a pack, and wish to consider the chance of s being of one suit, we may do so by drawing one card at a time, observing it and returning it, and then drawing again. Here there is not correlation between the successive contributions to r. Or we may draw the r cards, without replacing the individual; here the successive contributions are correlated with the previous contributions, and the third Gaussian principle is upset.

[†] "Mathematical Contributions to the Theory of Evolution, XIV. On the General Theory of Skew Correlation and Non-Linear Regression," *Drapers' Company Research Memoirs.* Biometric Series, II. (Dulau and Co. 1904) p. 6.

[‡] *Ibid.* p. 7.

(*b*) That what I term the "number of cause-groups" must be infinite in number, for without such infinity it is impossible to reach continuity.

(*c*) That looked at from the standpoint of binomial or hypergeometrical series the constants of some one or more of my curves may become unintelligible.

Now not one of these objections has any application to the method which has been used in this paper to deduce the differential equation to my curves. But I still think there are very grave objections to every one of the above statements.

To begin with (*a*). We meet in an immense variety of living forms with discrete variates. For example, the number of teeth on the rostrum of a prawn, the number of lips of a medusa, the number of veins in a leaf, the number of glands in a swine's foot, the number of tentaculocysts in *Ephyra*, the number of individuals in a litter, the number of bands on snails' shells, the number of somites in the body of an earthworm, the number of petals or sepals in a flower, etc. etc. Are we to put all these distributions of variation on one side because Ranke and Greiner hold that all distributions of variates are continuous? We have in these cases probably continuous causes producing discontinuous distributions. Are we not to use the areas of a continuous curve to give the frequency of such discrete variates?

Consider for example the function given by:

$$y = N \times \overset{n}{\underset{0}{S}} \left\{ p^{n-r} q^r \; \frac{\lfloor n}{\lfloor n-r \; \lfloor r} \; \frac{1}{\sqrt{2\pi}\,\sigma_r} \; e^{-\frac{1}{2}(x-rc)^2/\sigma_r^2} \right\}.$$

This is compounded of $\overline{n+1}$ normal curves, the area of the $(r+1)$th normal curve being $Np^{n-r}q^r \dfrac{\lfloor n}{\lfloor n-r \; \lfloor r}$, i.e. the $(r+1)$th term of the binomial $N(p+q)^n$, and this $(r+1)$th normal curve has σ_r for its standard deviation. The origin of the system is at the mode of the normal curve corresponding to $r=0$, and the means of these normal curves are spaced equal distances c apart.

When every $\sigma_r = 0$ we have discrete variation. When σ_r is small, less say than $\frac{1}{8}c$, it would probably be difficult to distinguish the result from discrete variation. Enlarging σ_r we pass on till we get a system which it would be practically impossible to distinguish from continuous variation, even if n were only moderate in magnitude. I lay no stress whatever on the above expression because I am in no sense pledged to any Gaussian curve, but it illustrates well what I want to express: namely, in actual nature the frequency might fundamentally fall on certain values of the character, but that the effect of nurture, environment, and growth may well scatter the values of the variable round the fundamental value, so that continuity of variation is all that can be actually observed. The number of somites in an annulose animal is discrete and probably inherited, but the length of the body may appear as a continuous variate. I do not think for a moment that the distinction made by Ranke between discrete and continuous variation,

and the further statement that variation in man is essentially continuous, is at all valid. Our units of grouping for the numbers available are not very fine, we can hardly successfully classify a few hundred observations into more than 20 groups, and with this unit of grouping it would be practically impossible to distinguish between the apparently continuous distribution and a discrete distribution of a similar number of classes with the variates modified by growth, nurture or any other scattering tendencies. Ranke appears entirely to have overlooked the current biological theory of inheritance summed up in the words *inheritance by determinants*. Such theories, whether they be those of Weismann or Mendel, lead us directly to discrete variation*. The discreteness of the variation will be more or less, in many cases probably entirely, obscured by the environmental influence. In such cases the number of fundamental cause-groups is not infinitely great, and Ranke is overlooking current biological views when he asserts that we *must* take "die Anzahl der Elementarursachen selbst als unendlich gross und die Grösse der Wirkung der einzelnen Ursache als unendlich klein." If the number of determinants which fix a character is finite, that character would correspond to a discrete variation of limited range. If the number of determinants be very large, the distribution would by Laplace's theorem be represented more and more closely by the normal curve.

I toss ten coins into the air and for every head in the result I pay a gramme of gold-dust, the frequency distribution of gold-dust would closely be given by the terms of the binomial $(\frac{1}{2} + \frac{1}{2})^{10}$ as in the points of our Fig. 1. But suppose instead of weighing my gramme of gold-dust accurately, I give a "handful" of sugar. If 6 heads turn up I give six handfuls of sugar, but each of these will not be exactly my standard mean handful. I am unlikely to give either five or seven standard handfuls as my six approximate handfuls, but in some cases even these might be possible; we pass in fact from discrete to continuous variation, and the multimodal character of the discrete variates will disappear with the roughness of the handfuls, or have the peaky appearance of random sampling. The total area up to any midpoint between two discrete groups s and $s+1$ will be given by the continuous integral which represents the first s terms of the binomial. If we have two such total areas, one up to the midpoint between groups s and $s+1$ and the other up to the midpoint between groups $s+1$ and $s+2$, then an interpolated area between these values as given by the continuous integral will be sensibly the same as if, c being the unit of discrete difference, we determined a curve corresponding to the mean binomial frequency in the *Spielraum c,* i.e.

$$\frac{N}{c} p^{n-r} q^r \frac{\lfloor n}{\lfloor n-r \lfloor r},$$

by simply fractionising r, i.e. we replaced the factorials by Stirling's theorem or used Γ functions, and supposed r to change continuously from s to $s+1$. This is

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* Thus I have shown that a generalised Mendelian theory leads directly to skew binomial distributions of characters in the general population. *Phil. Trans.* Vol. 203 A, pp. 53—86.

merely another way of looking at the change from discrete to continuous variation, due to the influence of a multitude of causes on the discreteness of the variates which fall into a given *Spielraum*. I still find nothing absurd in the statement that the actual effect of the scatter is sensibly equivalent to a fractionising of the indices. It is simply equivalent to the statements, (i) that the ordinates of the Gaussian curve closely give, even for small values of n, the terms of the binomial, (ii) that the ordinate of the Gaussian curve between two terms of the binomial closely gives a fractionised binomial term (owing to Stirling's theorem being true for fractionised factorials or Γ functions), (iii) that we have no knowledge of how the "scatter" within the *Spielraum* may be distributed so as to give a continuous effect*. Now these points are not in the least needful for the deduction of my skew curves, they are merely given here because in our complete ignorance of the nature of the causes, hereditary and environmental, which produce continuous variation, I think we have no warranty for saying that a limited number of cause-groups is impossible, or that no such limited number of fundamental cause-groups could give a continuous variation. In the present state of our knowledge we cannot agree with Ranke in sweeping away as impossible all the discreteness which follows from determinantal theories of inheritance. We cannot afford to be dogmatic as to the continuous or discontinuous character of the ultimate sources of variation and any effective theory must like the Laplace probability integral be equally applicable to the sum of a discontinuous series as well as to the areas of a continuous curve.

(*b*) Any finite series of cause-groups, Ranke tells us, must lead to discontinuity.

I have endeavoured to show above that the discontinuity may be as real and yet as undetectable as the distribution of lengths, say, of the vertebral columns of sharks which yet depends on the number of discrete vertebrae, with a scatter of their individual sizes. But Ranke's argument in itself is a false one, many discontinuous systems lead at once to continuous distributions. In our ignorance of the exact sources of variation, all we can do is to show that a limited number of cause-groups can quite well lead to continuous variation. To take a perfectly arbitrary illustration, suppose that a character can only take values lying between a_1 and a_2 and that this character is to be settled by the determinants derived from $s+1$ ancestors, i.e. suppose all but $s+1$ to be cast out in the successive divisions of the germ-cell. Then it by no means follows that the character will be a *blend* of these $s+1$ determinants, one or other of them may be *dominant*. It does not follow that the dominant one represents either the one with the least or the greatest value of the character. It might be the one with $s-r$ determinants below and r above it.

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* For example I have a variety of Binomial machines or "Quicunxes" like that figured in my memoir, *Phil. Trans.* Vol. 186 A, Plate I. Fig. 2, p. 414. It is quite possible to arrange a quicunx in which there are only a limited number of compartments, but in which the top of the seed in these compartments is not horizontal, but gives a continuous curve, e.g. the greater air draft of the greater frequency might be used to pile up the material in any receptacle on the side of the greater frequency.

In this case the frequency of individuals with character x would be given by the curve

$$y = y_0 \, (x - a_1)^{s-r} \, (a_2 - x)^r.$$

This is a perfectly continuous curve, and one of my series of skew curves. Thus it is quite conceivable that a finite number of fundamental cause-groups should lead to an absolutely continuous distribution*.

Now how does Ranke treat this illustration? He first states that all continuity must involve an infinite number of cause-groups, or variation in man being continuous, must be associated with an infinite number of cause-groups. He had this very case before him, and yet he writes:

Die Analyse der Elementarursachen ergibt uns also unweigerlich die bisher immer angenommene unendliche Anzahl derselben, die unendliche Kleinheit der Wirkung jeder einzelnen Ursache und die Kontinuität der moglichen Wirkungsgrade.

Sie ergibt also wirklich die Verhältnisse, die wir zum Verständniss der kontinuerlichen Variationskurve ganz unumgänglich nötig haben. Denn wie soll eine kontinuerliche Kurve sich aus der Kombination endlicher Bausteine ergeben (S. 321)?

Ranke only gets out of the difficulty by asserting that since the number of causes is finite, but must be infinite for variation, my continuous curve based on a finite number of cause-groups cannot represent variation! A more remarkable specimen of circular reasoning can hardly be conceived. The fact is that Ranke suffers from the old third Gaussian axiom, i.e. the supposition that the increments that go to build up the variate are independent of each other. The fundamental cause-groups are by no means *Bausteine* in the sense that the total variate is the sum of these *Bausteine* placed on top of each other! The causes determine the magnitude of the variate, but not at all necessarily by their sum.

(*c*) Ranke asserts that some of my curves have constants which if we endeavour to interpret them from the standpoint of the binomial give impossible or improbable values for the constants.

The answer to this is that the series were only the scaffolding to deduce the curves. The differential equation to the curves contains the limit to a good many other frequency systems which directly diverge from the fundamental axioms of Gauss. I used the original series as a means of dispensing with the Gaussian axioms in familiar cases, but the result reached involves a good deal more than can be interpreted by the original series. Ranke can only see absurdity in a binomial with a negative p or q. But the nature of the sources of variation is so little known to us that we cannot possibly assert the absurdity of such values. We may not indeed be able to directly interpret them in the case of man, say, but they occur and recur in chance investigations. I will illustrate this in one case only, but such will demonstrate the required possibility and dispose at once of Ranke's argument as to absurdity.

* Making r and a_2 infinite, but $s - r$ finite, we get the curve I have deduced as the limit to a binomial of finite power. In other words, that curve is also shown to correspond to a possible continuity.

Suppose an organ to require the conjunction of exactly n determinants of one kind to fix it, but that the size of the organ depends on how soon this conjunction takes place. Let D be the necessary kind of determinant and π the chance that it is left in the right position after each operation, say a cell-division or cell-fusion. Let D' be any other kind of determinant and κ its chance of appearing, so that $\kappa + \pi = 1$. Then if D appears n times in the first n operations, we have a certain size for the organ, if in the first $\overline{n+1}$ operations another size, and if only in the first $n + r$ operations a third size. But the chances of these respective appearances are the terms of the series

$$\pi^n + n\pi^n\kappa + \frac{n\,(n+1)}{2}\,\pi^n\kappa^2 + \ldots + \frac{n\,(n+1)\,(n+2)\,\ldots\,(n+r-1)}{\lfloor r}\,\pi^n\kappa^r + \ldots$$

$$= \pi^n\,(1-\kappa)^{-n} = \left(\frac{1}{\pi} - \frac{\kappa}{\pi}\right)^{-n} = (p-q)^{-n},\ \text{say},$$

where $p - q = 1$. Here p and q have lost the condition that they are both to be less than unity. I do not for a moment suggest that this is the real interpretation of a binomial with a negative q. I only assert that because we are as yet unable to certainly interpret such expressions, the absence of interpretation does not involve the absurdity which Ranke postulates.

There are many other matters to which I might justifiably take exception in Ranke and Greiner criticism*, but I think I shall have said sufficient to convince the impartial reader of the following points:

(i) The great bulk of modern statisticians are agreed that the Gaussian law is absolutely insufficient to describe observed facts. They may disagree as to the method of supplementing it. I do not think that the opinion of Ranke and Greiner can possibly weigh against those of Poisson, Quetelet, Galton, Edgeworth, Fechner and Kapteyn—all authorities who have had to deal for years with statistical data.

(ii) The original use of the probability integral (the areas of the Laplace-Gaussian curve) as introduced by Laplace was to represent the sum of terms of a discontinuous series. To the mathematical mind there is no absurdity in this replacement of discontinuity by continuity; it is the basis of the Euler-Maclaurin theorem.

(iii) The dogmatic assertion of Ranke that variation in man is due to an infinite number of infinitely small fundamental cause-groups, simply neglects the

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* For example, all the discrete variates mentioned on p. 205 have been dealt with by biometric writers, and many others besides, yet Ranke speaks as if such writers had not dealt with discrete variation. He speaks of Ludwig's multimodal curves for flowers as if there had been no controversy as to the actuality of the "Fibonaccizahlen" modes, when due regard is paid to homogeneity of season and environment. He speaks as if Johannsen had demonstrated normal variation in his "Erbsenspopulation," when he has really applied no valid criterion whatever to test for asymmetry, etc. In short he seems to me to have neglected a great deal of the modern literature of the subject, and, if I may venture to say so, to write over-dogmatically on what he has read.

whole determinantal theory of inheritance. A complete theory of asymmetrical frequency must describe in the manner of Laplace's probability integral either continuous or discontinuous variation.

(iv)   The apparent or practical continuity of many variation data may be due either to real continuity or to discontinuity effectually masked by: (a) the relative paucity of material and roughness of our measurements, which compels us to divide it into groups of the same order of number as the number of determinants, (b) the influence of age, nurture and environment superposed upon the pure hereditary results, or (c) the fact that many of the characters measured by us are built up of a larger or smaller, but not necessarily an infinite, number of simple organs or characters, which may possibly individually have discontinuous variation*.

(v)   The assertion of Ranke that a finite number of fundamental cause-groups *must* lead to discontinuity is disposed of by illustration. It is quite possible to invent a great variety of determinantal systems—the number of the determinants being finite—which lead to continuous variation. In our present ignorance of the sources of variation, especially of the mechanism of inheritance, it would be idle to lay weight on any special interpretation of this kind. It is important, however, to observe that continuity or discontinuity of variation are not, as Ranke asserts them to be, associated with the finite or infinite number of the cause-groups.

(vi)   The absurdity which Ranke finds in the values taken by some of the constants of my curves, exists only when a very narrow view is taken of the sources of organic variation. A binomial series with negative power or with negative p or q is capable, as is shown in this paper, of perfectly rational interpretation. But in the present state of our knowledge it would be idle to specify any particular interpretation as the correct one†.

(vii)   The problem of variation can be looked at in the following manner without the least loss of generality. Modify Gauss by replacing his third axiom, the independence of contributory increments to the variate, by the postulate that the increments are *correlated* with previous increments‡. Start with any binomial and we reach the generalised probability curve for an infinite number of cause-groups:

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2 f(x/\sigma_0)},$$

where f is an arbitrary function. This theory covers Galton, Edgeworth, Kapteyn and Fechner. Expanding $f(x/\sigma_0)$ in a series of ascending powers of x/σ_0 we have

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* Ranke has quite overlooked the work by Galton and myself on the *discontinuity* of the series of individuals even when the population obeys the Gaussian law. See *Biometrika*, Vol. I. pp. 289—299.

† Ranke apparently considers that $(p+q)^n$ with p, q and n positive *is* interpretable. A little philosophical consideration will show that it is merely "familiar," not really intelligible. There is no physiological meaning in p, q, n, and we cannot as yet associate them with any true organic mechanism.

‡ This postulate of course abrogates the first two axioms of the Gaussian theory as well.

my generalised probability curves*. A very few terms of the expansion, however, suffice for describing practical frequency distributions. If we keep only three terms, we see that the same system of curves suffices to describe continuous and discrete variates—an important point. If I lay stress upon this method here, it is because Ranke insists on an infinity of cause-groups and supposes no continuity can arise without them—

> ..."a truth
> Looks freshest in the fashion of the day."

(viii) The important physical constants of a frequency distribution are those which can be determined with the least probable errors. The probable errors of the moment coefficients increase rapidly with the moments. Hence the important physical constants are those which depend on the low moment coefficients, i.e. on the early terms of the expansion of $f(x/\sigma)$. Now these physical constants are (a) the mean, (b) the modal difference or distance of mean from mode, (c) the skewness, and (d) the kurtosis. We may replace either (b) or (c) by the standard deviation. Experience shows that these four physical constants are certainly independent. The constants of my skew curves directly give them and we are able to determine by their probable errors whether they are significant or not.

(ix) With regard to the other theories discussed I have shown:

(a) That the Galton-McAlister curve, ascribed by Ranke to Fechner, is not applicable to a great number of cases, for its kurtosis is a function of its skewness and its skewness cannot exceed ·21.

(b) That the double Gaussian curves due to Galton and Fechner are illogical, because they reach a Gaussian result by rendering invalid every one of the Gaussian principles. Further, the skewness is always a function of the kurtosis and the kurtosis cannot exceed ·87, a degree which is exceeded in a great variety of data.

(c) That the Edgeworth curves as developed by Kapteyn fail from the logical standpoint, for they appeal to a truncated Gaussian distribution which has never been observed in experience. They are not true graduation formulae, and are obtained in such manner that it is not possible to determine any one of the chief physical constants or evaluate their probable errors. Further in the examples given by Kapteyn they all sensibly reduce to the Galton-McAlister curve. But this curve has in every one of the cases dealt with by Kapteyn a skewness significantly less at a maximum than is required by every one of the statistical series involved.

Finally it seems to me that all discussion of asymmetrical frequency must turn in one form or another on the proper form to be given to $F(x)$ in the equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2 F(x)}.$$

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* See "Mathematical Contributions to the Theory of Evolution, XIV." Dulau and Co., London.

If we assume it to be $F(x) = \Sigma (a_r x^r)$ as I have done, we fall back on the normal curve when a_r for $r > 0$ is zero within the limits due to its probable error. Ranke, if he wishes to demonstrate the Gaussian law as general, must show this to be the case. It has been over and over again demonstrated that a_1, a_2, etc. differ significantly from zero for a great variety of series. Another advantage of the form $F(x) = \Sigma (a_r x^r)$ is that it covers as I have shown discrete as well as continuous variation. Considering $\sigma = \sigma_0 \sqrt{F(x)}$ as the standard deviation of the "instantaneous Gaussian curve," we see that the "instantaneous Gaussian curve" varies from one position to a second, like the "instantaneous ellipse" of the astronomers. A reasonable first hypothesis to make is that the local mean square deviation σ^2 is independent of x, we obtain the Gaussian curve. A next assumption is that it is a linear function of x—perhaps it would be better to say that its *mean* local value is a linear function of x, i.e. the mean square of the local variability σ^2 is *correlated* lineally with x. This gives my curve of Type III. The next easiest assumption is to suppose the regression line of σ^2 on x to be parabolic. In this case we obtain the remainder of the curves treated in my II. and XI. memoirs. If we stop at a_q we have what I have termed the skew frequency curves of the qth order*, and we see that this involves a regression curve between the square of the mean local variability and the character of the qth order*. I see, however, at present no *practical* necessity for proceeding beyond skew curves of the 2nd order, although I propose shortly to publish a discussion of skew curves of the 3rd order illustrating some *theoretical* points which arise in their discussion.

To sum up I think Ranke's criticism fails (α) because he has disregarded the universally recognised need of modern statistical science for asymmetrical frequency curves, (β) because he has not appreciated the mathematical transformation by which a number of finite terms are replaced by an integral expression, (γ) because he has not realised that modern theories of heredity lead directly to discontinuous skew distributions, (δ) because continuity does not depend upon infinity of fundamental cause-groups, and lastly (ϵ) because, and this may be due to my fault in the first deduction of my curves, he has quite failed to see either their scope or their real generality.

* "Mathematical Contributions to the Theory of Evolution, XIV. On the Theory of Skew Regression." Dulau and Co.

VARIATION AND CORRELATION IN THE EARTHWORM*.

By RAYMOND PEARL and WILBUR N. FULLER.

1. *Introductory.*

THE purpose of this paper is to present the results of a study of the variation and correlation exhibited in certain specific characters of the common garden earthworms.

The earthworm which occurs most abundantly in the region about Ann Arbor and with which we have dealt in the present work, has usually been identified as *Lumbricus herculeus*, Sav. or *L. agricola*, Hoffm. The counting and measuring and a portion of the computing were done by W. N. Fuller in connection with certain class-work in the University. The values of the constants were twice independently computed, and the paper written by R. Pearl.

There were two principal reasons which led to the undertaking of this work. In the first place we wished to determine if possible more exactly than had previously been done the typical condition of the form of earthworm commonly found in this portion of the United States. The earthworm is such a generally used type form, both for laboratory teaching and for investigation in a number of fields, notably regeneration, that an exact determination of the existing type and variation with reference to the several specific characters seemed decidedly desirable. Secondly, the writer hoped to get from this form some light on certain general biometrical problems on which he is collecting data.

The material for the work consisted of sexually adult specimens only. The criterion used to determine whether an earthworm was sexually adult, was the presence of a fully developed clitellum. That this is a justifiable criterion has been shown by Cole†.

The worms were collected on rainy nights, in gardens and lawns near the campus of the University of Michigan. On such nights the worms leave their burrows, and it is comparatively easy, after some experience, to pick up a large number in a reasonably short time.

The worms were brought to the laboratory, stupefied in weak alcohol, straightened to their full length, injected with weak chromic acid or formalin, and hardened and preserved in either alcohol or formalin. As the worms are stupefied in alcohol the muscles become relaxed, and it is easily possible to straighten them to their full length. They are hardened in this straightened position. The specimens used in this investigation were subsequently used by students for laboratory work. The measurements and counts were made during the first half of the fall semester of the college year, 1902–03.

Quantitative determinations of the following characters were made on each worm :

1. Total number of somites in the body.

2. Length of body in centimetres.

3. The number of somites from the anterior end of the worm to the most anterior somite of the clitellum.

4. The number of somites included in the clitellum.

5. The position of the genital openings.

The following points regarding the methods used in making the determinations may be noted. In all counts of somites which began at the anterior end of the worm the prostomium was not counted, but the first somite which was complete on the ventral side of the worm was counted as 1. All the other somites in the body were included in the count. All worms having spiral segments or other segmental anomalies were excluded in this work. Such worms were preserved for use in connection with a general study of teratology in the earthworm. In making the counts it was found very helpful to use a "tally register," such as is used in counting telegraph poles, and by gate keepers at places of amusement for counting people. This instrument has been found very useful in collecting biometrical data on integral variates for a variety of characters (e.g., scale rows and gill-rakers in fish, etc.). Anyone who has ever attempted to make a series of counts of repeated like characters knows how difficult it becomes in a short time to " keep the place," both mentally and on the specimen. All this difficulty is overcome by the use of a tally register, and its use makes it possible to work for a long time without becoming unduly fatigued. The only thing necessary to secure both speed and accuracy is that the operator have some preliminary practice.

With regard to the character "length of body" considerable care had to be exercised in order to get results in any way reliable. Of course, it would be nearly or quite impossible to get any reliable measure of the length of an earthworm

while it was alive, on account of the varying degrees of contraction of the muscles. This source of error is present to a certain degree in worms fixed and hardened in the way described above, but it is very greatly reduced in the fixed as compared with the living worm. Since, however, there was present in all cases a source of error on account of different degrees of contraction, no attempt was made to obtain close measurements of length. What was desired was to get all the worms classified into a comparatively few, rather large length groups, for purposes of correlation. This end was attained by using as our smallest unit of measurement a distance of 2·5 centimetres. A worm was laid on a measuring stick divided into 2·5 cm. spaces, its length observed, and the record made by writing on the record sheet a letter with an algebraic sign, denoting the division of the measuring stick within which the end of the worm fell. Thus a record of $C -$ indicated that that particular worm fell, in length, somewhere between 10 cm. and 12·5 cm. In making the computations all worms falling within the class $C -$ were considered to have been 11·25 cm. long, or in other words it was assumed that within a group the lengths were arranged symmetrically about the centre of that group. The assumption is, of course, the same as is usually made in dealing with the frequency distribution of graduated variates.

In the counts for the position and number of somites in the clitellum the following arbitrary rules were followed. The number of somites from the anterior end of the worm back to the *first somite on which any clitellar tissue was present*, was recorded as "Number of somites to clitellum." *All somites on which any clitellar tissue was present* were recorded as "Number of somites in clitellum." Some such arbitrary rules were necessary in dealing with the cases in which the clitellum included only a part of one or both of its terminal somites. A record was kept, according to a plan which will be described later, of the exact extent and form of the clitellum in each particular worm.

The only variations which were found in the collection of worms with reference to the position of the genital openings were regarded as teratological and the specimens exhibiting such variations were excluded from our tabulations. Abnormality in position of the genital pores in the very few cases in which it appeared was found associated with other abnormalities. The position of these openings is perfectly constant in normal worms. There is no "fluctuating" or continuous variation. On this account these characters will not be discussed further in this paper.

It should be stated that the collection was made entirely at random. No selection on the basis of any characteristic of the worm, with the exception of the presence or absence of the clitellum as noted above, was exercised in forming the collection. The question of the homogeneity of the material will be considered farther on in the paper.

We may turn now to the results.

2. *Total Number of Somites and Length of Body.*

In Table I. is shown the frequency distribution for the total number of somites in the body.

TABLE I.

Total Somites.

Number of Somites	Frequency	Number of Somites	Frequency
79	1	130	1
80—88	—	131	4
89	2	132	7
90—92	—	133	4
93	1	134	3
94	—	135	5
95	1	136	6
96—98	—	137	9
99	2	138	10
100	1	139	6
101	—	140	5
102	2	141	17
103	—	142	27
104	3	143	21
105	1	144	24
106	1	145	37
107	2	146	32
108—112	—	147	31
113	1	148	29
114	3	149	26
115	3	150	51
116	—	151	15
117	—	152	15
118	2	153	12
119	—	154	9
120	5	155	9
121	—	156	10
122	1	157	4
123	2	158	—
124	2	159	2
125	5	160, 161	—
126	2	162	2
127	1	163	—
128	5	164	1
129	4		
		Total	487

This distribution is shown graphically in Fig. 1. There is hardly any doubt that the apparently multimodal condition of this histogram is due simply to the fact that we have a limited amount of material and a very wide range of variation. Our sample is not sufficiently large to cover smoothly the whole of the range.

The values for the mean, standard deviation (with Sheppard's correction), and coefficient of variation deduced from this frequency distribution are as follows:

Mean = 142·715 ± ·362 somites.

Standard Deviation = 11·850 ± ·256 somites.

Coefficient of Variation = 8·303 ± ·181 °/₀.

FIG. 1.   Variation in number of Somites in body of Earthworm.

Number of Somites.
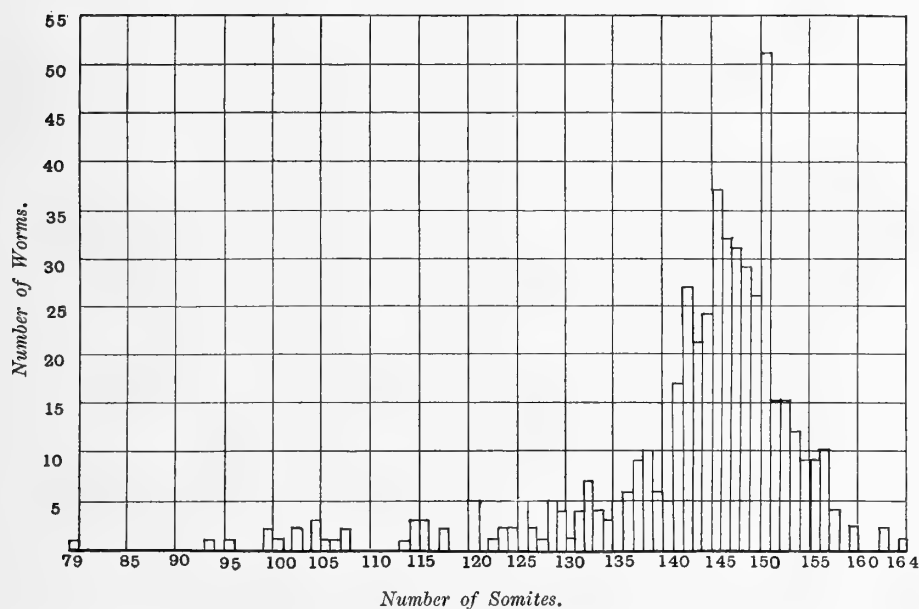
Turning to the character "length of body" we have the frequency distribution given in Table II.

TABLE II.

Length of Worms.   Class unit = 2·5 cms.

Length in centimetres	Frequency
10 —12·5	7
12·5—15	50
15 —17·5	75
17·5—20	153
20 —22·5	137
22·5—25	55
25 —27·5	9
27·5—30	1
Total	487

The constants deduced from this distribution have the following values:

Mean $= 19\cdot171 \pm \cdot094$ cms.

Standard Deviation $= 3\cdot077 \pm \cdot067$ cms.

Coefficient of Variation $= 16\cdot049 \pm \cdot356$ $^\circ/_\circ$.

From these values the following points are to be noted:

(*a*) The range of variation in respect to number of somites in the body is very great. The explanation of this fact is simple. The sample of worms under consideration is not homogeneous with respect to age. Up to a certain age at least, *Lumbricus* adds somites at the posterior end of the body with growth. It is clear from either Table I. or Figure 1, that the great extent of the total range in the variation in this character arises from the presence of a comparatively small number of individuals with few somites which unduly extend the lower end of the range. It seems reasonably certain from what we know of the method of growth in the earthworm that if this sample of worms could have been allowed to go on growing the range of variation in total number of somites would have steadily decreased with lapse of time.

The fact that there is no apparent tendency towards extreme extension of the range at the upper end indicates that the bulk of the worms included in the sample have either completed the process of adding somites or at least that the process is going on very slowly if at all. So then the sample serves to define the existing typical condition of the worms in the region with respect to this character with a reasonable degree of accuracy. It is to be regretted that an entirely homogeneous sample with respect to age cannot be obtained, but in the nature of the case this is practically impossible. It would involve following each individual worm from the time that its development began.

(*b*) The earthworm is apparently more variable in respect to length of body than in respect to number of somites. This would indicate that the observed variation in length is not due primarily to variation in *number* of somites, but rather to variations in the *lengths* of the individual somites in different worms. Of course, it immediately occurs to one that possibly some of the greater variation in length is due to varying degrees of contraction in individual worms. As has been stated above, however, this source of error has been largely eliminated by taking large length classes. The range of variation in length due to degree of contraction, when the worms are fixed in the way described, is very probably considerably less than one length class unit of $2\cdot5$ cm. The conclusion, that the earthworm is more variable in length than in number of somites, may seem at first sight paradoxical, but it is really no more so than to conclude, what is very obviously the fact, that man is more variable in respect to sitting height than in respect to number of vertebrae. It simply indicates that in the earthworm we have two kinds of growth occurring together: one the addition of somites, the other increase in size of the individual somites. That these two kinds of growth are acting together will be clear when the correlation surface between total number of somites and length of body is examined.

(c). The average length of the individual somite in this sample of worms is evidently

$$\frac{191 \cdot 71}{142 \cdot 715} = 1 \cdot 34 \text{ mm.}$$

This value so obtained accords very well with observation on the single somites. Its significance from a taxonomic standpoint will be considered later.

We may turn next to the question of the correlation between total number of somites and length of body. The correlation surface is exhibited in Table III.

Computing the value of the coefficient of correlation from this table by the usual equation

$$r = \frac{S(xy)}{N \sigma_1 \sigma_2},$$

we get $r = \cdot 260 \pm \cdot 028.$

This is a rather low coefficient and indicates that, so far as can be judged from this sample, there is no very close relationship between length and number of somites in the earthworm.

It is seen at once by mere inspection that the relationship between the two characters is not the same in all parts of the correlation table. Thus the worms in the two lowest length classes (i.e. from 10 to 15 cm. in length) are not, in general, individuals having few somites, but instead these arrays centre well towards the right end of the table. The five worms having the smallest number of somites all fall in or above the third length class.

This apparently paradoxical result of low correlation between length and number of somites is evidently to be explained as a consequence of the fact that the length of a worm depends not only on the number of somites in the body, but also on the size of the individual somites. It is the same sort of result as would be expected if the sitting height and number of vertebrae in man were correlated.

This factor of increase in length of the individual somite can now be analysed somewhat more precisely, through the medium of the regression equations. For the regression of length on number of somites we have the coefficient of regression:

$$b_1 = \cdot 260 \times \frac{3 \cdot 077}{11 \cdot 850} = \cdot 0675,$$

and for the regression of number of somites on length the regression coefficient

$$b_1 = \cdot 260 \times \frac{11 \cdot 850}{3 \cdot 077} = 1 \cdot 001.$$

The characteristic equations, referred to the means as origin, are

(i) $L = 9 \cdot 538 + \cdot 0675 \ S,$ and

(ii) $S = 123 \cdot 525 + 1 \cdot 001 \ L,$

TABLE III.

Correlation between Length of Body and Number of Somites.

Number of Somites.

Length in centimetres	79	89	93	95	99	100	102	104	105	106	107	113	114	115	118	120	122	123	124	125	126	127	128	129	130	131	132	133
10 —12·5																2												
12·5—15	1		1		1		1	1	1		1		2	2						2				1			2	
15 —17·5		2		1		1	1	1		1	1					1		1	1	1	1	1					3	1
17·5—20					1							1	1	1	2	2	1	1	1	2	1		2		1	3	1	1
20 —22·5								1															2	2		1	1	2
22·5—25																							1	1				
25 —27·5																												
27·5—30																												
Totals	1	2	1	1	2	1	2	3	1	1	2	1	3	3	2	5	1	2	2	5	2	1	5	4	1	4	7	4

Continued

Length in centimetres	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	159	162	164	Totals
10 —12·5					1								1				1	1										7
12·5—15		1	1	1			1	1	1	2	1	1	4	4	4	2	3	1	3	1			1				1	50
15 —17·5			1		2	1		1	1	1	3	3	6	5	4	5	8	3	1		2					1		75
17·5—20	1	4	3	3	6	3	2	6	7	6	8	10	8	8	7	6	18	6	5	2	1	3	3	2		1		153
20 —22·5	2		1	4	1	2	2	6	7	8	10	14	10	7	8	11	11	3	3	6	6	1	2	2	1			137
22·5—25				1				3	8	4	2	7	2	4	5	2	10	1	2	1		4	4		1			55
25 —27·5									3			2	1	3					1	2		1						9
27·5—30															1													1
Totals	3	5	6	9	10	6	5	17	27	21	24	37	32	31	29	26	51	15	15	12	9	9	10	4	2	2	1	487

where L denotes the length of the body in centimetres, and S the total number of somites. The probable error made in estimating length from number of somites is 2·004 cms., and in estimating somite number from length the probable error is 7·718 somites.

From (i) it is seen that an increase of one somite is associated, on the average, with an increase in length of the worm of ·6 mm. But, as has been shown above, the average length of an individual somite is more than twice this amount (1·34 mm.). Now if the individual somites did not increase in size after they are added the value given in equation (i) and the average value obtained by dividing the total length by the number of somites should agree. The values actually obtained lead us to conclude that the individual somite after being laid down about doubles in length. This result is of interest as showing the quantitative relation of the two factors considered in the growth of this simple segmented animal.

The regressions are sensibly linear.

3. *Variation and Correlation in the Clitellum.*

In the discussion of this subject the term "position of the clitellum" will be used to indicate the number of somites from the anterior end of the body back to the first somite which bears any clitellar tissue. It will be recalled that "number of somites in the clitellum" is taken to include all somites on which any clitellar tissue is present.

The frequency distributions for these two characters are given in Tables IV. and V. respectively.

<table>
<tr><td colspan="2">TABLE IV.
Position of Clitellum.</td><td colspan="2">TABLE V.
Somites in Clitellum.</td></tr>
<tr><td>Number of Somites</td><td>Frequency</td><td>Number of Somites</td><td>Frequency</td></tr>
<tr><td>29</td><td>1</td><td>6</td><td>394</td></tr>
<tr><td>30</td><td>117</td><td>7</td><td>84</td></tr>
<tr><td>31</td><td>376</td><td>8</td><td>17</td></tr>
<tr><td>32</td><td>1</td><td></td><td></td></tr>
<tr><td>Total</td><td>495</td><td>Total</td><td>495</td></tr>
</table>

The very small range of variation in both cases is noteworthy. In the case of the position of the clitellum practically all the individuals are included within a range of two classes. The number of somites in the clitellum with a total range of variation of only three somites has approximately four-fifths of the individuals in a single class.

The variation constants for position of clitellum deduced from Table IV. have the following values:

Mean = 30·762 ± ·013 somites.

Standard Deviation = ·436 ± ·009 somites.

Coefficient of Variation = 1·416 ± ·030 °/₀.

For the number of somites in the clitellum the values are:

Mean = 6·238 ± ·015 somites.

Standard Deviation = ·500 ± ·011 somites.

Coefficient of Variation = 8·019 ± ·173 °/₀.

It is at once evident that the variation in respect to number of somites in the clitellum is significantly greater, as indicated both by the standard deviation and the coefficient of variation, than the variation in the position of the clitellum. The difference between the coefficients of variation in the two cases is very large (6·603 °/₀). The difference between the two standard deviations is absolutely small (·064), but relatively it is large enough to be significant, being more than three times its probable error (which is ± ·014). The probable reason for the difference in variability between these two characters will be discussed later.

Turning now to the correlation between position and number of somites in the clitellum, we have the following table:

TABLE VI.

Correlation between Number of Somites in Clitellum and Position of that Organ.

Number of Somites in Clitellum.

		6	7	8	Totals
Position of Clitellum.	29	—	1	—	1
	30	39	62	16	117
	31	354	21	1	376
	32	1	—	—	1
	Totals	394	84	17	495

The value of the coéfficient of correlation deduced from this table is

$$r = - ·629 ± ·018.$$

This rather high negative value for the coefficient is of considerable interest from the morphogenetic standpoint. It shows that as the number of somites anterior to the clitellum increases, the number included in the clitellum itself tends to decrease, and *vice versâ*. Or in other words, it means that the posterior end of the clitellum is a definitely fixed morphological point, subject to very little variation.

That the posterior end of the clitellum shows the least variation of any point in connection with this organ is further shown by direct measurement. The frequency distribution concerned is as follows:

Number of Somites to posterior end of Clitellum	Frequency
36	40
37	416
38	38
39	1
Total	495

The mean deduced from this is 37 somites and the standard deviation ·407, giving a coefficient of variation of 1·100 °/$_\circ$, which is significantly lower than that for the position of the anterior end of the clitellum.

This relatively fixed position of the posterior end of the clitellum is very probably associated with the fixed position of the openings of the male ducts and the other parts of the reproductive system in this form. The clitellum is to be regarded as an accessory organ, at least, of the reproductive system. It has been shown by teratological work that the various portions of the reproductive system proper vary, so far as position is concerned, as a unit[*]. Whenever one part develops in another somite than that in which it normally occurs all the other parts appear in abnormal positions such that the normal relations between them are preserved. Beddard[†] has shown that in the highly variable form *Perionyx* the same sort of relationship holds between the position of the clitellum and the other organs of the reproductive system proper.

We may now turn to the consideration of the exact form which the clitellum takes in different cases. It was found during the collection of the data, that the clitellum did not, in all cases, completely include all the somites which it touched. In other words, it was rather frequently observed that clitellar tissue would only partially cover a somite at the anterior or the posterior end of the organ, or, in rare cases at both ends. It is of interest to know how frequently this condition occurs, and accordingly we have prepared a table to bring out this point. The following method was used in recording clitellum form: in case the clitellar tissue ended at both extremities of the organ, precisely at intersegmental grooves, two parallel, unbroken vertical lines were made, with a figure indicating the number of somites in the clitellum between them. Thus | 6 | indicates that the clitellum

[*] Cf. Bateson, *Materials for the Study of Variation*, pp. 159—167, and a paper by the present writer in the *Anat. Anz.* Bd. xviii. pp. 123—127 describing a case in which this close interconnection of the parts in respect to position is very clearly shown.

[†] *Proc. Zool. Soc.* 1886, pp. 298—314.

contains six somites and ends anteriorly and posteriorly in an intersegmental groove. Whenever the clitellar tissue overlapped so as to cover only a part of a somite at one end or the other, either the left or right vertical line was dotted, according as the overlapping occurred at the anterior or posterior end of the clitellum. Thus ⋮6| indicates that the clitellar tissue is present on six somites, and that it only covers a portion of the most anterior one of these six somites. |6⋮ signifies that it is the most posterior somite which is only partially covered, and ⋮6⋮ indicates that overlapping occurs at both ends.

In Table VII. are given the results of a determination of clitellum form in 495 worms, the form classes being arranged in the descending order of the frequency of their occurrence.

TABLE VII.

Clitellum Form.

Form Class	|6|	|7|	|8|	⋮7|	⋮6|	|7⋮	⋮8⋮	|8⋮	⋮7⋮	|6⋮	Total
Frequency	389	72	14	9	4	2	2	1	1	1	495

In 95·9 $\%$ of all the worms the clitellum ends exactly in an intersegmental groove at both ends. In the remaining 4·1 $\%$ it will be seen that overlapping occurs much more frequently at the anterior than at the posterior end of the clitellum. Thus in only 1·4 $\%$ of all the worms is there any overlapping at the posterior end, including the three cases, or ·6 $\%$ of the whole number, in which overlapping occurs at both ends of the clitellum.

So far as may be judged from the present sample of worms, then, it appears that (1) clitellar tissue does not extend over more than eight whole somites; (2) in a majority of cases it extends over exactly six whole somites; (3) in the great majority of cases clitellar tissue ends exactly on intersegmental grooves, both anteriorly and posteriorly, and (4) when overlapping does occur, it is more apt to be at the anterior than at the posterior end of the clitellum.

Now this spreading out or "overlapping" of the clitellum beyond the limits set by intersegmental grooves would apparently indicate a growth in longitudinal extent of the clitellum after it is first laid down. It seemed very desirable to determine exactly whether any such growth actually does occur, because it is usually supposed that the clitellum is an organ of definite extent which does not change so far as size is concerned. The matter can be definitely settled by determining whether large worms have extensive clitella, or, in other words, whether extent of clitellum and length of worm are sensibly correlated.

In order to test this matter we have made use of Pearson's recently published *Method of Contingency**. This extension of the theory of correlation makes it

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* " Math. Cont. to the Theory of Evolution, XIII." *Drapers' Company Research Memoirs*, Biometric Series I. pp. 35, Two plates.

possible to deal with a variety of problems which could not before be attacked by biometric methods on account of inherent peculiarities of the data.

At the outstart the following contingency table (Table VIII.) between length of worm and extent of clitellum was formed:

TABLE VIII.

Length.   Class unit 2·5 cm.

		$C-$	$C+$	$D-$	$D+$	$E-$	$E+$	$F-$	$F+$	Totals
	|6|	5	35	43	129	120	38	8	1	379
	：6|	—	—	1	—	2	—	—	—	3
	|6：	—	—	—	—	1	—	—	—	1
Form class—Clitellum.	|7|	2	14	23	19	8	6	—	—	72
	：7|	—	—	—	3	1	3	1	—	8
	|7：	—	—	—	—	—	2	—	—	2
	：7：	—	—	—	—	—	1	—	—	1
	|8|	—	—	6	4	2	2	—	—	14
	|8：	—	—	—	—	—	1	—	—	1
	：8：	—	—	—	—	1	—	—	—	1
Totals		7	49	73	155	135	53	9	1	482

This gives a table of 80 sub-contingencies, a comparatively large number for this method. Following the notation of Pearson in the memoir referred to, we find for the square contingency from this table

$$\chi^2 = 101\text{·}5385,$$

whence the mean square contingency

$$\phi^2 = \text{·}2107.$$

The mean square contingency coefficient

$$C_1 = \sqrt{\frac{\phi^2}{1 + \phi^2}},$$

hence equals   ·4172.

From the same grouping

$$\psi = \text{·}1167,$$

whence, interpolating by the diagram in Pearson's Table I., the mean contingency coefficient

$$C_2 = \text{·}34.$$

It is at once evident that these values do not approach very closely the relation $C_1 = C_2$, which should be the case if the proper grouping is used in the

table and the correlation is normal. It seemed probable that the discrepancy was due to too fine grouping in the table. In this particular case with the grouping adopted a considerable number of compartments have no observed occurrences falling in them.

With the hope of bettering the results a somewhat coarser grouping was tried, the table then taking the form shown in Table IX.

TABLE IX.

Length. Class unit 2·5 cm.

Form class—Clitellum.		$C-$	$C+$	$D-$	$D+$	$E-$	$E+$	$F-$	$F+$	Totals
	|6| and :6|	5	35	44	129	122	38	8	1	382
	|6: and |7|	2	14	23	19	9	6	—	—	73
	:7| and |7:	—	—	—	3	1	5	1	—	10
	:7: and |8|	—	—	6	4	2	3	—	—	15
	|8: and :8:	—	—	—	—	1	1	—	—	2
	Totals	7	49	73	155	135	53	9	1	482

This gives only 40 sub-contingencies and a correspondingly larger number of the compartments are filled with observations. With this grouping we find the following values:

$$\chi^2 = 70\cdot615,$$
$$\phi^2 = \cdot1465,$$
$$C_1 = \cdot3575.$$

For the mean contingency from the same grouping we find:

$$\psi = \cdot1093,$$

whence
$$C_2 = \cdot33.$$

This evidently comes fairly close to satisfying the relation $C_1 = C_2$, and further manipulation of the grouping seems hardly worth while. We may conclude, then, that there is a very sensible degree of correlation between the size of the worm and the extent of the clitellum, the value of r being about ·357 with a probable error of somewhat less than $\pm$ ·054.

This result implies that growth occurs in the clitellum after it is formed. Putting all the results together we are able to form a fairly complete and connected picture of the morphogenetic phenomena connected with the position and extent of this organ. These phenomena we conceive to have probably the following relations: (*a*) the clitellum develops at such a point on the body that its posterior end lies a definite and, except in teratological cases, a fixed number of somites behind the openings of the genital ducts. (*b*) The clitellum when formed has a

variable number of somites but as a consequence of (*a*) any increase in the number of somites in this organ must necessarily entail a decrease in the number of somites between the anterior end of the worm and the anterior end of the clitellum. Hence a negative correlation between these two characters arises. (*c*) There is a tendency for the clitellum after being formed to increase its extent by growth. This growth tends to extend the clitellum both anteriorly and posteriorly. In a sample of worms non-homogeneous with respect to age any extension in the posterior direction tends to increase the variation in the number of somites in the clitellum without correspondingly affecting the amount of variation in the number of somites *lying in front of* this organ. (*d*) The increase in extent of the clitellum by growth is more marked at the anterior than at the posterior end of the organ.

If this analysis be correct the factors concerned in the form taken by the clitellum after it is first laid down are fairly simple and understandable. The problem as to what causes the clitellum to originally appear in the precise region where it does is not so simple however. It is a particular case falling under the general problem of the "localization of morphogenetic phenomena." While natural selection is capable of accounting for the facts when the worm is entirely normal, this explanation fails to account for such cases as those decribed by Bateson for *Perionyx* and referred to above, where the position of the clitellum bears the same relation to the other organs of the reproductive system, whatever its absolute position.

4. *The Taxonomic Position of the Form Studied.*

It was supposed that the sample of worms studied in connection with this work was sufficiently large to enable a very precise determination of the systematic position of the form to be made. The constants obtained for the various characters from this collection were compared with the values given by Beddard* for the same characters in the various species of *Lumbricus*. Somewhat to our surprise it was found that our specimens differed widely in some points from the condition found in any described species of the genus. The worms come nearest to *L. herculeus*, Sav., and should evidently be classed with, or at least very close to this species. The points of distinction and agreement are shown in the following table:

Character	Ann Arbor form	*L. herculeus* Sav. (as given by Beddard)
Number of Somites ...	Mean 142·715	180
Length	„ 191·7 mm.	360 mm.
Somites to Clitellum ...	„ 30·76	31
Somites in Clitellum ...	„ 6·24	6

(Other diagnostic characters in agreement in the two cases.)

* *A Monograph of the Order of Oligochaeta.* Oxford, 1895, pp. 721—724.

In position and extent of the clitellum there is essential agreement between the Ann Arbor form and the typical *L. herculeus*. In size and number of somites, however, there is a striking discrepancy. It would appear that the earthworms typical for this region approach towards a dwarf condition of the type of the species, being on the average only a little more than half as long and having about 40 fewer somites. The typical *L. herculeus*, according to the above figures, has for the single somite an average length of 2 mm., as against 1·34 mm. for our sample. Applying equations (i) and (ii) (p. 219) we find the mean length of an array of worms each having 180 somites, 216·9 mm. instead of 360 mm., as given for *L. herculeus*, and for the mean number of somites for an array having a length type of 360 mm., 159·6 as against 180.

Is this Ann Arbor form to be regarded as a true dwarf variety, or is the small size to be explained in some other way? Several possible explanations occur to one but none of them seems adequate. In the first place it might be maintained that the specimens in our sample were young, and had not completed their growth. This is, undoubtedly, true for some of the specimens in the sample. The earthworm continues to add somites for a long period, possibly throughout its life. If such addition does occur throughout the life of the worm it must, after a time, be at a very slow rate. This is evidenced by the fact that in the present sample there is no tendency towards extreme extension of the range at the upper end. The form of the frequency polygon for these worms, as has been pointed out above, indicates that the majority have either stopped growing or are growing at a very slow rate.

Another possibility is that in the collection of these worms an unconscious and unavoidable selection was made on the basis of size. This might arise in the following way. Suppose it were a habit of very large (old) worms not to leave the burrows on rainy nights. Then it would necessarily result that any collection made in the way the present one was made, would contain no or few very large worms. Now it is possible that this is the case, but it seems to me hardly probable, and for the following reason: in the large number of earthworms which have been collected for class-work in this laboratory during the last five years, including several thousand individuals at least, specimens have not been found of twice the size of the average of the present collection. Furthermore worms collected by digging are of no larger size than those collected in the way described in this paper*. Finally no student of earthworm habits has recorded, so far as I am able to find, any tendency towards a habit of the sort mentioned.

Again it might be thought that the relatively small size of the worms in this sample as compared with the type of the species was due to the method of killing and fixing. That this cannot be the case is shown, first by comparison with living

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* These last two facts also argue strongly against the first point raised; viz., that the worms in the present sample are all young specimens. Surely in five years collecting a fair number at least of old worms would be found.

worms from this same locality, and secondly because such an explanation would not in any way account for the smaller *number* of somites.

Altogether, it must be concluded that the typical form of *Lumbricus* of this locality is very considerably smaller and contains fewer somites than the type of the species to which it is most closely related; viz. *L. herculeus**. There is no evidence available as to what is the cause of this dwarfing, but it is not unlikely that it is due to direct environmental influence.

5. *Summary.*

A study of variation and correlation in several characters of the earthworm *Lumbricus herculeus*, Sav., leads to the following conclusions:

1. The earthworm is more variable in length of body than in number of somites.

2. There is a rather low degree of correlation ($r = \cdot 260$) between these two characters.

3. The results just stated (1 and 2) arise from the fact that the organism increases in length as a consequence of the combined action of two processes, (*a*) the addition of new somites, and (*b*) the growth of somites already added.

4. There is greater variability in the number of somites in the clitellum than in the number of somites lying in front of that organ. The factors concerned in this greater variability are analysed.

5. The number of somites included by the clitellum is negatively correlated to a relatively high degree with the number of somites making up the portion of the body anterior to that organ.

6. This implies that the posterior end of the clitellum is a relatively fixed point. This conclusion is confirmed by direct measurement.

7. There is a strongly pronounced tendency for the clitellum to end both anteriorly and posteriorly exactly at intersegmental grooves. Overlapping occurs more frequently at the anterior than at the posterior end of the organ.

8. The clitellum increases in extent as the worm grows in length.

9. The form of *Lumbricus* found at Ann Arbor is shorter and contains fewer somites than the typical *L. herculeus*, Sav.

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* There remains, of course, still another possibility, namely, that the values for the type of the species are based on a few unusually large specimens, but if this is the case it is difficult to understand why some one has not called attention to the matter. Especially does this seem remarkable when we remember that there is no organism more universally used as a type in zoological laboratories.

MISCELLANEA.

I.   Per la risoluzione delle curve dimorfiche.

Nota del Dr FERNANDO DE HELGUERO, Roma.

Il Prof. Karl Pearson ha trattato il problema della decomposizione di una curva dimorfica risultante da componenti normali in una memoria inserita nelle *Phil. Trans.* Vol. 185 a, pp. 71–110.

Egli riduce tutta la difficoltà del problema alla risoluzione del sistema:

$$\begin{cases} \mu_3{}^2 - 4\mu_3 p_3 - 2p_3{}^2 - \lambda_4 p_2 + 6p_2{}^3 = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots (24) \\ 5\mu_3{}^2 p_3 - 2p_3{}^3 + 4p_3 p_2{}^3 - 20\mu_3 p_2{}^3 - \lambda_5 p_2{}^2 = 0 \dots\dots\dots\dots\dots\dots (25) \end{cases}$$

dove μ_3, λ_4, λ_5 sono dedotti dai dati empirici e p_2, p_3 sono le incognite. Ovvero eliminando p_3 si trova l'unica equazione risolvente:

$$24p_2{}^9 - 28\lambda_4 p_2{}^7 - 36\mu_3{}^2 p_2{}^6 - (24\mu_3\lambda_5 - 10\lambda_4{}^2)\, p_2{}^5 - (148\mu_3{}^2\lambda_4 + 2\lambda_5{}^2)\, p_2{}^4$$
$$+ (288\mu_3{}^4 - 12\lambda_4\lambda_5\mu_3 - \lambda_4{}^3)\, p_2{}^3 + (24\mu_3{}^3\lambda_5 - 7\mu_3{}^2\lambda_4{}^2)\, p_2{}^2 + 32\mu_3{}^4\lambda_4 p_2 - 24\mu_3{}^6 = 0\dots(29)$$

Studiando questo problema io sono giunto a formule un po' più semplici di quelle sopra scritte, che da quelle possono direttamente dedursi colle posizioni

$$\lambda_4 = -3\mu_3{}^{\frac{4}{3}}\nu_4, \qquad \lambda_5 = -3\mu_3{}^{\frac{5}{3}}\nu_5, \qquad p_3 = \mu_3 p_3', \qquad p_2 = \mu_3{}^{\frac{2}{3}}p_2'.$$

Le mie formule risolventi sono:

o il sistema:

$$\left.\begin{array}{l} 1 - 4p_3' - 2p_3'^2 + 6p_2'^3 + 3p_2'\nu_4 = 0 \\ 5p_3' - 20p_3'^3 - 2p_3'^3 + 4p_3' p_2'^3 + 3p_2'^2\nu_5 = 0, \end{array}\right\} \dots\dots\dots\dots\dots \text{(A)}$$

o l'unica equazione:

$$24p_2'^9 + 84\nu_4 p_2'^7 + 36p_2'^6 + 8\,(4\nu_5 + 5\nu_4{}^2)\, p_2'^5 + 6\,(74\nu_4 + 3\nu_5{}^2)\, p_2'^4$$
$$+ 9\,(32 - 12\nu_4\nu_5 + 3\nu_4{}^3)\, p_2'^3 - 9\,(8\nu_5 + 7\nu_4)\, p_2'^2 - 96\nu_4 p_2' - 24 = 0. \dots\text{(B)}$$

Il modo di usare praticamente di queste formule è il seguente:

Siano
$$y_1 = \frac{c_1}{\sigma_1\sqrt{2\pi}}\, e^{-\frac{(x-b_1)^2}{2\sigma_1{}^2}}, \qquad y_2 = \frac{c_2}{\sigma_2\sqrt{2\pi}}\, e^{-\frac{(x-b_2)^2}{2\sigma_2{}^2}}$$

le equazioni delle due componenti prendendo l'asse delle ordinate passante per il baricentro della curva complessa; si vogliono determinare c_1, c_2, σ_1, σ_2, b_1, b_2.

Siano μ_2, μ_3, μ_4, μ_5 i momenti della curva complessa rispetto all' asse delle ordinate, a l' area.

Si calcolino:
$$\nu_4 = \frac{a\mu_4 - 3\mu_2^2}{\mu_3^{\frac{4}{3}} a^{\frac{2}{3}}}, \quad \nu_5 = \frac{a\mu_5 - 10\mu_2\mu_3}{\mu_3^{\frac{5}{3}} a^{\frac{1}{3}}}.$$

Si trovino p_2', p_3' per mezzo del sistema (A), o p_2' per mezzo della nonica (B) e p_3' per mezzo della

$$p_3' = \frac{-8p_2'^3 + 3p_2'(2\nu_4 + p_2'\nu_5) + 2}{2p_2'^3 + 3\nu_4 p_2' + 4}. \dots\dots\dots\dots\dots\dots\dots (C)$$

Noti p_2', p_3' si calcoli $p_1' = \dfrac{p_3}{p_2'}$, poi γ_1 e γ_2 che sono le radici della

$$\gamma^2 - p_1'\gamma + p_2' = 0.$$

I parametri delle curve normali componenti sono allora date dalle formule:

$$b_1 = \gamma_1 \sqrt[3]{\frac{\mu_3}{a}}, \quad b_2 = \gamma_2 \sqrt[3]{\frac{\mu_3}{a}},$$

$$c_1 = \frac{a\gamma_2}{\gamma_2 - \gamma_1}, \quad c_2 = \frac{a\gamma_1}{\gamma_1 - \gamma_2},$$

$$\sigma_1^2 = \frac{\mu_2}{a} + b_1^2\left\{\frac{1}{3\gamma_1}\left(2p_1 - \frac{1}{p_2}\right) - 1\right\}, \quad \sigma_2^2 = \frac{\mu_2}{a} + b_2^2\left\{\frac{1}{3\gamma_2}\left(2p_1 - \frac{1}{p_2}\right) - 1\right\}.$$

L' equazione (B) può ammettere più radici reali, così il sistema (A). Per il nostro problema interessano solo le soluzioni con p_2' negativo. A ciascuna soluzione corrisponde una coppia di curve normali componenti.

Riguardo al numero di queste soluzioni si osservi che esso è sempre pari: se $\nu_4 > -7$ si hanno o nessuna o due soluzioni; se $\nu_4 < -7$ esistono, al più, quattro soluzioni, due sono certo reali se $\nu_4 < -5{\cdot}94587$.

La dimostrazione di questi risultati, come pure dei metodi per la risoluzione del sistema (A), si troveranno in una speciale memoria che verrà pubblicata altrove.

[In this *Note* μ_n stands for the nth moment about the centroid of the frequency system—the notation of my memoir of 1893—and not $a\mu_n$ the more usual notation now. K.P.]

II. Albinism in Sicily. A further Correction.

In *Biometrika*, Vol. III. Pt. IV. I pointed out that Dr Arcoleo states explicitly that he never met with any case of an albino child being born to an albino parent, and consequently that Professor Weldon's use of this authority to prove the production of two pigmented children (+3 albinos) by albino parents was erroneous.

Professor Pearson replied that the meaning of the explicit statement I quoted from Arcoleo was not clear to him, and that "Dr Arcoleo states explicitly that the mother of the four albinos in Family No. 6 was ' una albina di belle forme,' &c."

Happily we need not examine the various discrepancies and self-contradictions which Professor Pearson's version would presuppose in Arcoleo's table and text. It is merely a

mis-translation. The beautiful "*albina*" was not the mother of Family No. 6, but the *friend* of the mother. The text reads :—

> ......" Il Cav. N. N. sentì grande simpatia per una albina di belle forme e desiderò avere una figlia che le rassomigliasse. Con tale scopo indusse la moglie a farlese amica, e già ambedue vagheggiarono la medesima idea. Il desiderio fu soddisfatto colla nascita di una bella bambina; ma moriva a due anni, &c."

The words "indusse......amica" mean that he induced *la moglie*, his wife to make friends with the "*albina*"; literally, *far*, to make, *le*, her [the *albina*], *amica*, a friend, *si*, to herself. Arcoleo evidently attributes the albino births to "sympathy," or other mysterious influence, in which case he might well call the occurrence a "fatto curiosissimo." Nevertheless his memoir is perfectly consistent, save for a numerical slip in one place, and when he wrote that he knew no instance of hereditary albinism he meant no more or less than he said, doubtful as this has seemed to Professor Pearson.

<div align="right">W. BATESON.</div>

Cambridge, 27 *Feb.* 1905.

[I am sorry that I overhastily read Arcoleo's paragraph and thus stated that Cav. N. N. made an *amica* of the "fair albino." It is clear that he caused his wife to make a friend of her with the object that his wife and not the albino lady should provide the albinotic daughter. Arcoleo's consistency is thus established, and I can only hope that his powers of scientific observation are not detrimentally influenced by the capacity thus exhibited for archaic belief. K.P.]

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Dr F. de Helguero has sent the following corrections to his Note, *Biometrika*, Vol. IV. p. 230. It is desirable that they should be made at once, or they may give much trouble to a computator using the formulae on that page.

Equation (29) *for the third term* $-36\mu_3{}^2 p_2{}^6$ *read* $+36\mu_3{}^2 p_2{}^6$,

,,   (B) *for the fourth term* $+8\,(4\nu_5+5\nu_4{}^2)\,p_2'{}^5$ *read* $+18\,(4\nu_5+5\nu_4{}^2)\,p_2'{}^5$,

,,   ,,   *for the fifth term* $+6\,(74\nu_4+3\nu_5{}^2)\,p_2'{}^4$ *read* $+6\,(74\nu_4-3\nu_5{}^2)\,p_2'{}^4$,

,,   ,,   *for the seventh term* $-9\,(8\nu_5+7\nu_4)\,p_2'{}^2$ *read* $-9\,(8\nu_5+7\nu_4{}^2)\,p_2'{}^2$.

DATA FOR THE PROBLEM OF EVOLUTION IN MAN.

ON FERTILITY, DURATION OF LIFE AND REPRODUCTIVE SELECTION.

By A. O. POWYS, Statist's Office, Melbourne.

A. *On the Correlation between Duration of Life and Number of Offspring.*

IN a previous paper on this subject* the data were derived from the published records of New South Wales for the two years 1898 and 1899, and referred only to the deaths of married women. As the experience of the subsequent three years with regard to married women, and that of 1901 and 1902 with regard to married men, are now available†, I have used the complete experience for each sex as the basis of a fresh investigation of the correlation between the duration of life and number of offspring, and as the number of observations is large, viz. 15,548 deaths of married women and 7303 of married men, the results should be of some value in the determination of this subject. In Tables I. and II. are

TABLE I.

Ages of Married Men at Death and Average Number of Offspring (Living and Dead). Experience of New South Wales, 1901 and 1902.

Ages at Death	Deaths of Married Men	Total Offspring	Average Offspring
20 and under 25	43	39	·91
25 ,, 30	173	212	1·23
30 ,, 35	296	691	2·33
35 ,, 40	477	1454	3·05
40 ,, 45	574	2329	4·06
45 ,, 50	603	2927	4·85
50 ,, 55	578	3073	5·32
55 ,, 60	642	3908	6·09
60 ,, 65	731	4452	6·09
65 ,, 70	913	5804	6·36
70 ,, 75	849	5651	6·66
75 ,, 80	591	3653	6·18
80 ,, 85	428	2774	6·48
85 ,, 90	267	1747	6·54
90 ,, 95	96	640	6·67
95 ,, 100	33	218	6·61
100 and over	9	43	4·78
Total	7,303	39,615	5·42

* *Biometrika*, Vol. I. Part I. pp. 34—38.

† Vide "Vital Statistics" of New South Wales for 1900, 1901 and 1902.

contained the complete data, and also the average number of offspring to the married of each sex in each quinquennial age-group at death.

These averages will be found plotted in Figs. 1 and 2. On reference to Fig. 1, it will be seen that the male statistics divide into two well-defined series,

TABLE II.

Ages of Married Women at Death and Number of Offspring (Living and Dead). Experience of New South Wales, 1898 to 1902.

Ages at Death	Deaths of Married Women	Total Offspring	Average Offspring
Under *20*	69	54	·78
20 and under *25*	480	598	1·25
25 ,, *30*	937	1970	2·10
30 ,, *35*	1131	3711	3·28
35 ,, *40*	1311	6004	4·58
40 ,, *45*	1101	6011	5·46
45 ,, *50*	1040	6048	5·82
50 ,, *55*	958	5669	5·92
55 ,, *60*	1116	6955	6·23
60 ,, *65*	1343	8657	6·45
65 ,, *70*	1485	9741	6·56
70 ,, *75*	1410	8628	6·12
75 ,, *80*	1269	7921	6·24
80 ,, *85*	1073	6539	6·10
85 ,, *90*	544	3201	5·88
90 ,, *95*	204	1225	6·00
95 ,, *100*	63	355	5·63
100 and over	14	75	5·36
Total	15,548	83,362	5·36

Fɪɢ. 1. Length of Life of Married Men and Size of Family. Experience of New South Wales, 1901 and 1902.

Equation to Regression Straight Line *aa'*, $y = -2·6570 + ·15391x$ } Origin at Birth.

,,   ,,   ,,   *bb'*, $y = 5·39489 + ·013284x$ } Unit of $x = 1$ year.

,,   ,,   Cubical Parabola *cc'*, $y = 6·13673 + ·34808x - ·04913x^2 - ·00078x^3$ {Origin at 62·5 years. Unit of $x = 5$ years.}

Fig. 2. Length of Life of Married Women and Size of Family. Experience of New South Wales, 1898—1902.

Equation to Regression Straight Line aa', $y = -3\cdot7179 + \cdot21729x$ $\Big\}$ Origin at Birth.
" " " bb', $y = 3\cdot6866 + \cdot04335x$ $\Big\}$ Unit of $x = 1$ year.
" " " cc', $y = 8\cdot1556 - \cdot025366x$ $\Big)$
" " Cubical Parabola dd', $y = 6\cdot29315 + \cdot15595x - \cdot04839x^2 + \cdot00199x^3$ $\Big\{$ Origin at 60 years. Unit of $x = 5$ years.

(a) from age period 20—25 to 50—55, and (b) from 55—60 to 95—100. The observations beyond age 100 are too few to attach much weight to the results. From Fig. 2 it is seen that the female statistics divide into three well-defined series, (a) from age group 15—20 to 40—45, (b) from 40—45 to 65—70, and (c) from 65—70 to 100—105. These divisions in each sex are practically in agreement with those of Professor Pearson's English and American Quaker families *—more especially the former—although owing to the comparative paucity of data, his series are not so well defined. The New South Wales data for ages beyond 75 are sufficiently extensive to furnish reliable results up to age 100.

The correlation between the duration of life and number of offspring and the regression straight lines have been determined for each of the five series and the general results are set forth in Tables III. and IV.

TABLE III. *General Results.*

Parent	Period of Life	Mean Age at Death (years)	Mean Size of Family	Standard Deviation		Correlation between Duration of Life and Fertility	Regression	Probable error of y (Size of Family)
				Age at Deaths†	Size of Family			
Father	Under 60	45·3424	4·3216	1·8434	1·4266	·99436 ± ·00013	+ ·76954	± ·1020
„	60 and over	70·90975	6·3369	1·9050	·2304	·54915 ± ·00698	+ ·06642	± ·1299
Mother	Under 45	33·6208	3·6484	1·3226	1·4436	·99541 ± ·00009	+1·08645	± ·0931
„	45 to 70	56·0605	6·1168	1·7563	·3877	·98178 ± ·00029	+ ·21675	± ·1450
„	70 and over	76·4400	6·2166	1·4935	·2205	− ·85910 ± ·00824	− ·12683	—

* "Data for the Problem of Evolution in Man. V. On the Correlation between the Duration of Life and the Number of Offspring." Beeton, Yule and Pearson, *R. S. Proc.* Vol. LXVII. pp. 161—179.
† Unit of Standard Deviation of Age at Death is 5 years.

TABLE IV.

Table of Regression Formulæ.

$x=$Duration of Life.　　$y=$Size of Family.

(1)　For lives of Fathers under 60 years of age
$$y = -2 \cdot 6570 + \cdot 15391x.$$

(2)　For lives of Fathers, 60 years of age and over
$$y = 5 \cdot 39489 + \cdot 01328x.$$

(3)　For lives of Mothers under 45 years of age
$$y = -3 \cdot 71792 + \cdot 21729x.$$

(4)　For lives of Mothers, 45 and under 70 years of age
$$y = 3 \cdot 6866 + \cdot 04335x.$$

(5)　For lives of Mothers, 70 years of age and over
$$y = 8 \cdot 1546 - \cdot 02537x.$$

Note :—Origin at birth, and unit of $x=1$ year.

These lines, which are shown on the diagrams by broken lines, must be regarded as admirable representations of the statistics in all the series. Cubical parabolas were also computed for both sexes, but in neither case can it be said that the curves nearly as accurately represent the statistics as the regression straight lines. Adopting Professor Pearson's method* (mid-ordinate formula) the following equations for parabolas of the third order were obtained :

$$\text{For Males} \quad y = 6 \cdot 13673 + \cdot 34808x - \cdot 04913x^2 - \cdot 00078x^3,$$
$$\text{,,} \quad \text{Females} \ y = 6 \cdot 29315 + \cdot 15595x - \cdot 04839x^2 + \cdot 00199x^3,$$

the origin in the male curve being at 62·5 years, and in the female 60 years. The theoretical maximum fecundity in males was found to be at 78·925 years, and that in females at 69·075 years. Referring to the regression straight lines it is seen that in the case of males the coefficient of correlation in both series is positive; in the females in the first two periods it is positive, but in the period beyond 70 years of age it is negative. In all cases the coefficient is high and the probable error is very small. Thus it is seen that in man increased longevity is associated with increased fertility, whilst in women this association only exists up to about age 70, after which increased longevity is associated with diminished fertility, thus confirming Professor Pearson's suspicion that extreme fertility is not in all cases favourable to a maximum longevity. In man up to age 55—60, as might of course have been suspected, the association is well defined, but beyond that period it is not so pronounced. The association during the first period requires no explanation, being so intimately connected with the first or child-bearing period of women. But in the second period the connection between longevity and fertility is possibly more apparent than real. Many men who become widowers during that period contract subsequent marriage with

* *Biometrika*, Vol. II. Part I. pp. 16 and 17.

young women and thus increase their issue, whilst other married men, equal or superior in physique, whose wives still living survive the reproductive period, are necessarily unable to do so*. I have shown elsewhere† that the reproductive period in man, though diminishing in intensity from age 30—35, is practically not exhausted till about 75—80. Too much weight must not therefore be attached to this apparent association of extreme fertility and extreme longevity in man. There may be such association, but it is impossible to determine it from the available statistics. With females that increase of life should connote increased fertility during the period up to age 45 is too obvious to need comment; beyond that period it would seem that at all events so far as New South Wales is concerned, Weismann's views are supported—married women dying at ages 65—70 have had more offspring than either those who die younger or those who die older. Certainly extreme fertility is not associated with extreme longevity in females, and this phenomenon is probably brought about by the physical strain in bearing and the mental strain in rearing large families. Now the New South Wales statistics admit of this subject being analysed in another and perhaps in a more satisfactory manner. For the five years 1898 to 1902, the " Vital Statistics" contain tables showing the ages at death of married women (by single years), together with the number of children born to each woman. From these we are therefore enabled to ascertain the average duration of life of mothers of each sized family, and from any assigned period of life. Here it may be stated that throughout this investigation the term "mother" will be used instead of "married woman," notwithstanding that it may be regarded as an Hibernicism to speak of mothers of no children. From this data then I have determined the average duration of life of mothers beyond 46 years of age by survivors of age 45, and beyond ages 60 and 70 by survivors of those ages respectively. Obviously it would be prejudicial to the computed longevity of mothers of small families if the duration were determined as from any year prior to the termination of the reproductive period. That this period in New South Wales practically terminates with the close of the forty-fifth year is evident from the fact that of 335,642 confinements of married and single women which took place during the nine years 1893 to 1900 and 1902 (the tabulation in 1901 did not permit of the numbers of births to women aged 46 and upwards being obtained), I find that only 1013 or about three per 1000 were due to women who had attained the age of 46, and of these it may be of interest to mention that seventeen are recorded as due to women aged 50, six to women of 52, two to women of 53, one to a woman of 54, three to a woman of 55 (one of these was unmarried), and one each to women of 56 and 58 years of age respectively! In Table V. will be found the ages at death and number of offspring of the 10,276 women who died in New South Wales during the five years 1898 to 1902, and who had passed the reproductive period.

[* It seems desirable, as in the paper referred to on p. 235, to consider only single marriages, and not the children of two wives. Ed.]

† *Biometrika*, Vol. I. Part I. p. 34.

TABLE V.

Ages at Death of Married Women and Number of Offspring.   Experience of New South Wales, 1898—1902.

Ages at Death	Number of Children in Family																									Total Deaths of Mothers
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	24	30	
46	19	15	12	21	18	17	28	21	18	17	11	10	7	1	2	—	—	1	—	—	—	—	—	—	—	217
47	23	14	12	19	11	14	11	13	14	16	15	9	8	5	2	1	—	1	—	1	—	—	—	—	—	189
48	24	8	14	13	22	23	19	24	33	17	12	10	6	2	4	1	—	—	—	—	—	—	—	—	—	232
49	11	12	6	13	10	15	12	19	12	18	13	7	6	2	2	1	1	—	—	—	—	—	—	—	—	160
50	22	16	17	18	18	15	18	17	10	13	14	9	3	3	2	1	—	—	—	—	—	—	—	—	—	196
51	24	12	14	11	15	11	18	9	13	13	14	8	10	6	2	1	2	1	—	—	—	—	—	—	—	184
52	24	10	12	10	12	17	13	21	22	17	14	11	7	5	1	2	1	1	—	—	—	—	—	—	—	200
53	21	14	17	8	11	13	17	18	17	16	10	18	11	3	1	1	—	—	—	—	—	—	—	—	—	196
54	18	15	7	9	20	13	15	13	19	14	13	8	10	3	3	1	—	—	—	—	—	—	—	—	—	181
55	31	10	8	12	14	17	17	21	22	19	11	14	8	3	—	1	1	—	—	—	—	—	—	—	—	209
56	34	12	16	16	10	13	10	26	24	26	18	19	15	3	7	—	1	1	—	—	—	—	—	—	—	251
57	15	9	9	19	17	17	18	25	21	11	15	14	10	8	2	—	—	—	—	—	—	1	—	—	—	211
58	28	17	14	13	12	15	17	22	21	24	21	15	6	4	1	2	—	—	—	—	—	—	—	—	—	232
59	22	7	11	12	13	17	21	24	14	23	20	12	7	7	1	—	2	—	—	—	—	—	—	—	—	213
60	50	17	18	20	25	17	18	23	28	15	29	19	3	10	6	2	4	—	1	—	—	—	—	—	—	305
61	24	11	11	8	7	13	16	16	21	13	20	10	13	9	3	1	1	1	—	—	—	—	—	—	—	198
62	25	15	15	20	6	24	26	26	26	20	23	20	14	2	4	2	1	1	1	—	—	—	—	—	—	271
63	25	9	13	13	14	21	24	31	33	18	30	15	15	8	5	4	1	1	—	—	—	—	—	—	—	280
64	24	19	12	9	19	25	29	35	24	27	28	18	7	7	2	5	—	—	—	—	—	—	—	—	—	290
65	41	18	12	15	25	27	22	23	30	18	23	12	14	7	3	2	3	1	—	—	—	—	—	—	—	296
66	33	13	13	20	15	22	16	33	25	39	28	14	15	9	2	1	3	—	—	—	—	—	—	—	—	301
67	32	12	14	20	23	24	16	30	31	31	30	20	12	7	9	2	3	—	—	—	—	—	—	—	—	316
68	30	8	9	18	21	20	23	34	27	28	24	23	21	11	5	2	1	1	—	—	—	—	—	—	—	306
69	31	5	14	15	12	18	21	27	24	33	28	15	12	5	3	1	1	—	—	—	—	—	—	—	—	265
70	46	13	26	15	24	28	32	33	28	28	26	15	9	11	2	1	—	—	—	—	—	—	—	—	—	337
71	19	15	17	15	23	20	9	27	17	20	15	16	13	7	1	2	1	—	—	—	1	—	—	—	—	238
72	31	13	20	25	18	27	26	23	24	20	16	18	14	6	4	5	1	1	—	—	—	—	—	—	—	292
73	28	22	12	23	17	19	29	30	24	18	5	22	9	9	5	1	1	—	1	—	1	—	—	—	—	275
74	28	13	11	14	20	20	25	30	27	15	27	15	7	6	4	4	—	1	—	—	—	—	1	—	—	268
75	31	17	10	20	21	26	36	21	17	19	24	15	14	8	4	1	1	—	—	—	—	—	—	—	—	285
76	24	7	12	10	15	24	17	20	29	25	16	13	12	5	5	—	1	—	—	—	—	—	—	—	—	235
77	16	10	17	16	15	15	22	22	25	20	14	15	12	5	2	1	1	—	—	—	—	—	—	—	—	228
78	28	8	18	20	15	27	27	29	23	17	25	21	12	7	2	2	1	—	—	—	—	—	—	—	—	282
79	36	13	16	13	20	16	20	22	18	14	20	9	15	3	2	—	1	—	—	—	—	—	—	—	1	239
80	32	17	25	21	13	26	23	28	35	21	17	15	11	2	3	1	1	—	—	—	—	—	—	—	—	291
81	22	8	5	9	17	10	13	16	21	21	15	8	8	4	3	1	2	—	—	—	—	—	—	—	—	183
82	19	11	15	15	17	18	13	21	21	17	15	9	6	4	4	—	1	—	—	1	—	—	—	—	—	207
83	24	10	12	10	12	14	14	14	15	13	14	9	8	4	3	2	—	—	1	—	—	—	—	—	—	179
84	18	12	13	12	13	20	18	20	31	12	13	14	6	4	5	1	—	—	1	—	—	—	—	—	—	213
85	13	9	5	5	14	11	14	8	14	15	14	6	4	3	2	—	—	—	1	—	—	—	—	—	—	138
86	14	5	7	7	8	10	11	13	9	15	9	2	6	2	1	—	—	1	—	—	—	—	—	—	—	120
87	13	7	8	5	10	10	12	8	12	10	11	2	5	—	2	—	—	—	—	—	—	—	—	—	—	115
88	9	5	9	8	11	10	9	10	11	6	7	2	2	1	—	1	—	—	—	—	—	—	—	—	—	101
89	7	7	3	4	7	3	10	4	8	9	4	1	1	2	—	—	—	—	—	—	—	—	—	—	—	70
90	8	4	5	8	6	5	9	9	7	8	5	3	1	1	—	—	—	—	—	—	—	—	—	—	—	79
91	2	—	2	3	2	3	4	1	4	3	—	1	1	—	—	—	1	—	—	—	—	—	—	—	—	27
92	3	2	2	2	4	4	4	8	2	3	2	3	3	—	—	—	—	—	—	—	—	—	—	1	—	43
93	2	3	2	4	2	2	3	1	2	4	2	2	1	—	1	—	—	—	—	—	—	—	—	—	—	31
94	—	—	1	4	2	5	2	1	3	2	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	24
95	1	—	3	2	1	—	2	—	2	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	14
96	2	3	1	—	—	7	3	3	—	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	24
97	1	—	1	—	—	3	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
98	1	—	—	1	2	1	2	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	8
99	—	—	1	—	1	1	1	1	1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	8
100	—	—	2	—	—	—	—	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
101	—	—	—	1	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
102	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
103	—	—	—	··	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
104	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
105	1	1	—	··	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1110	533	581	644	702	813	855	976	963	847	786	568	422	226	129	57	39	12	5	2	2	1	1	1	1	10,276

Although the last column shows the usual aggregations at the decennial years of age, it was not found practicable to graduate or adjust the numbers in consequence of the number of offspring also being involved. This will not however vitiate the comparison between the mothers of the different sized families, as the error is common to all. The average duration of life was determined by means of the single years and a half-year added to each result to allow for the life lived in the year of death on the assumption of the equal distribution of deaths throughout that year. The statistics relating to families of more than seventeen children were not used, being altogether too small to furnish reliable results. The average duration of life beyond age 45 of the mothers of various sized families who had survived their forty-sixth year, and of those beyond 60 and 75 by survivors of those ages, is given in Table VI.

TABLE VI.

Mean Duration of Life of Mothers.

Number of children in Family	After 45 years of age		After 60 years of age		After 75 years of age	
	Number of Deaths	Average Life (years)	Number of Deaths	Average Life (years)	Number of Deaths	Average Life (years)
0	1110	22·63	794	13·33	327	6·90
1	533	22·29	362	14·15	159	7·63
2	581	23·41	412	14·76	195	7·64
3	644	22·82	450	14·36	200	7·46
4	702	23·39	499	14·75	230	7·73
5	813	23·68	596	14·42	271	7·51
6	855	23·61	621	14·53	289	7·33
7	976	22·91	703	13·54	282	7·18
8	963	23·41	703	14·06	314	7·38
9	847	23·14	603	14·10	260	7·57
10	786	22·96	585	12·91	233	6·95
11	568	22·26	404	12·75	152	6·15
12	422	22·94	308	13·29	130	6·19
13	226	22·68	171	12·06	57	6·22
14	129	23·62	99	13·42	41	7·16
15	57	21·94	45	10·72	10	6·20
16	39	23·24	31	10·60	10	6·20
17	12	18·58	8	9·38	—	—

Guided approximately by the statistics, each of these series was broken up into two sections—from 0 to 8 children and from 9 to 17 in the first two, and from 0 to 8 and 9 to 16 children in the last series. The correlation and the regression straight lines were then determined for the six sections with the results set forth in Tables VII. and VIII.

Although the correlation cannot be regarded as high in any of the series there are some significant facts in connection with the results—first, the correlation in each of the series 0 to 8 children is positive, and from 9 children upwards it is negative; that is, that fertility up to about 8 children is apparently

TABLE VII.

General Results.

Period of Life	Size of Family	Mean Duration of Life (years)	Mean Size of Family	Standard Deviation		Correlation. Duration of Life and Fertility	Regression
				Duration of Life (Unit =1 year)	Size of Family		
After 45 years	0 to 8 children	23·1369	4·2031	12·2551	2·7189	+ ·02144 ± ·00796	+ ·09666
,, ,,	9 to 17 ,,	22·8396	10·7654	11·4486	1·6648	− ·01285 ± ·01214	− ·08834
After 60 years	0 to 8 ,,	14·1321	4·2379	8·7413	2·7189	+ ·009205 ± ·00941	+ ·02959
,, ,,	9 to 17 ,,	13·1207	10·7289	8·3649	1·6825	− ·07192 ± ·01413	− ·35757
After 75 years	0 to 8 ,,	7·3796	4·2541	5·3257	2·6810	+ ·00710 ± ·01420	+ ·01410
,, ,,	9 to 16 ,,	6·8203	10·6685	5·0405	1·5830	− ·08687 ± ·02240	− ·27662

TABLE VIII.

Regression Formulæ.

y = Duration of Life ; x = Size of Family (Origin at 0 children).

(1)   For lives of Mothers of 0 to 8 children, after 45 years of age, $y = 22·7306 + ·09666x$.

(2)   ,,   ,,   9 to 17   ,,   ,,   ,, $y = 23·7906 − ·08834x$.

(3)   ,,   ,,   0 to 8   ,,   60   ,, $y = 14·0067 + ·02959x$.

(4)   ,,   ,,   9 to 17   ,,   ,,   ,, $y = 16·9808 − ·35757x$.

(5)   ,,   ,,   0 to 8   ,,   75   ,, $y = 7·3196 + ·01410x$.

(6)   ,,   ,,   9 to 16   ,,   ,,   ,, $y = 9·7714 − ·27662x$.

favourable to longevity, whilst unfavourable beyond that number; secondly, the mean family of the 0 to 8 children groups increases with each period, whilst the means of the 9 to 17 children groups as regularly diminish; and thirdly, that the mean duration of lives of mothers of the 0 to 8 children groups is consistently greater than that of the mothers of larger families. These statistics have also been fitted with cubical parabolas (true ordinate formula) with the following results :

For life after age 45, $y = 23·32576 − ·08538x − ·01864x^2 − ·00071x^3$,

,, ,, ,, ,, 60, $y = 13·94679 − ·21964x − ·02765x^2 − ·00032x^3$,

,, ., ,, ,, 75, $y = 7·20525 − ·15409x − ·00816x^2 + ·00135x^3$.

The origin of the first two curves is at 9 children, and of the latter at 8·5 children. The maxima were found to be at 5·790, 4·208, and 3·527 children respectively. The statistics, regression straight lines and curves are plotted on Fig. 3.

Although as has been previously remarked the correlation is not high in any of the series, yet in the case of life beyond 60 and beyond 75 for mothers of more than eight children, it is sensibly negative within the limits of probable error, and this is sufficient to justify the conclusion that even in the case of survivors attaining those ages, their future expectation of life has been prejudiced by every additional child that had been born to them. But if excessive fertility

FIG. 3. Number of Offspring and Average Duration of Life of Mothers.

A, Mean Lifetime after 45 Years of Age, by Survivors of Age 46.

B, ,, ,, 60 ,, ,, ,, 60.

C, ,, ,, 75 ,, ,, ,, 75.
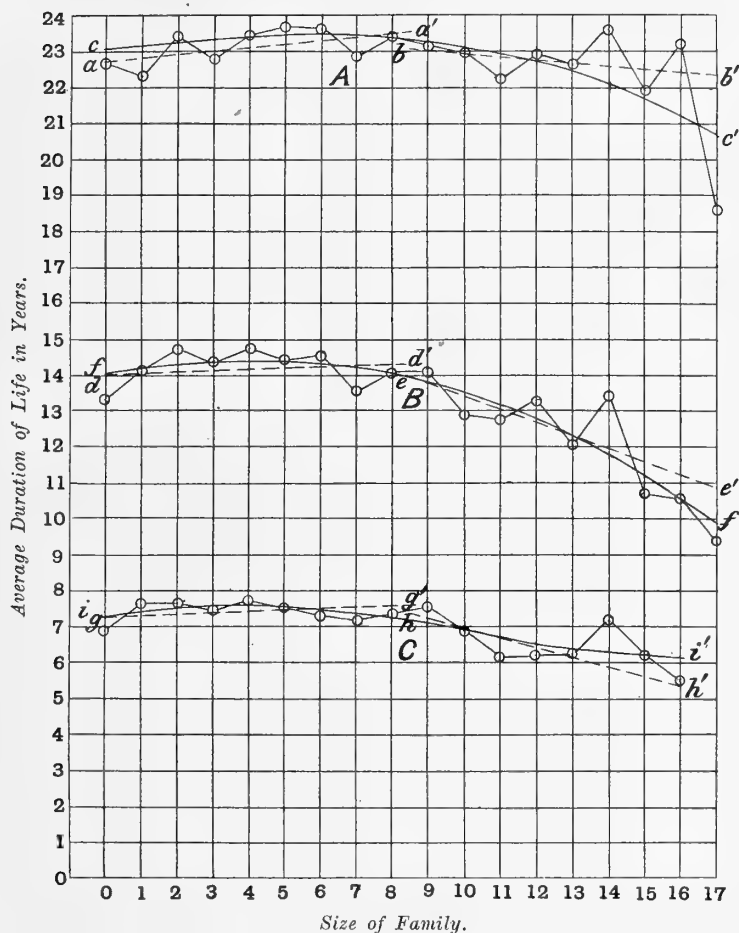
Equations to Regression Straight Lines:

aa', $y = 22 \cdot 7306 + \cdot 09666x$

bb', $y = 23 \cdot 7906 - \cdot 08834x$

dd', $y = 14 \cdot 0067 + \cdot 02959x$ $\Big|$ Origin at 0 children.

ee', $y = 16 \cdot 9808 - \cdot 35757x$ $\Big|$ Unit of $x = 1$ child.

gg', $y = 7 \cdot 3196 + \cdot 01410x$

hh', $y = 9 \cdot 7714 - \cdot 27662x$

Equations to Cubical Parabolas:

cc', $y = 23 \cdot 32576 - \cdot 08538x - \cdot 01864x^2 - \cdot 00071x^3$ $\Big\{$ Origin at 8·5 children.

ff', $y = 13 \cdot 94679 - \cdot 21964x - \cdot 02765x^2 - \cdot 00032x^3$ $\Big\{$ Unit of $x = 1$ child.

ii', $y = 7 \cdot 20525 - \cdot 15409x - \cdot 00816x^2 + \cdot 00135x^3$ $\Big\{$ Origin at 8 children. Unit of $x = 1$ child.

be prejudicial to longevity at all, it must be only those of exceptional physique who have attained the ages of 60 and 75 years, and the more satisfactory method of determining the correlation between fertility and longevity would be to take into account the deferred expectation beyond those ages by survivors of age 46. (It may be remarked that the expectation here referred to is not the actuarial expectation of life—but of that hereafter.)   This expectation has been calculated for mothers of each sized family, and the results are shown in Table IX.

<div align="center">TABLE IX.</div>

<div align="center">*Average Duration of Life by Mothers Surviving Age 46.*</div>

Number of children in Family	Duration beyond 60 years of age	Duration beyond 75 years of age	Number of children in Family	Duration beyond 60 years of age	Duration beyond 75 years of age
0	9·53 years	2·03 years	9	10·03 years	2·32 years
1	9·61 ,,	2·28 ,,	10	9·61 ,,	2·06 ,,
2	10·47 ,,	2·56 ,,	11	9·07 ,,	1·65 ,,
3	10·04 ,,	2·32 ,,	12	9·70 ,,	1·91 ,,
4	10·49 ,,	2·53 ,,	13	9·13 ,,	1·57 ,,
5	10·57 ,,	2·50 ,,	14	10·30 ,,	2·28 ,,
6	10·56 ,,	2·48 ,,	15	8·46 ,,	1·09 ,,
7	9·75 ,,	2·08 ,,	16	8·43 ,,	1·41 ,,
8	10·26 ,,	2·41 ,,	17	6·25 ,,	0·96 ,,

These results were obtained by dividing the total life beyond 60 and beyond 75 years of age by the number of mothers who had survived their forty-sixth year and adding a half year for the year of death.   Taking advantage of the knowledge derived from the parabolas of the preceding investigation that the maximum duration of life was about six children, these results were broken up at that number and the correlation and regression straight lines were determined at each period as from 0 to 6 children and from 6 to 17 children, with the results set forth in Tables X. and XI.

<div align="center">TABLE X.</div>

<div align="center">*General Results.*</div>

Period of Life	Size of Family	Mean Duration of Life (years)	Mean Size of Family	Standard Deviation		Correlation. Duration of Life and Fertility	Regression
				Duration of Life (Unit = 1 year)	Size of Family		
After 60 years	0 to 6 children	10·1636	2·9840	·44058	2·1561	+ ·89004 ± ·00194	+ ·18187
,, ,,	6 to 17 ,,	9·8640	8·9946	·24559	2·2957	− ·68479 ± ·00467	− ·14782
After 45 years	0 to 6 ,,	2·3647	2·9840	·19396	2·1561	+ ·64963 ± ·00539	+ ·05844
,, ,,	6 to 17 ,,	2·1366	8·9946	·30305	2·2957	− ·74144 ± ·00396	− ·09788

TABLE XI.

Table of Regression Formulæ.

$y =$ Duration of Life ; $x =$ Size of Family.

(1) Life after 60 years of age, Mothers of 0 to 6 children, $y = 9\cdot6209 + \cdot18187x$
(2) „ „ „ „ „ 6 to 17 „ $y = 11\cdot1936 - \cdot14782x$
(3) „ 75 „ „ „ 0 to 6 „ $y = 2\cdot1903 + \cdot05844x$
(4) „ „ „ „ „ 6 to 17 „ $y = 3\cdot0170 - \cdot09788x.$

Origin at 0 children.   Unit of $x = 1$ child.

Cubic parabolas were also determined for these two series, and were found to be—the origin of both curves being at $8\cdot5$ children—

For life after 60 years of age

$$y = 10\cdot465864 - \cdot099753\,x - \cdot033096\,x^2 - \cdot000352\,x^3.$$

For life after 75 years of age

$$y = 2\cdot245851 - \cdot089609\,x - \cdot007900\,x^2 + \cdot000422\,x^3.$$

The theoretical maxima were found to be at $6\cdot955$ and $5\cdot268$ children respectively. The statistics, regression straight lines and curves are plotted on Fig. 4.

Fig. 4.   Number of Offspring and Average Duration of Life of Mothers surviving Age 46.

A, Beyond 60 years of Age.

B, „ 75 „
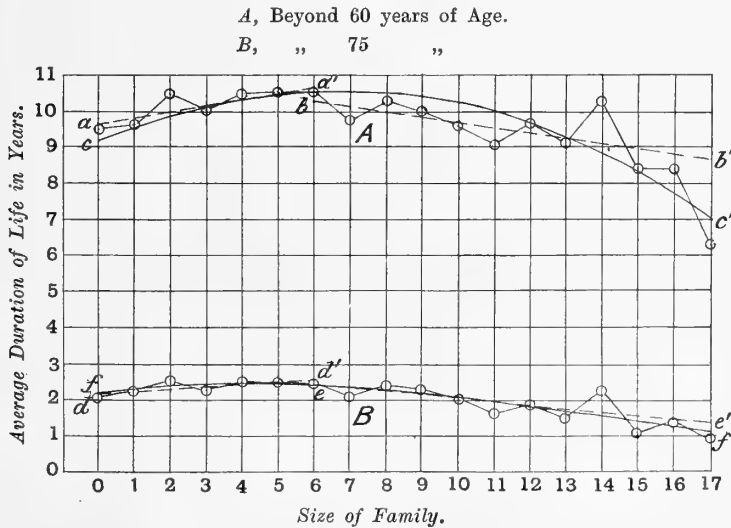
Equations to Regression Straight Lines :

aa', $y = 9\cdot6209 + \cdot18187x$
bb', $y = 11\cdot1936 - \cdot14782x$ Origin at 0 children.
dd', $y = 2\cdot1903 + \cdot05844x$ Unit of $x = 1$ child.
ee', $y = 3\cdot0170 - \cdot09788x$

Equations to Cubic Parabolas :

cc', $y = 10\cdot465864 - \cdot099753x - \cdot033096x^2 - \cdot000352x^3$ Origin at 8·5 children.
ff , $y = 2\cdot245851 - \cdot089609x - \cdot007900x^2 + \cdot000422x^3$ Unit of $x = 1$ child.

In these deferred expectations the correlation is very high and within very small limits of error, and again it is positive in both series for mothers of small families and negative for mothers of large families. We may therefore safely conclude that the advantage in longevity is distinctly in favour of mothers of moderate sized families of about six children, and that in New South Wales at all events, maximum fertility is extremely unfavourable to extreme longevity. I think the reasons are sufficiently obvious—and what applies to New South Wales must apply with equal force elsewhere—the incessant strain upon the physique of women who bear large families during the periods of gestation, parturition and lactation must be very prejudicial to longevity, whilst the mental strain involved in the rearing of such families cannot be regarded as other than detrimental. But we are also presented with the fact that childless women and mothers of extremely small families have shorter expectation of life than mothers of moderate sized families. This I consider must arise partly from the inferior physique of unprolific women generally, and partly—perhaps chiefly—in consequence of the prejudicial effects of the efforts to prevent and to limit families. But whatever may be the explanation, the shortened life thus occasioned is by no means equal to that occasioned by unrestrained excessive fertility. In New South Wales there is but little Malthusian restraint when compared with the old countries of the world—notwithstanding that the Government of that State, alarmed at the decline in the birthrate, appointed a Royal Commission to investigate the causes operating to produce such decline. In its Report, relying upon medical evidence and on statistics which I think were wrongly interpreted, it declared that there had been a serious decline and that this was chiefly to be attributed to artificial restraint. That there has been a decline in the birthrate, whether measured as per 1000 of the population, or per 1000 married women of the reproductive period, or in age groups within that period, is obvious from the figures produced in evidence, but it was entirely overlooked when dealing with the latter method—the only correct one—that there had been a considerable change in the physique of the population. In the earlier days the community consisted chiefly of specially selected immigrants drawn for the most part from the most prolific classes of the British community, viz. the agricultural and artisan. Now that immigration has practically ceased, the birthrate has declined from an abnormal condition due to a differentiated community to that of a more normal population. That there is restriction to some extent amongst certain classes of the community is doubtless true—although it is equally probable that this has always been the case—but, as we shall subsequently see in another part of this paper*, the extent of this practice is very limited.

It was previously remarked that the expectation or mean duration of life used throughout the investigation should not be mistaken for the actuarial expectation of life at the ages referred to. Thus according to the English Registrar-General's life table†, the expectation of females aged 46 is 23·34 years,

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* See "Distribution of Fertility," p. 247.

† Supplement to *Fifty-fifth Annual Report*, Part i. pp. xviii. and xix.

so that the expectation beyond 45 of those surviving age 46 is 24·34 years; the expectation at age 60 is 14·10 and at age 75, 6·68 years*. The complete expectations at similar ages computed by the present writer from the New South Wales census experience of 1891 are 26·17, 14·37, and 7·02 years respectively. The average durations of life determined from the experience of married women in the present investigations are 23·05, 13·82, and 7·22 years respectively. It is seen that at the first two ages the averages for married women are less than the expectations according to both the English and New South Wales life tables for the general population of females, although in excess at the later period. At first sight it might appear that the married condition was somewhat unfavourable to longevity; but this is not so. We are able to compare, not the actuarial expectation, but the mean duration of life of married and celibate men and women upon exactly the same basis. The ages at death of each sex are tabulated in quinquennial age groups, and by subtracting the married, the numbers of bachelors and spinsters have been obtained as shown in Table XII.

TABLE XII.

Number of Deaths of each Sex, Single and Married.

Age at Death	Deaths of Females (1898 to 1902)			Deaths of Males (1901 and 1902)		
	All Females	Married	Single	All Males	Married	Single
45 and under 50	1181	1050	131	939	616	323
50 „ 55	1093	972	121	851	588	263
55 „ 60	1218	1126	92	931	652	279
60 „ 65	1484	1362	122	1083	750	333
65 „ 70	1609	1491	118	1382	937	445
70 „ 75	1513	1423	90	1276	871	405
75 „ 80	1356	1281	75	830	606	224
80 „ 85	1132	1080	52	589	433	156
85 „ 90	579	551	28	310	271	39
90 „ 95	219	206	13	129	98	31
95 „ 100	69	65	4	41	33	8
100 „ 105	14	14	—	13	9	4
Totals ...	11,467	10,621	846	8374	5864	2510

Note:—The numbers given in the columns headed "Married" do not agree with those given in Tables I. and II., as in this table are included those mothers whose ages at death were known but the numbers of whose offspring were unknown. The latter were of course excluded from Tables I. and II.

From the data in this table the average durations of married and celibate life were determined, the central age of each age group being adopted as the mean age of that group. The results for females were as follows†:

* I cannot tell from the table referred to whether the expectations given are complete or curtate.

† It will be noticed that the averages determined by the adoption of central ages of a quinquennial group differ but slightly from those determined by the adoption of single years.

Average duration of life after 45 years of age :

                    Married Women   22·60 years.
                    Single   do.   18·98   „

Average duration of life after 60 years of age :

                    Married Women   13·91 years.
                    Single   do.   12·23   „

Average duration of life after 75 years of age :

                    Married Women   7·40 years.
                    Single   do.   7·24   „

At each period there is exhibited a sensible superiority of the married over the unmarried state, and this difference being undoubtedly real, the natural inference is that the exercise of the functions incidental to the married state is conducive to longevity—at all events after the reproductive period. It can hardly be urged that an unmarried woman who survives the age of 45 suffers generally from any constitutional infirmity which, from motives of prudence or otherwise, deterred her from contracting marriage or which physically incapacitated her. The greatest enemy of mankind—phthisis—has at that age practically exhausted itself. A life at that age may be regarded as a selected life, but more and more undoubtedly so at ages 60 and 75. For males the results were * :

Average duration of life after 45 years of age :

                    Married Men   21·59 years.
                    Single   do.   19·98   „

Average duration of life after 60 years of age :

                    Married Men   13·07 years.
                    Single   do.   11·68   „

Average duration of life after 75 years of age :

                    Married Men   7·49 years.
                    Single   do.   6·60   „

Again we find the differences consistently in favour of the married state—not so pronounced at the first two periods as amongst females, but somewhat more so at the latest. In view of the results obtained for both sexes, we are warranted in concluding that marriage, at all events so far as longevity is concerned, is not a failure.

It may be remarked that the material we have been dealing with may be said to be homogeneous—the deaths are those of persons almost entirely of British stock or origin. Of the 11,396 deaths of females which occurred during the years 1898 to 1902 aged 45 and upwards, 2572 or 22·5 per cent. were of Australasian birth; 8406 or 73·8 per cent. were born in the United Kingdom, whilst only 418 or 3·7

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* It will be noticed that the averages determined by the adoption of central ages of a quinquennial group differ but slightly from those determined by the adoption of single years.

per cent. were born elsewhere. Of the 6457 who died at age 65 or over these figures are 665 or 10·3 per cent., 5545 or 85·9 per cent., and 247 or 3·8 per cent. respectively.

From the results of this section of the investigation we are amply justified in concluding:

(*a*) That in females extreme fecundity is unfavourable to extreme longevity.

(*b*) That mothers of moderate sized families of about six live on the average longer than those with smaller or larger families.

(*c*) That although in males increased fertility is apparently associated with increased longevity throughout the whole of life, such association in part at least is due to second and subsequent marriages.

(*d*) That the married of both sexes have a greater expectation of life than the unmarried, and that this superiority is not all due to physical infirmities which prevented marriage.

B. *Distribution of Fertility.*

Professor Pearson, in his essay "Reproductive Selection*," has dealt with this subject rather exhaustively, using the data derived from certain sections of the Anglo-Saxon community and from the upper and middle classes of the city of Copenhagen tabulated by Messrs Rubin and Westergaard. The statistics now to be presented are for the whole state of New South Wales—urban and rural— and will exhibit the distribution of fertility under varying conditions of ages of women and durations of marriage. Taking first of all the women used in the last investigation, i.e. those who died beyond the reproductive period irrespective of the age at marriage or of the duration of marriage, the distribution is as shown in Table XIII.

From this table we see that nearly 11 per cent. of the women were sterile, and it would appear from Fig. 5 (broken lines) that this is unduly high. The excess might at first sight be attributed to absolute prevention of family, but as these statistics relate to married women dying beyond the reproductive period, they comprise widows—whether widowed before or after the reproductive period— and of women who may have married late in that period and whose fertility had but little opportunity to display itself†, and of women who may have married beyond that critical period and consequently no opportunity whatever. Therefore although this percentage of barren women agrees fairly closely with that obtained by Rubin and Westergaard for marriages of duration of 15 years and upwards with both husband and wife alive, viz. 12·87, this ratio is distinctly too high for New South Wales. For marriages under the last-mentioned conditions, as we shall subsequently see, the barren marriages are but 5·563 per cent., which

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* *Chances of Death*, pp. 63 *et seq.*

† Vide *Biometrika*, Part I. Vol. I. p. 33, see curve showing initial fertility in deferred marriages.

TABLE XIII.

Distribution of Fertility. Women dying beyond the Reproductive Period.
Experience of New South Wales.

Size of Family	Deaths of Married Women	Per Cent. of Total	Sum of Percentages	Number of Children	Per Cent. of Total	Sum of Percentages
0	1110	10·802	10·802	—	—	—
1	533	5·187	15·989	533	·837	·837
2	581	5·654	21·643	1162	1·826	2·663
3	644	6·267	27·910	1932	3·035	5·698
4	702	6·831	34·741	2808	4·411	10·109
5	813	7·912	42·653	4065	6·230	16·339
6	855	8·320	50·973	5130	8·216	24·555
7	976	9·498	60·471	6832	10·733	35·288
8	963	9·371	69·842	7704	12·103	47·391
9	847	8·243	78·085	7623	11·976	59·367
10	786	7·649	85·734	7860	12·348	71·715
11	568	5·527	91·261	6248	9·816	81·531
12	422	4·107	95·368	5064	7·956	89·487
13	226	2·199	97·567	2938	4·615	94·102
14	129	1·255	98·822	1806	2·838	96·940
15	57	·555	99·377	855	1·343	98·283
16	39	·380	99·757	624	·980	99·263
17	12	·116	99·873	204	·320	99·583
18	5	·049	99·922	90	·142	99·725
19	2	·020	99·942	38	·060	99·785
20	2	·019	99·961	40	·063	99·848
21	1	·010	99·971	21	·033	99·881
22	1	·010	99·981	22	·034	99·915
24	1	·010	99·991	24	·038	99·953
30	1	·009	100·000	30	·047	100·000
Totals	10276	100·000	—	63653	100·000	—
Median	5·8831			8·2179		
Mean Family = 6·1943 children						

agrees remarkably close with that computed by Professor Pearson for the Danish statistics, viz. 5·717. This is of course for all women at whatever age married, but whose marriage had existed 15 years or more at the time of the New South Wales census of 1901. From the returns of that census I have been enabled to compile not only the durations of marriage but the ages of the women at the time of the census together with the number of childless women. The results are shown in Table XIV.

This table shows that under suitable conditions of age at marriage and duration of marriage, the naturally sterile women do not exceed $2\frac{1}{2}$ per cent. of the total, and even of that small percentage some part is probably due to impotent husbands. We see also from this table that barrenness increases with the age at marriage,

Fig. 5. Distribution of Fertility. Married Women, New South Wales.

Broken Lines—Deaths over 46 years of Age.
Firm Lines—All married Women at Census.

Size of Family.
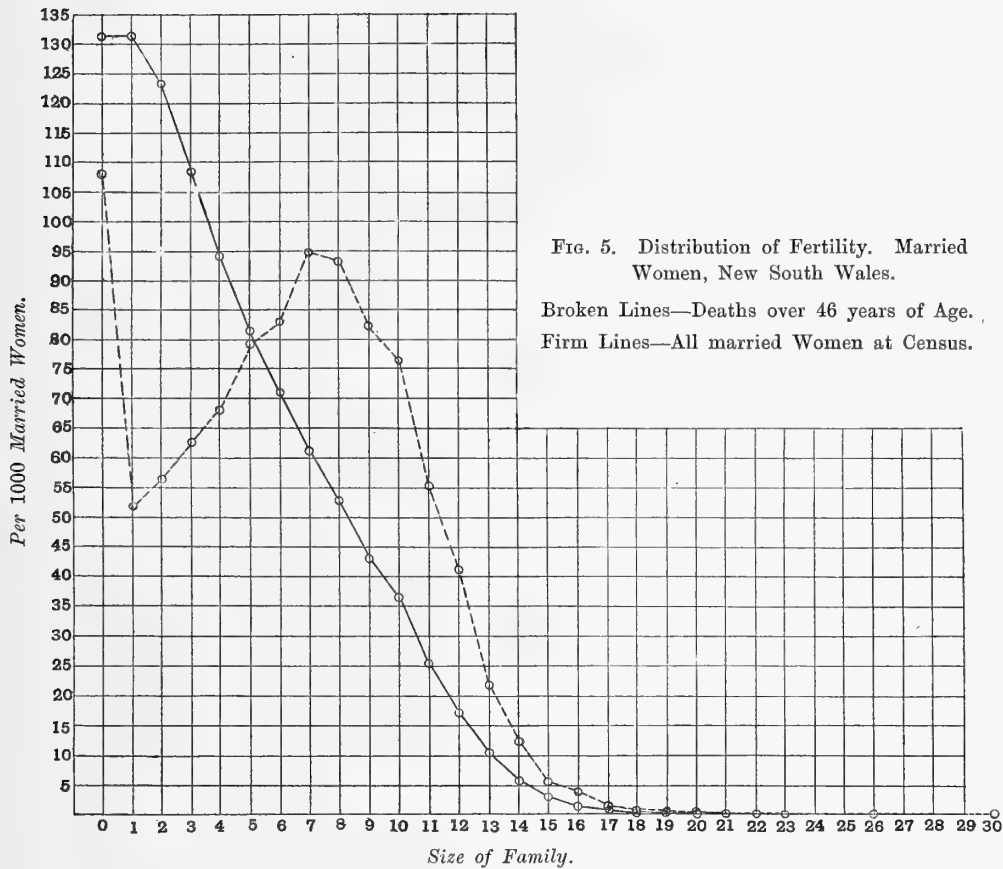
TABLE XIV.

Age of Wife at Census 1901	Duration of Marriage at Census 1901								
	10 Years and Over			15 Years and Over			20 Years and Over		
	Total Wives	Without Issue		Total Wives	Without Issue		Total Wives	Without Issue	
		Number	Per Cent.		Number	Per Cent.		Number	Per Cent.
20 and under 25	6	1	16·67	—	—	—	—	—	—
25 ,, 30	3712	119	3·21	5	—	—	—	—	—
30 ,, 35	18000	726	4·03	3438	89	2·59	7	—	—
35 ,, 40	26520	1281	4·83	16458	536	3·26	2799	68	2·43
40 ,, 45	23904	1183	4·95	20681	740	3·58	12672	309	2·44
45 ,, 50	17169	1165	6·79	16137	878	5·44	13771	598	5·98
50 ,, 55	35294	3503	9·93	34338	2818	8·21	32848	2127	6·47
Totals ...	124605	7978	6·40	91057	5061	5·56	62097	3102	5·00

and that for the same age at marriage it decreases with its duration. Thus of women who married at age under 20, and whose duration of marriage was 10 years and upwards, 3·21 per cent. were barren; of those whose duration was 15 years and upwards, 2·59 per cent., whilst for a duration of 20 years and upwards the percentage was only 2·43. These differences are I think really due to first births occurring from 10 to 15 and from 15 to 20 years after marriage, rather than to increased restraint in the more modern marriages, for on reference to the "Vital Statistics" of New South Wales for the 10 years 1893—1902, I find that the average annual first births from 10 to 15 years after marriage were 24, from 15 to 20 years after marriage 5, and from 20 to 25 years ·5. When the number of marriages and the proportion of those marrying at such ages as to be able to bear children at the intervals referred to are considered, as also the ruptured marriages caused by the death of husband or wife, there is little or nothing left to be accounted for by restraint in the more modern marriages. It seems safe to assume then that the naturally sterile women cannot exceed 2½ per cent. So far as I am aware these statistics are the first from which an accurate determination of the value of this ratio can be made. Again referring to Table XIII., it is seen that the condition of things existing after the close of the reproductive period is that the average family is 6·19 children, the median 5·88, the mode 7, and that 28·36 per cent. of such women produce 50 per cent. of the children. If, however, the married women at all ages at death be taken, it is found that the average family is 5·31 children, the median 4·55, and that 25·61 per cent. of the women produce 50 per cent. of the children. This latter result—based on what I regard as the true method, being on deaths at all ages—is almost in exact accordance with Prof. Pearson's conclusion from the Anglo-Saxon and Danish statistics, viz. " that one quarter of the popula-

TABLE XV.

Distribution of Fertility.   Duration of Marriage 15 years and upwards.
Experience of New South Wales Census 1901.

Size of Family	Families		Size of Family	Families		Size of Family	Families	
	Number	Per Cent.		Number	Per Cent.		Number	Per Cent.
0	5074	5·563	*9*	8420	9·231	*18*	67	·073
1	3309	3·628	*10*	7260	7·959	*19*	32	·035
2	4079	4·472	*11*	5076	5·565	*20*	10	·011
3	5416	5·938	*12*	3550	3·892	*21*	9	·010
4	6909	7·575	*13*	2078	2·278	*22*	4	·004
5	8273	9·070	*14*	1171	1·284	*23*	3	·003
6	9551	10·471	*15*	591	·648	*26*	1	·001
7	10022	10·987	*16*	283	·310			
8	9907	10·861	*17*	119	·131	Totals	91214	100·000

tion produce one half of the next generation*." This additional corroboration affords strong evidence of its universal application.

Next taking the marriages, the duration of which is 15 years and upwards, the distribution of fertility is as shown in Table XV.

A comparison of these statistics with the similar statistics of Messrs Rubin and Westergaard's quoted by Prof. Pearson will show an immense superiority of fertility in favour of the New South Wales woman, but it must be borne in mind that the Danish statistics refer only to the professional and middle classes of the city of Copenhagen, and which necessarily exclude the most prolific sections (at least it is so in New South Wales, as will be subsequently shown in the section "Reproductive Selection") of the community, viz. the artisan and the agricultural and pastoral. Here we find 5·56 of the marriages are childless, and as this appears unduly high I have, in calculating the fertility curve, altered the value to 2 per cent., and the computed curve justified the alteration. The following were found to be the constants:

$\mu_2 =$	11·32501	Mean Family	=	6·95103
$\mu_3 =$	4·88272	Modal Family	=	5·68098
$\mu_4 =$	350·82375	Median Family	=	8·03020 (Observed)
$\beta_1 =$	·016414	Range	=	29·51447
$\beta_2 =$	2·735343	Standard Deviation =		3·3653
$\epsilon =$	77·7792	y_0		= 109·9507

and the equation to the curve of fertility

$$y = 109 \cdot 9507 \left(1 + \frac{x}{12 \cdot 3500}\right)^{6 \cdot 6225} \left(1 - \frac{x}{17 \cdot 1645}\right)^{9 \cdot 2039}.$$

In this series it is found that 31·16 of the women produce one half of the children. The statistics and curve will be found plotted on Fig. 6.

On comparing the constants of this curve with those of the Danish or Anglo-Saxon curves, it is seen that we are dealing either with a naturally more prolific community or one in which there is little resort to Malthusian restraint; for although the artisan section of the community is not included in the former statistics, yet the mean family of the New South Wales woman married 15 years and upwards, viz. 6·951, is far in excess of the mean Danish artisan family for marriages of 25 years and upwards duration, viz. 5·26 †. As the Scandinavian race is unquestionably a very prolific race, and the Australian mothers are, as shown in Section A, of essentially British stock—which, as Prof. Pearson remarks ‡, is markedly less fertile than the Danish—it must be concluded that but little Malthusian restraint is practised in New South Wales: yet, as previously stated, the Government of that State deemed it necessary to appoint a Royal Commission to enquire into the causes of the so-called decline in the birthrate. That Com-

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* *Chances of Death*, p. 92.

† *Loc. cit.* p. 98.   ‡ *Loc. cit.* p. 89.

mission concluded "That there is a diminution of fecundity and fertility in recent years which is due to:

(*a*) Deliberate prevention of conception and destruction of embryonic life;

(*b*) Pathological causes consequent upon the means used and the practices involved therein;

"and from the evidence of medical witnesses, police officers and others, we are led to the conclusion that:

(*a*) The practice of preventing conception by artificial means is common among all classes of the community and in all parts of the State;

(*b*) This practice has greatly increased during the last 15 years; and

(*c*) The extension of the deliberate and intentional avoidance of procreation by means used to prevent conception is a main factor in producing the decline in birthrate, which we have ascertained to have taken place in New South Wales since the year 1888."

Fig. 6. Curve of Fertility. Duration of Marriage 15 years and upwards. Experience of New South Wales Census 1901.

Number of Children in Family.
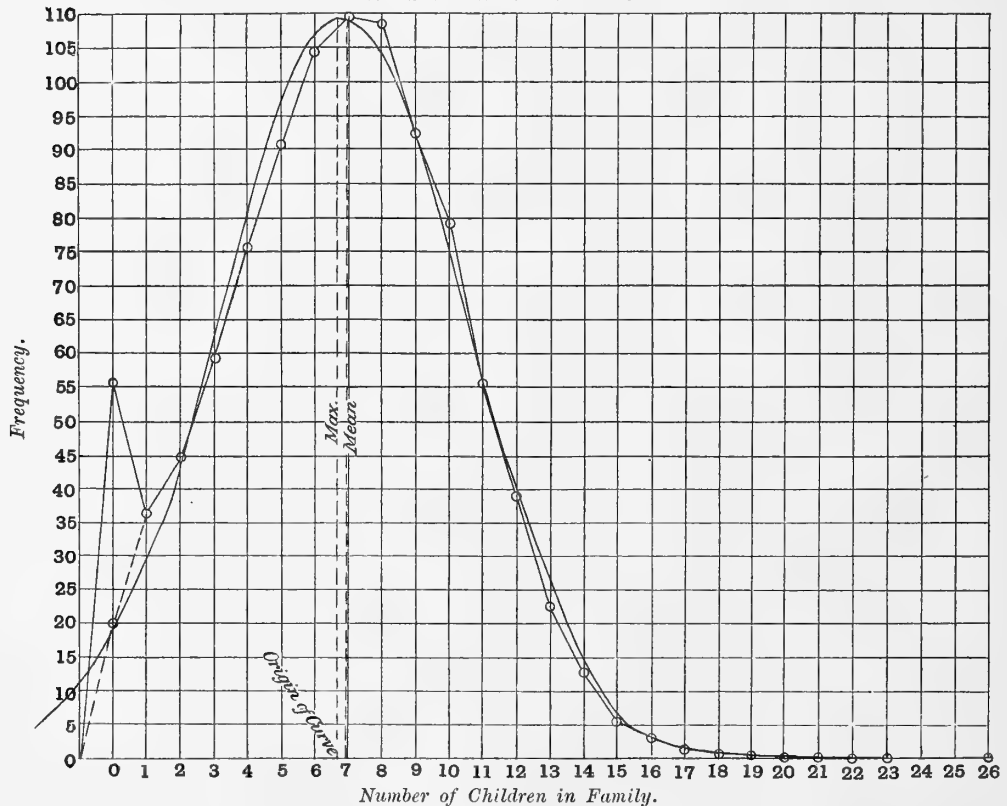
Equation to Curve $y = 109 \cdot 9507 \left(1 + \dfrac{x}{12 \cdot 3500}\right)^{6 \cdot 6225} \left(1 - \dfrac{x}{17 \cdot 1645}\right)^{9 \cdot 2039}$.

Mean Family = 6·95103.
Modal   ,,   = 6·68020.

It is not my intention to deal with this subject from the ethical standpoint, but a thorough statistical analysis compels me to disagree not only with the conclusions of the Commission but the unscientific reasons in support thereof. It is not necessary here to traverse the evidence given, but it will suffice to produce the actual facts as recorded at the census. It is clearly seen from the distribution of fertility as revealed in Table XV. that we are dealing with a naturally most fertile race. It may be reasonably argued that the distribution shown in that table refers only to marriages which were contracted at least fifteen years ago, and not to present marriages. But we can well judge of the effect of the alleged fifteen years' excessive restriction if we examine the women who were married from 15 to 20 years at the time of the census and who were then under 40 years of age. The results I find to be as follow :

Size of Family	Number of Families	Size of Family	Number of Families
0	553	6	2662
1	637	7	2570
2	948	8	2063
3	1402	9	1125
4	1967	10	539
5	2235	11 and over*	369

Here it is seen that (allowing only 11 children to the 369 women who had 11 and over) the mean family is 5·61 children and only 3·24 per cent. who had so far proved sterile, and consequently these figures utterly refute the Commission's conclusions. In view of these figures, we can assert that there cannot have been much restraint practised in New South Wales during the last fifteen years. I wish to emphasise this point in order that it may be recognised that the curves of fertility for various durations of marriage in New South Wales are as nearly as possible those free from restraint. That restraint is practised by certain sections of the community I am well aware, as also that it always has been so in some form or other, but in view of the figures which have been produced and those to be produced, I think it is altogether erroneous to assert that the practice is common to all sections of the community and to all parts of the State.

Now let us ascertain the distribution for durations of marriage of less than 15 years. I have taken those women who were under 20 years of age at marriage, and have divided the duration of marriage into three periods—from 4 to 5 years, from 5 to 10 years, and from 10 to 15 years—the ages at the census being under 30 for the first two, and under 35 for the latter period. The statistics and percentages are shown in Table XVI.

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* The returns do not show the distribution beyond 10 children.

TABLE XVI.

Duration of Marriage and Number of Offspring. Women's Age at Marriage under 25. Experience of New South Wales Census 1901.

Number of Offspring	Duration 4 to 5 Years		Duration 5 to 10 Years		Duration 10 to 15 Years	
	Married Women	Percentage	Married Women	Percentage	Married Women	Percentage
0	541	9·47	1094	6·05	757	4·14
1	1386	24·27	2230	12·33	1039	5·68
2	2731	47·81	4811	26·60	1874	10·25
3	954	16·70	5567	30·78	2931	16·03
4	96	1·68	3124	17·27	3603	19·71
5	4	·07	996	5·51	3715	20·32
6	—	—	226	1·25	2542	13·91
7	—	—	36	·20	1233	6·75
8	—	—	2	·01	422	2·31
9	—	—	—	—	118	·65
10	—	—	—	—	38	·21
11	—	—	—	—	7	·04
Totals	5712	100·00	18086	100·00	18279	100·00

These percentages have been plotted on Fig. 7, and the fertility curves computed. The constants were :

Duration of Marriage.

	4 to 5 years	5 to 10 years	10 to 15 years
μ_2	·72604	1·6271	3·6399
μ_3	− ·11092	+ ·1283	− ·5179
μ_4	1·51477	7·7554	36·6377
β_1	·032146	·003821	·005562
β_2	2·873581	2·929381	2·765345
Range	10·0683	22·5456	18·2893
Mean Family	1·7706	2·6348	4·1229
Modal Family	1·8631	2·5932	4·2086
Standard Deviation	·8521	1·2756	1·9078
y_0	45·9164	30·9904	20·2118

It is worthy of notice that for marriages of an average duration of 4·5 years the mean family is 1·77, and that for an average duration of 7·5 years it is 2·63— the difference of 3 years' duration being responsible for ·86 child or ·288 annually. The difference in mean families for marriages of $7\frac{1}{2}$ and $12\frac{1}{2}$ years' mean duration is 1·488 child or ·298 annually. It is thus seen that the average annual rate of increase is very nearly uniform, viz. about 290 or 300 children per 1000 married women. It would thus appear that the initial rate of fertility is maintained at all events for 15 years of marriage by women marrying at under 25 years of age. Whilst such a rate as that prevails in a community I do not think

FIG. 7. Distribution of Fertility amongst Women—Married at Age Under 25. Experience of New South Wales Census 1901.

A, Duration of Marriage 4 to 5 Years.
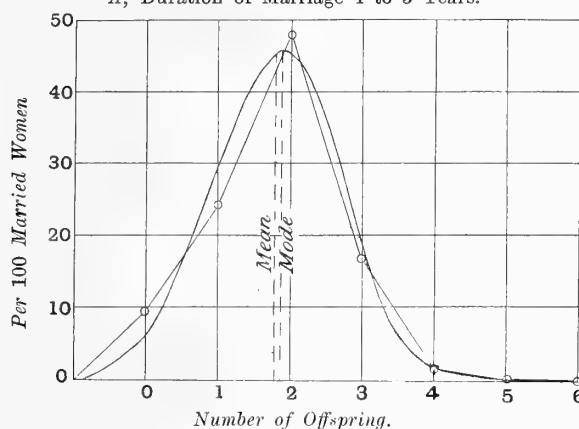
Mean at 1·7706 Children.
Mode at 1·8631 Children (Origin of Curve).

Equation to Curve $y = 45 \cdot 9164 \left(1 + \dfrac{x}{6 \cdot 40540}\right)^{18 \cdot 85225} \left(1 - \dfrac{x}{3 \cdot 66295}\right)^{10 \cdot 78065}$.

B, Duration of Marriage 5 to 10 Years.
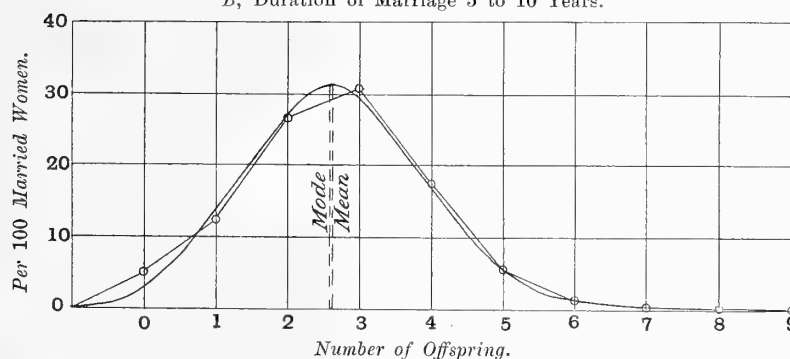
Mean at 2·6348 Children.
Mode at 2·5932 Children (Origin of Curve).

Equation to Curve $y = 30 \cdot 9904 \left(1 + \dfrac{x}{9 \cdot 7003}\right)^{31 \cdot 6924} \left(1 - \dfrac{x}{12 \cdot 8453}\right)^{41 \cdot 9672}$.

C, Duration of Marriage 10 to 15 years.
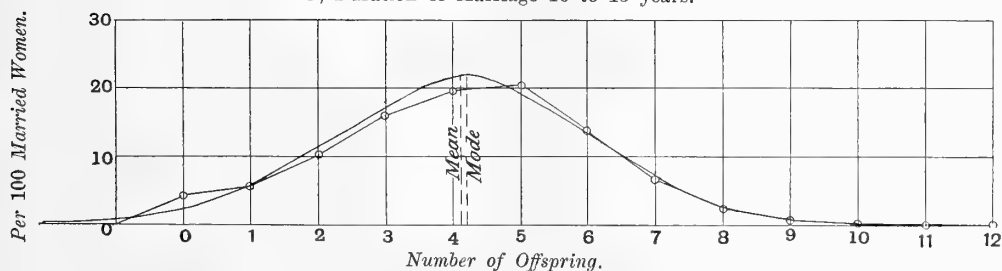
Mean at 4·1229 Children.
Mode at 4·2086 Children (Origin of Curve).

Equation to Curve $y = 20 \cdot 2118 \left(1 + \dfrac{x}{10 \cdot 0752}\right)^{10 \cdot 86664} \left(1 - \dfrac{x}{8 \cdot 2141}\right)^{8 \cdot 85926}$.

it can be seriously held that there has been very much Malthusian restraint; especially is this so when it is borne in mind that the potential embryonic life has not of course been taken into account. It may therefore I think be held that the distributions of fertility as set forth in Tables XV. and XVI. are as nearly as possible those of a non-restricting population.

C. *On the Correlation between Age at Marriage and Duration of Life beyond Age* 46.

Although in Section A of this investigation we found that the number of offspring born to a woman is highly correlated with the duration of life beyond the reproductive period, irrespective of the age at marriage, yet it is possible that the fact that maximum fertility is not favourable to extreme longevity may be due to the generally early contraction of marriages, which is of course generally productive of large families—before the proper development of the reproductive organs. The period of mature or maximum development has been shown elsewhere to be about age 25*. Although it is clear that early marriage has not the effect of arresting development (perhaps it tends to too rapid development), this may be accomplished to the subsequent prejudice of life. Had it been possible to have examined each age at marriage with each sized family and subsequent life incidental to each sized family, this could readily have been ascertained, but the voluminous statistics involved present an almost insuperable obstacle to its accomplishment. However, statistics are published in the "Vital Statistics" of New South Wales for the two years 1901 and 1902 which show the age at marriage, the number of offspring, and the duration of life after marriage, but we are unable to connect mothers of each sized family with the duration of marriage. One table in the "Vital Statistics" shows the ages at marriage in quinquennial groupings, together with the durations of marriage by single years and the total offspring. In order to obtain from this table the average number of children born to women marrying in each quinquennial group, and who survived the reproductive period, I have taken the earliest duration such that if added to the latest year of such group the sum would be 49. Thus for age at marriage under 20 the duration of marriage was 29 years and upwards; for age at marriage 20—25 years the duration was 24 years and upwards; and so on. From the table referred to I have compiled the information shown in columns 2, 3 and 4 in Table XVII. Another table in the "Vital Statistics" shows the ages at marriage in single years and the duration of marriage also in single years, but not the number of offspring, and from this table I have computed for each single year of age at marriage the mean duration of life beyond age 46, and the results appear in Table XVIII. The means of the quinquennial groups are shown in the fifth column of Table XVII. Although the two sets of women shown in the

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* *Biometrika*, Vol. I. Part I. p. 33.

two tables are not necessarily the same, it will be noticed that there is but little difference in the numbers within the groups. It must also be noted that all the figures refer to age at first marriage, if more than one has been contracted.

TABLE XVII.

Age at Marriage	Number of Deaths	Total Children	Average Children	Average Life beyond Age 46 by Survivors of that Age
Under *20* years	1226	9892	8·06	20·66 years
20 and under *25* years	1661	11462	6·90	20·70 ,,
25 ,, *30* ,,	843	4399	5·22	23·54 ,,
30 ,, *35* ,,	326	1096	3·36	23·44 ,,
35 ,, *40* ,,	121	253	2·09	25·20 ,,
40 ,, *45* ,,	92	60	0·65	

TABLE XVIII.

Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration after Age 46	Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration after Age 46
15 years	57	1277	22·40 years	*28* years	156	3723	23·87 years
16 ,,	162	3401	20·99 ,,	*29* ,,	93	2013	21·65 ,,
17 ,,	238	4795	20·15 ,,	*30* ,,	143	3487	24·39 ,,
18 ,,	399	8203	20·56 ,,	*31* ,,	44	902	20·50 ,,
19 ,,	378	7793	20·62 ,,	*32* ,,	56	1383	24·70 ,,
20 ,,	417	8616	20·66 ,,	*33* ,,	42	915	21·79 ,,
21 ,,	395	8246	20·88 ,,	*34* ,,	39	907	23·26 ,,
22 ,,	319	6855	21·49 ,,	*35* ,,	38	928	24·42 ,,
23 ,,	272	5224	19·21 ,,	*36* ,,	29	760	26·21 ,,
24 ,,	251	5298	21·11 ,,	*37* ,,	24	593	24·71 ,,
25 ,,	269	6339	23·57 ,,	*38* ,,	21	514	24·48 ,,
26 ,,	183	4425	24·18 ,,	*39* ,,	10	279	27·90 ,,
27 ,,	132	3107	23·54 ,,				

From the statistics contained in Table XVIII. the correlation between the age at marriage and the duration of life after age 46 was determined. The following were the constants:

Mean Age at Marriage.................................. 22·6806 years

Mean Duration of Life after Age 46 .... 21·5942 ,,

Standard Deviation $\begin{cases} \text{Age at Marriage} & 4·8835 \\ \text{Duration of Life} & 1·6004 \end{cases}$,,

Correlation .... + ·70679 ± ·00523

Coefficient of Regression .... + ·23163

Regression Straight Line (Origin at Age at Marriage 14 years.

Unit of $x = 1$ year), $y = 19·5835 + ·23163\,x$.

The statistics and regression straight lines are shown on Fig. 8.

The correlation is high and within very small limits of probable error, and we are therefore justified in concluding that early marriage, whether in consequence of generally larger families (see Section A) or in consequence of immature development, is distinctly prejudicial to longevity beyond the reproductive period. If any weight is to be attached to the results of the comparatively meagre observations of the marriages contracted between ages 35 and 40, we would be

Fig. 8. Age at Marriage and Duration of Life beyond Age 46 by Survivors of that Age.

Females.
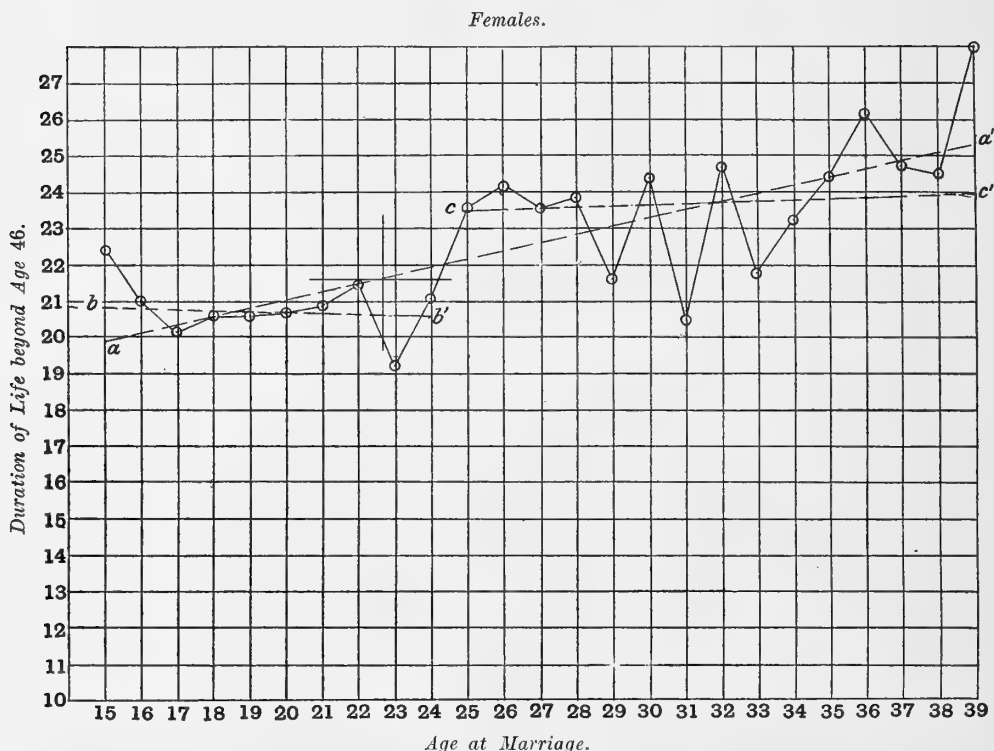
Equations to Regression Straight Lines:

aa' (Origin at 14 years of Age. Unit of $x = 1$ year)
$$y = 19 \cdot 5835 + \cdot 23163x.$$

bb' (Origin at 14 years of Age. Unit of $x = 1$ year)
$$y = 20 \cdot 8118 - \cdot 0228x.$$

cc' (Origin at 24 years of Age. Unit of $x = 1$ year)
$$y = 23 \cdot 4057 + \cdot 05629x.$$

further justified in concluding that the longer marriage is deferred the greater the probable lifetime after the reproductive period is passed. If, however, we examine Fig. 8, we see that the statistics divide into two fairly well defined sections, viz. from age 15 at marriage to age 24, and from age 25 upwards. The dividing age (24—25) is remarkable as that found to be that of maximum initial

fertility and also that of greatest physical development, at least so far as stature is concerned*. I have therefore determined the correlation for each of these sections with the following results:

	Age at Marriage	
	15 to 24 years	*25 to 39 years*
Mean Age at Marriage	20·0104 years	28·7099 years
Mean Duration of life beyond Age 46	20·6745 ,,	23·6708 ,,
Standard Deviation {Age at Marriage	2·3557 ,,	3·5277 ,,
{Duration of Life	·6221 ,,	1·0769 ,,
Correlation	− ·0865 ± ·0124	+ ·1844 ± ·01822
Coefficient of Regression ...	− ·02284	+ ·05629
Regression Straight Line ...	{Origin at Age at Marriage 14 years $y = 20·8118 − ·0228x$	{Origin at Age at Marriage 24 years $y = 23·4057 + ·05629x$

In these two series we see that the correlation is low, though sensibly within the limits of probable error. The coefficient of regression shows that there is little difference in the average duration of life beyond the reproductive period by marriage at any age under 25 years—the mean being 20·01 years; also that there is but little difference by marriage at any age between 25 and 39—the mean of which is 23·67 years. But there is a most decided advantage in longevity by deferring marriage until after age 25—a difference of $3\frac{2}{3}$ years—and as these are the results of over 4000 observations, they must be accepted as reliable and not merely as accidental. In comparing these results with those of Section A, it must be remembered that in these observations the average lifetime is curtate, i.e. only completed years have been counted—the year of death is not considered— and that the age 46 is the origin. To bring these into line, therefore, with those of Section A, $1\frac{1}{2}$ years must be added to the present results. We see from Table XVII. that the average number of offspring to marriages contracted under age 25, and in which the wife at least survives to age 46, is 7·40, and that of marriages contracted between ages 25 and 40, 4·46. If now we turn to Fig. 3 of Section A we find from the parabola that the average durations of life after age 45 to mothers of 7·40 and 4·46 children are almost identical, viz. 23·4 years, whilst (adding the corrections referred to above, viz. $1\frac{1}{2}$ years) the average durations of those who marry before and after 25 years of age are 22·2 and 25·2 years respectively. These differences I think show that the age at marriage, as well as the number of offspring, is a factor in connection with the duration of life beyond the reproductive period. During the reproductive period it is of course obvious that as each successive birth has its risk, mothers of large families imperil their lives more than those of smaller ones—not simply in proportion to the number of births, but after the first confinement in a constantly increasing ratio. From the "Vital Statistics" of New South Wales, I have been enabled to deter- mine the risk incidental to each confinement. In one table is to be found the order of confinement—first, second, etc., whilst in another are to be found the

deaths in childbirth in order of confinement. This information is available for the nine years 1894 to 1902 and, together with the crude probability of death. in each confinement, is shown in Table XIX.

<div align="center">TABLE XIX.</div>

Probability of Death in Confinement. Experience of New South Wales, 1894—1902.

Order of Confinement	Births	Deaths in Confinement	Probability of Death in Confinement	Order of Confinement	Births	Deaths in Confinement	Probability of Death in Confinement
1	65778	552	·00839	11	4698	46	·00979
2	52818	258	·00488	12	2716	34	·01252
3	43462	240	·00552	13	1427	24	·01682
4	35510	213	·00600	14	703	9	·01280
5	28920	204	·00705	15	338	6	·01775
6	23527	161	·00684	16	129 ⎫	1 ⎫	·00775 ⎫
7	18729	130	·00694	17	68 ⎬ 236	1 ⎬ 4	·01471 ⎬ ·01695
8	14659	132	·00900	18	21 ⎭	2 ⎭	·09524 ⎭
9	10580	114	·01078	Beyond 18	18 ⎰	— ⎰	— ⎰
10	7230	61	·00844				

Rejecting the statistics of the sixteenth and subsequent confinements as being too limited to yield satisfactory results, the probabilities of death were graduated by means of a parabola of the fourth order, the equation to which was found to be

$$y = 8\cdot384571 + \cdot846415\,x + \cdot040384\,x^2 - \cdot002525\,x^3 + \cdot000568\,x^4,$$

where $y =$ the probability of death, and x the number of confinement—the origin of the curve being at the eighth confinement. There are no points of inflexion, and the curve is convex throughout. The theoretical minimum occurs at the $3\cdot0954$ confinement. The statistics and curve will be found plotted on Fig. 9. Obtaining the probabilities of death from the parabola, we are enabled to ascertain the number of survivors of any given number of fertile married women at each confinement—presuming the mortality of childbirth to be the only operative cause of death. Thus adopting the symbols used in life tables*, calling x the number of confinement, q_x the probability of death in the xth confinement, p_x the probability of surviving it, and l_x, we have (taking 10,000 fertile married women):

x	q_x	p_x	l_x	x	q_x	p_x	l_x
1	·00667	·99333	9933	9	·00927	·99073	9387
2	·00604	·99396	9873	10	·01027	·98973	9291
3	·00583	·99417	9815	11	·01127	·98873	9186
4	·00595	·99405	9757	12	·01240	·98760	9072
5	·00632	·99368	9695	13	·01367	·98633	8948
6	·00691	·99309	9628	14	·01511	·98489	8831
7	·00758	·99242	9555	15	·01679	·98321	8665
8	·00838	·99162	9475				

* *Institute of Actuaries Text Book*, Part II, p. xxi.

Thus if no other causes of death were in operation, we see that no less than 134 out of every 1000 women would perish from accidents of childbirth in accomplishing the fifteenth confinement, and each subsequent confinement would be accompanied by an ever-increasing risk. It thus appears that granting that economic reasons (such as in sparsely populated territories as Australia, where there is but little more than one individual to the square mile) demand that there

Fig. 9. Probability of Death in Confinement. Experience of New South Wales, 1894—1902.
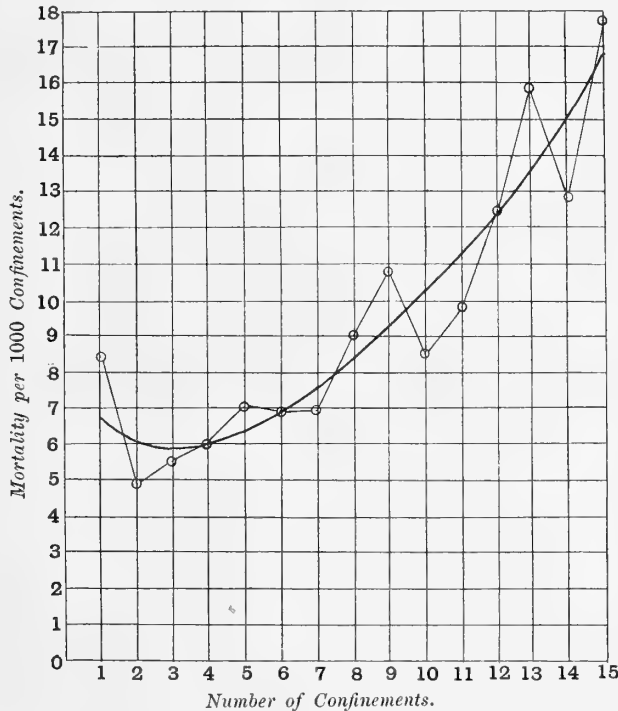
Equation to Curve (Origin at 8 Children):

$$y = 8\cdot384571 + \cdot846415x + \cdot040384x^2 - \cdot002525x^3 + \cdot000568x^4.$$

should be but little if any restriction in families, it is distinctly prejudicial to the individual, both during and beyond the reproductive period, to bear a large family. It may of course be urged that it is prejudicial within the reproductive period even to bear but one, but it will be seen from the last table that to bear a family of six the risk cannot be said to be large—over 96 per cent. of the women will survive this—whilst the reproductive period passed almost a maximum expectation is attained.

Turning now to married men marrying at various ages, I have computed from similar tables to those from which the information was obtained for married women, the mean duration of life beyond age 46 by married men surviving that age and marrying at the various ages. The age of 46 is not of course a critical age in man as it is in woman, but has been adopted so that comparison

might be effected with the duration of life in women. The results are shown in Table XX.

<div align="center">TABLE XX.</div>

Mean Duration of Life of Married Men beyond Age 46 by survivors of that age. Experience of New South Wales, 1901 and 1902.

Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration of life after Age 46	Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration of life after Age 46
20	222	4290	19·32 years	33	137	3075	22·45 years
21	342	6595	19·28 ,,	34	158	3152	19·95 ,,
22	358	7021	19·61 ,,	35	136	3407	25·05 ,,
23	336	6510	19·38 ,,	36	109	2409	22·10 ,,
24	342	6712	19·63 ,,	37	91	1937	21·29 ,,
25	410	7958	19·41 ,,	38	86	1903	22·13 ,,
26	351	6777	19·31 ,,	39	69	1390	20·14 ,,
27	300	5894	19·65 ,,	40	102	2504	24·55 ,,
28	339	6489	19·14 ,,	41	39	821	21·05 ,,
29	241	4990	20·71 ,,	42	52	957	18·40 ,,
30	285	5864	20·58 ,,	43	27	483	17·90 ,,
31	158	3223	20·40 ,,	44	35	824	23·54 ,,
32	180	4075	22·64 ,,				

The correlation between age at marriage and duration of life beyond age 46 was determined, and also the regression straight line. The following are the general results:

Mean Age at Marriage ....27·94414 years.

Mean Duration of Life beyond Age 46 ....?.......20·2365 　,,

Standard Deviation $\begin{cases} \text{Age at Marriage .... 7·1415 ,,} \\ \text{Duration of Life .... 1·4623 ,,} \end{cases}$

Correlation .... + ·51973 ± ·00703

Coefficient of Regression .... + ·10642

Regression Straight Line (Origin at Age at Marriage 19 years.
Unit of $x = 1$ year), $y = 19·2847 + ·10642\,x$.

The statistics and regression straight line will be found plotted on Fig. 10. Here we see that early marriage is again apparently unfavourable to longevity in man, though not so pronounced as in woman, as perhaps *a priori* might have been suspected. It has been suggested that two causes operate against longevity in excessively fertile women, viz. the physical strain in bearing and the mental strain in rearing large families. The latter factor only is operative in the case of males, and probably contributes to the shortening of the duration of life in men marrying at the early ages. Again we see that the statistics divide into two well-defined series, the demarcation being at age 28 at marriage. Up to and inclusive of that age the results are remarkably uniform, the mean of the mean durations of life beyond age 46 being 19·41 years. For marriages contracted beyond age 28 the results are rather irregular, but distinctly exhibit an increase

in longevity, the mean duration being 21·53 years—an advance of over 2 years—
and there can be no doubt that the increase is a real one, whatever may be the
cause. It of course may be urged, and with much force, that amongst those
following hazardous occupations, such as the industrial classes, marriage is generally

Fig. 10. Age at Marriage and Duration of Life beyond Age 46 by Survivors of that Age.

Males.

Age at Marriage.
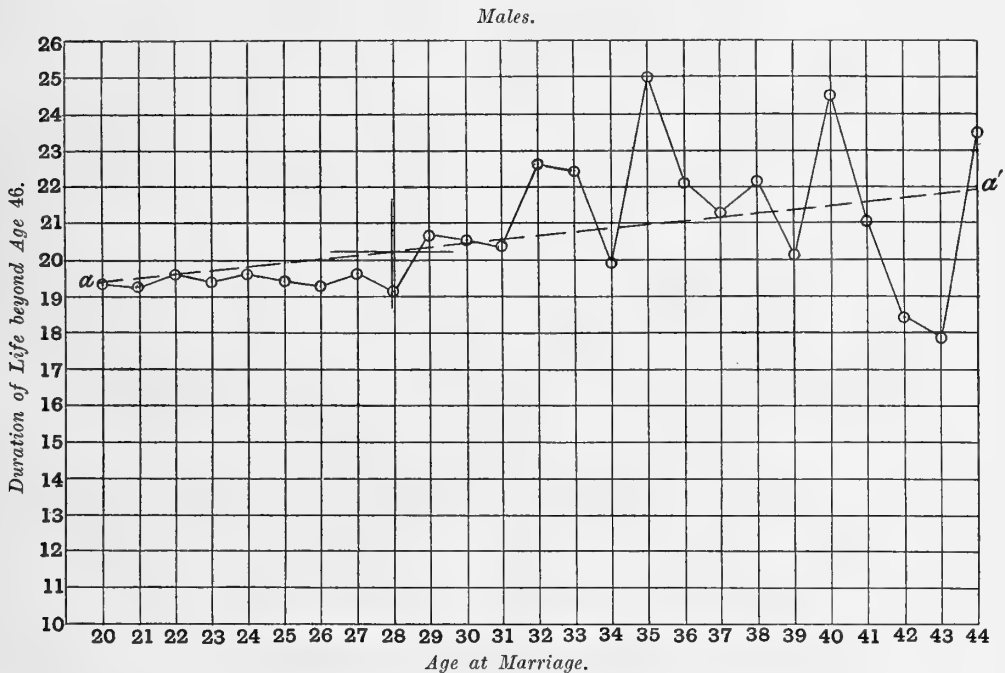
Equation to Regression Straight Line:
aa' (Origin at 19 years of Age. Unit of $x=1$ year)
$$y = 19·2847 + ·10642x.$$

contracted early in life, whilst amongst those following the least hazardous—
the professional—marriage is contracted much later. This is true, as will be
seen from Section E, though the difference in age at marriage does not exceed
$1\frac{2}{3}$ years in the two classes. The adults' death rates are also shown in the same
section, that of the professional class being 15·73 and that of the industrial 22·72
per 1000 exposed to risk. But notwithstanding this great difference in the
mortality in view of the slight difference in the age at marriage, it is doubtful
whether if all allowances be made the difference in the duration of life can
be thus accounted for. Some portion is probably due to the struggle to maintain
the large family generally connected with early marriage *.

[* It will be seen from *Biometrika*, Vol. I. p. 46 *et seq.*, that a man's prime is about 28 in the same
sense as a woman's is about 24. May not therefore the lessened expectation of life in the case of man
as in the case of woman be due to exercise of sex-functions before the prime? Further is not the
association of extreme fertility with shorter duration of life in women due in part to the fact that very
large families mean as a rule early marriages? K. P.]

D. *On the Correlation between Duration of Marriage of Females and Number of Offspring—at various ages at Marriage.*

There are two methods by which the necessary information may be obtained : first, by noting at death the date of and age at marriage and the total offspring ; second, at a census obtaining the information direct from the living married women.   In New South Wales the materials requisite for both methods are available, but the information for the first method refers to women who at the time of death were either married or widows, or who had been separated from their husbands.   This information will be found in the "Vital Statistics" of

TABLE XXI.

Deaths of Females, Age at Marriage, Duration of Life after Marriage, and Number of Offspring.

Duration of life after Marriage (years)	Age at Marriage											
	Under 20			20 and under 25			25 and under 30			30 and under 35		
	Deaths	Offspring		Deaths	Offspring		Deaths	Offspring		Deaths	Offspring	
		Total	Average		Total	Average		Total	Average		Total	Average
0	11	7	·64	26	13	·50	13	8	·62	5	4	·80
1	17	13	·76	66	53	·80	41	34	·83	10	2	·20
2	9	14	1·56	37	43	1·16	19	20	1·05	10	13	1·30
3	16	26	1·62	29	44	1·52	23	37	1·61	8	13	1·62
4	16	31	1·94	48	90	1·87	18	24	1·33	8	14	1·75
5	11	35	3·18	37	72	1·95	15	29	1·93	6	8	1·33
6	22	50	2·27	51	125	2·45	26	66	2·54	4	9	2·25
7	15	38	2·53	27	77	2·85	17	37	2·18	7	13	1·86
8	18	49	2·72	32	92	2·88	22	55	2·50	6	19	3·17
9	27	87	3·22	34	98	2·88	14	49	3·50	9	24	2·67
10	27	99	3·67	42	153	3·64	15	41	2·73	9	31	3·44
11	27	103	3·82	37	141	3·81	27	79	2·93	6	14	2·33
12	17	65	3·82	46	192	4·17	23	104	4·52	3	17	5·67
13	31	120	3·87	29	130	4·48	20	85	4·25	6	13	2·17
14	23	99	4·30	41	179	4·36	13	48	3·69	4	8	2·00
15	28	122	4·36	51	230	4·51	17	82	4·82	—	—	—
16	23	111	4·83	42	234	5·57	20	104	5·20	—	—	—
17	29	136	4·69	39	239	6·13	9	38	4·22	—	—	—
18	34	184	5·41	44	244	5·55	16	72	4·50	—	—	—
19	26	124	4·77	55	281	5·11	19	87	4·58	—	—	—
20	37	206	5·57	40	217	5·43	—	—	—	—	—	—
21	34	208	6·12	36	233	6·47	—	—	—	—	—	—
22	23	186	8·09	44	234	5·32	—	—	—	—	—	—
23	28	185	6·61	41	281	6·85	—	—	—	—	—	—
24	28	204	7·29	41	230	5·61	—	—	—	—	—	—
25	32	243	7·59	—	—	—	—	—	—	—	—	—
26	34	244	7·18	—	—	—	—	—	—	—	—	—
27	23	171	7·43	—	—	—	—	—	—	—	—	—
28	18	143	7·94	—	—	—	—	—	—	—	—	—
29	26	187	7·19	—	—	—	—	—	—	—	—	—

New South Wales for the two years 1901 and 1902, whilst that relating to married women living with their husbands at the time of the census of 1901 will be found in Part "Conjugal Condition and Families" of the Census returns. The former tables show for married women dying in those years the age at marriage (in quinquennial groups), the duration of marriage in single years, and total offspring. Limiting the information to the reproductive period, the statistics are shown in Table XXI.

The correlation and regression straight lines were determined for each of these series with the results set forth in Tables XXII. and XXIII.

TABLE XXII.

General Results.

Age at Marriage	Mean Duration of Life after Marriage (within Reproductive Period)	Mean Size of Family	Standard Deviation		Correlation. Duration of Life after Marriage and Size of Family	Regression
			Duration of Life after Marriage	Size of Family		
Under 20	16·7648 years	4·9155	7·9165 years	2·0168	·97328 ± ·00133	·24796
20 and under 25	12·6961 ,,	3·8670	7·2585 ,,	1·8226	·96554 ± ·00143	·24245
25 ,, 30	9·3217 ,,	2·8398	5·7459 ,,	1·4459	·94568 ± ·00362	·23797
30 ,, 35	6·8762 ,,	2·0000	4·1622 ,,	1·0963	·65700 ± ·03812	·17304

TABLE XXIII.

Regression Formulæ.

$x =$ Duration of Life after Marriage (within Reproductive Period). $y =$ Number of Offspring.

Age at Marriage,   Under *20* years, $y = ·7585 + ·24796x$
,,   ,,   *20* and under *25* years, $y = ·7888 + ·24245x$
,,   ,,   *25*   ,,   *30*   ,, $y = ·6215 + ·23797x$
,,   ,,   *30*   ,,   *35*   ,, $y = ·8101 + ·17304x$

Note:—Origin at marriage and unit of $x = 1$ year.

The statistics and regression straight lines will be found plotted on Figs. 11, 12, 13 and 14; the average number of offspring for each year of life after marriage is shown by small circles, and that for quinquennial periods by triangles. It is seen that in each series although the statistics are somewhat irregular they are well represented by the straight lines, and thus it would appear that the initial fertility incidental to the age at marriage is preserved almost unchanged throughout the reproductive period. It must be clearly understood that the foregoing are the results of women dying who had been married and whose husbands may or may not have survived them. Although it is seen that the initial fertility is preserved throughout the whole of the reproductive period, yet the age at marriage is the determinant of that fertility. Up to age 30 there would seem to be but little difference in this initial fertility, although it is slightly less at

Age at Marriage, Duration of Marriage and Number of Offspring.   Experience of New South Wales, Deaths of Married Women, 1901 and 1902.

Fig. 11.   Age at Marriage, Under 20 years.

Duration of Marriage.
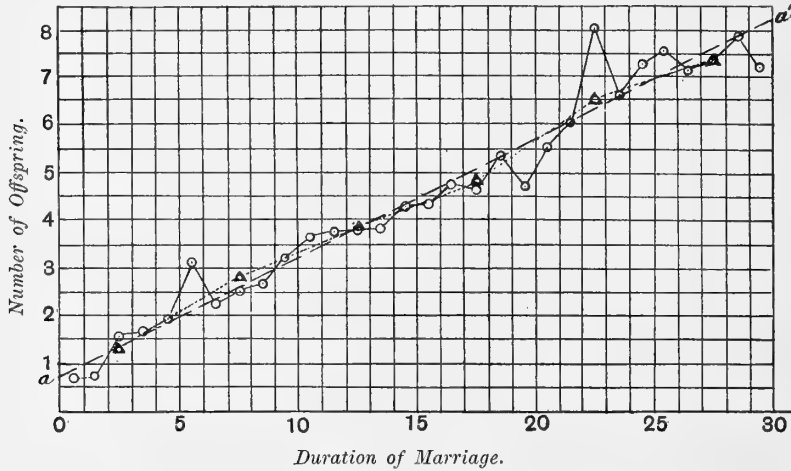
Fig. 12.   Age at Marriage, 20 to 25 years.

Duration of Marriage.
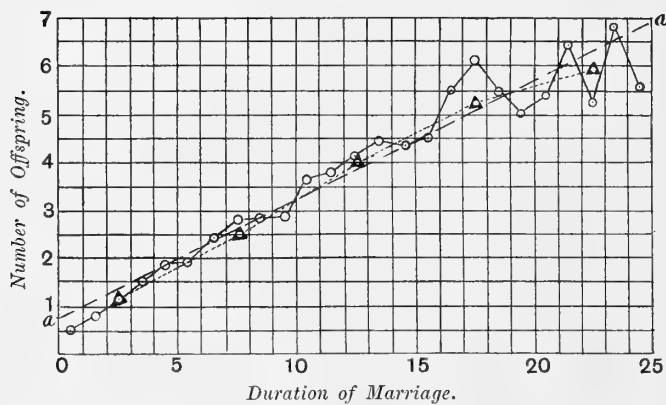
Fig. 13.   Age at Marriage, 25 to 30 years.   Fig. 14.   Age at Marriage, 30 to 35 years.
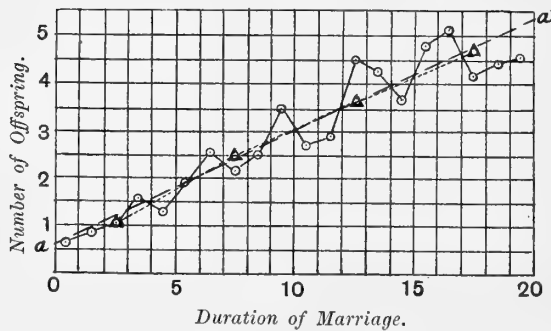
Duration of Marriage.   *Duration of Marriage.*
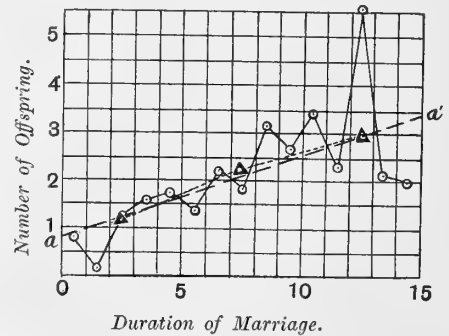
Regression Straight Lines *aa'*.
Averages of Quinquennial Period shown thus Δ........................Δ.............

each advance in the age at marriage. At age 30—35 there is a most decided fall in the initial fertility, and the extent of the fall when marriage is deferred to age 35—40 may be judged from the fact that 20 women who married at that age, and whose duration of marriage was under 5 years, bore only 14 children, and 7 whose duration was 5 to 10 years bore 10 children. After age 40 at marriage—by unprejudiced* spinsters—the initial fertility may be said to have reached zero. Calculating the probable average number of children at the end of the reproductive period (end of forty-fifth year) by means of the regression formulæ, we obtain

For Age at Marriage under 20 ....7·55
  ,, ,, ,, ,, 20 and under 25.........6·51
  ,, ,, ,, ,, 25 ,, ,, 30.........5·12
  ,, ,, ,, ,, 30 ,, ,, 35.........3·21

—results which are somewhat less than the actual results of those women who survived the reproductive period, and which are shown in Table XVII.

Turning now to the census results of New South Wales for 1901, we are able to obtain—for wives who were living with their husbands on the night of the census—the age, the duration of marriage, and the number of offspring. From the nature of the tabulation we are unable to obtain this information for all wives, but this will best be understood from the following specimen of the tabulation :

SPECIMEN OF TABULATION.

Number of Wives without Issue.

Age of Wife	Total Wives	Duration of Marriage								
		Under 1 year	1 year	2 years	3 years	4 years	5 and under 10 years	10 and under 15 years	15 and under 20 years	And so on in quinquennial periods
14 years 15 ,, 16 ,, &c. ,, 20 ,, *21* and under *25* *25* ,, *30* *30* ,, *35* &c. &c.										

These tabulations appear for each sized family up to 10 children, 11 and over being amalgamated. Thus if we wish to trace women married, say, at age under 20, and with duration of marriage under 5 years, we can only make use of those who were specifically stated at the census to be under 20 and who had been married less than 5 years. This of course does not give us all such women, since

* Those for whom conception was not antenuptial.

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some who had been married under 1 year, and between 1 and 2 years, and were aged 20 at census, belong to the age at marriage group under 20. So also do some women aged 21 to 25 years, and who had been married 2 and under 5 years. If we take duration of marriage 5 to 10 years with age at census 20—25, we see that the whole of these women must have been married at under age 20. But so too do some who were aged 25—30 with the same duration of marriage, but we are unable to determine how many. I have therefore been obliged to confine the investigation to such groups as are absolutely certain. Another difficulty has to be faced when we reach the duration of marriage sufficient to yield 11 or more children. To effect the distribution of families beyond 10

TABLE XXIV.

Distribution of Fertility amongst Wives Married at Age under 20 Years.
Experience of New South Wales Census, 1901.

Number of Children in Family	Duration of Marriage											
	Under 5 years		5 and under 10 years		10 and under 15 years		15 and under 20 years		20 and under 25 years		25 and under 30 years	
	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children
0	1171	—	166	—	120	—	89	—	68	—	44	—
1	1166	1166	438	438	228	228	158	158	77	77	41	41
2	188	376	1026	2052	376	752	195	390	88	176	63	126
3	22	66	1034	3102	624	1872	289	867	152	456	65	195
4	—	—	422	1688	812	3248	429	1716	170	680	83	332
5	—	—	83	415	864	4320	490	2450	231	1155	105	525
6	—	—	17	102	484	2904	583	3498	312	1872	141	846
7	—	—	4	28	155	1085	520	3640	330	2310	167	1169
8	—	—	—	—	39	312	398	3184	396	3168	237	1896
9	—	—	—	—	9	81	170	1530	367	3303	272	2448
10	—	—	—	—	1	10	72	720	314	3140	333	3330
11	—	—	—	—	—	—	36	396	260	2860	271	2981
12	—	—	—	—	—	—	—	—	34	408	208	2496
13	—	—	—	—	—	—	—	—	—	—	131	1703
14	—	—	—	—	—	—	—	—	—	—	77	1078
15	—	—	—	—	—	—	—	—	—	—	46	690
16	—	—	—	—	—	—	—	—	—	—	19	304
17	—	—	—	—	—	—	—	—	—	—	9	153
18	—	—	—	—	—	—	—	—	—	—	6	108
19	—	—	—	—	—	—	—	—	—	—	3	57
20	—	—	—	—	—	—	—	—	—	—	1	20
Totals	2547	1608	3190	7825	3712	14812	3429	18549	2799	19605	2322	20498
Average Family	·63		2·45		3·99		5·41		7·00		8·83	
Per cent. without issue	46·0		5·20		3·23		2·60		2·43		1·90	

children, we are obliged to call to our assistance another table in the census which shows the number of wives with each sized family and the ages at marriage, but not the duration of marriage. In the manner thus shown—having regard to the age at marriage—the distribution of families of 11 and more children has been effected, and to this extent the columns of the last three durations in Table XXIV. are vitiated, though the averages obtained cannot possibly be affected to any appreciable extent. This table shows the distribution of fertility to wives married at under 20 years for various durations of marriage within the reproductive period.

The average issue for under 5 years' duration of marriage is evidently too small and the percentage of women without issue too large, owing to the large number of women married under 1 year to be fair criteria of the results of a mean duration of $2\frac{1}{2}$ years' marriage. The results of this duration were obtained from the census and were found to be 422 women with 468 children (56 had 0, 270 had 1, 90 had 2, and 6 had 3), or an average of 1·11 each with 13·3 per cent. without issue. As, however, the result was 1·32 in the case of deaths of married women, which I think is more probably the truth in view of the fact that so many of those marrying under 20 being undetermined in the ages 20 and 21—25, I adopted this value in the calculation of the correlation and regression straight line, with the following general results:

Mean Age at Marriage ....14·6415 years.
„   Size of Family .... 4·6967 children.
Standard Deviation $\begin{cases} \text{Duration of Marriage} ...... 7\cdot9585 \text{ years.} \\ \text{Size of Family} .... 2\cdot3841 \text{ children.} \end{cases}$
Correlation ....·99790 ± ·00002
Coefficient of Regression ....·29894
Regression Straight Line (Origin at Marriage. Unit of $x = 1$ year)
$$y = \cdot3198 + \cdot29894\,x,$$

where $x =$ duration of marriage, and y the number of offspring.

The statistics and regression straight line are plotted on Fig. 15, on which is also plotted for the purpose of comparison the regression straight line as determined by deaths.

The methods of tabulation of the census do not admit of tracing the marriages contracted between intervals of 5 years, say, between 20 and 25, but does admit of them being dealt with as under 25 years at marriage*. The information for

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* It might at first sight be thought possible to have effected the desired computations for quin-quennial periods. For the results are ascertainable for marriages contracted as under each quinquennial age: as for instance, under age 20, under age 25, under age 30, &c., and the difference between any two succeeding ages would appear to give the necessary results. Thus, take the duration of marriage 5 and under 10 years, marrying under age 20 there were 3190 such women who had 7825 children; marrying under age 25 there were 18,086 such women who had 47,653 children; but it is not true that the difference between these two series, viz. 14,896 women having 39,828 children married between the ages 20 and 25, for many (and it is impossible to say how many) of the latter were married at ages under 20. The number shown as marrying under 20 are those we are certain of from the returns but are not all. The same applies to all the groups.

the latter period I have also taken from the census returns, and the results are shown in Table XXV.

TABLE XXV.

Distribution of Fertility amongst Wives Married at Age under 25 Years.
Experience of New South Wales Census, 1901.

Number of Children in Family	Duration of Marriage									
	Under 5 years		5 and under 10 years		10 and under 15 years		15 and under 20 years		20 and under 25 years	
	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children
0	6152	—	1094	—	757	—	553	—	332	—
1	8133	8133	2230	2230	1039	1039	637	637	344	344
2	3668	7336	4811	9622	1874	3748	928	1856	477	954
3	597	1791	5567	16701	2931	8793	1402	4206	669	2007
4	43	172	3124	12496	3603	14412	1967	7868	858	3432
5	2	10	996	4980	3715	18575	2235	11175	1122	5610
6	—	—	226	1356	2542	15252	2662	15972	1383	8298
7	—	—	36	252	1233	8631	2570	17990	1548	10836
8	—	—	2	16	422	3376	2063	16504	1735	13880
9	—	—	—	—	118	1062	1125	10125	1670	15030
10	—	—	—	—	38	380	539	5390	1368	13680
11	—	—	—	—	7	77	200*	2200	652†	7172
12	—	—	—	—	—	—	90*	1080	454†	5448
13	—	—	—	—	—	—	50*	650	264†	3432
14	—	—	—	—	—	—	29*	406	149†	2086
15	—	—	—	—	—	—	—	—	74†	1110
16	—	—	—	—	—	—	—	—	36†	576
17	—	—	—	—	—	—	—	—	15†	255
18	—	—	—	—	—	—	—	—	8†	144
Totals	18595	17442	18086	47653	18279	75345	17050	96059	13158	94294
Average Family	·94		2·64		4·12		5·64		7·17	
Per cent. without issue	33·1		6·05		4·14		3·24		2·52	

Owing to the number of wives (4114) who had been married under 1 year, the statistics of under 5 years' duration of marriage were not used to determine the average number of offspring for the mean of that duration, but the actual result of $2\frac{1}{2}$ years' duration, viz. 3968 wives (563 without issue, 2570 with 1 child, 814 with 2, and 21 with 3) having 4261 children, or an average of 1·07. The percentage of childless women was 14·2. With this substitution the correlation

* Estimated—Total 11 and over, 369.   † Estimated—Total 11 and over, 1652.

Fig. 15. Women Married under 20.

Duration of Marriage (Years).
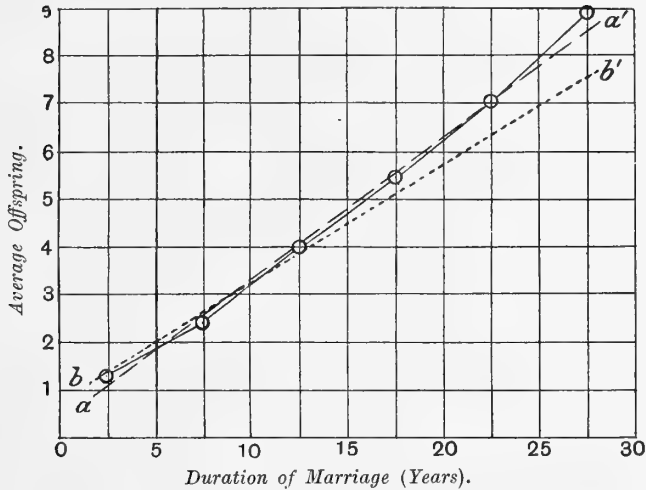
Regression Straight Lines:

aa' (Census), $y = \cdot3198 + \cdot29894x$ $\}$ Origin at Marriage.
bb' (Deaths), $y = \cdot7585 + \cdot24796x$ $\}$ Unit of $x = 1$ year.

between duration of marriage and number of offspring was determined from the figures in the above table with the following results:

Mean Duration of Marriage ....13·7295 years.
„   Size of Family .... 4·5031 children.
Standard Deviation $\{$Duration of Marriage ...... 5·9345 years.
$\{$Size of Family.............. 1·7972 children.
Correlation .... 1·0000
Coefficient of Regression .... ·30286
Regression Straight Line (Origin at Marriage.   Unit of $x = 1$ year)

$$y = \cdot3450 + \cdot30286\,x,$$

where $x =$ duration of marriage, and y the number of offspring.

For the purpose of comparison the regression straight line was determined from the deaths on a similar basis (age at marriage under 25 years, and duration of marriage in quinquennial groupings), and was found to be $y = \cdot7360 + \cdot2484\,x$.

On comparing the results from the census figures of those marrying at under 20 with those marrying under 25, we see that in both cases, as in the results obtained from the deaths, that the initial fertility is preserved throughout the reproductive period, and that it is slightly greater when marriage is contracted under age 25 than when contracted under age 20; that is, that it is greater at age 20 to 25 than under 20. This is in accordance with results arrived at elsewhere* in connection with first births. It is also seen that the mean annual

* *Biometrika*, Vol. I. Part I. p. 33.

births are about 300 per 1000 married women marrying under 25 years. But if marriage be delayed beyond that age, not only is the initial fertility reduced, but it apparently diminishes with the duration of marriage. As has already been shown, it is impossible to obtain from the census returns the results of marriages within quinquennial groups of age at marriage, but it can be obtained in decennial groupings, though, for reasons previously set forth, not for all such marriages. Thus, those women whose ages at the census were between 30 and 35 years, and whose duration of marriage was under 5 years, must have been married between the ages of 25 and 35, as also those who were aged 35 to 40, and had been married 5 to 10 years, and so on. In this way was obtained the information relating to women marrying at ages 25 to 35, and also upon the same principle that of women marrying at ages 35 to 45. The results are shown in Table XXVI.

Fig. 16.   Women Married under 25.

Duration of Marriage (Years).
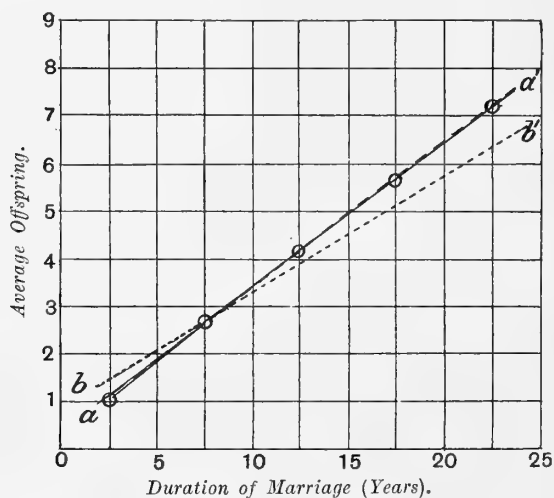
Regression Straight Lines :

aa' (Census), $y = \cdot3450 + \cdot30286x$ ⎞ Origin at Marriage.
bb' (Deaths), $y = \cdot7360 + \cdot24840x$ ⎠ Unit of $x = 1$ year.

It would now be well to show side by side the average families and proportion of wives without issue for each age at marriage and each duration of marriage. This is done in Table XXVII. The second column in that table shows the mean age at marriage of those marrying within the groups shown in the first column according to the experience of the years 1897 to 1902, when the ages were shown in single years.

These results are shown graphically in Figure 17—except for marriages under 20. The table shows in a striking manner the effects of delayed marriage and how the generative organs in females tend to become atrophied through non-use. For marriages contracted under age 25 we see that only $2\frac{1}{2}$ per cent. are sterile at the end of the reproductive period, whilst for marriages contracted between 25 and 35 or at a mean age of 28·4 years the percentage is nearly 12, and for ages 35 to 45

TABLE XXVI.

Distribution of Fertility amongst Wives Married at Ages 25 to 35, and 35 to 45. Experience of New South Wales Census, 1901.

Size of Family	Age at Marriage 25 to 35 years					Age at Marriage 35 to 45 years		
	Duration of Marriage					Duration of Marriage		
	$2\frac{1}{2}$ years	Under 5 years	5 to 10 years	10 to 15 years	15 to 20 years	$2\frac{1}{2}$ years	Under 5 years	5 to 10 years
0	312	1879	654	445	281	133	619	345
1	614	1938	548	293	162	41	193	100
2	139	1220	879	400	232	2	59	81
3	1	217	953	510	277	1	16	30
4	2	30	644	517	311	—	—	14
5	—	—	261	448	328	—	—	5
6	—	—	80	323	302	—	—	2
7	—	—	20	198	216	—	—	—
8	—	—	2	65	139	—	—	—
9	—	—	—	20	72	—	—	—
10	—	—	—	3	25	—	—	—
11	—	—	—	2	11	—	—	—
12	—	—	—	1	6	—	—	—
13	—	—	—	1	3	—	—	—
14	—	—	—	—	1	—	—	—
15	—	—	—	—	1	—	—	—
Total Wives	1068	5284	4041	3226	2367	177	887	577
Total Children	903	5149	9682	11032	9936	48	359	445
Average Family	·85	·97	2·39	3·42	4·20	·27	·40	·77
Per cent. without issue	29·21	35·56	16·18	13·79	11·87	75·14	69·79	59·79

TABLE XXVII.

Average Families and Issueless Wives.

Age at Marriage (Years)	Mean Age at Marriage (Years)	Duration of Marriage												
		$2\frac{1}{2}$ years		Under 5 years		5 to 10 years		10 to 15 years		15 to 20 years		20 to 25 years		25 to 30 years	
		Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %
Under 20	18·6	1·10	12·7	1·03	46·0	2·45	5·20	3·99	3·23	5·41	2·60	7·00	2·43	8·83	1·90
„ 25	21·4	1·07	14·2	·94	33·1	2·64	6·05	4·12	4·14	5·64	3·24	7·17	2·52	—	—
25 to 35	28·4	·85	29·2	·97	35·6	2·39	16·18	3·42	13·79	4·20	11·87	—	—	—	—
35 to 45	38·4	·27	75·1	·40	69·8	·77	59·79	—	—	—	—	—	—	—	—

or a mean of 38·4 years nearly 60 per cent. are sterile. Although it is noticeable that for marriages contracted under 25 years the average offspring is greater for each duration of marriage than those contracted under 20—except for the duration $2\frac{1}{2}$ years—yet the percentage of issueless wives is also greater. This I think more an apparent than a real physiological fact and is due to the larger proportion of

FIG. 17. Average Issue and Proportion of Issueless Wives. Average Issue shown by Firm Lines. Issueless Wives shown by Broken Lines.
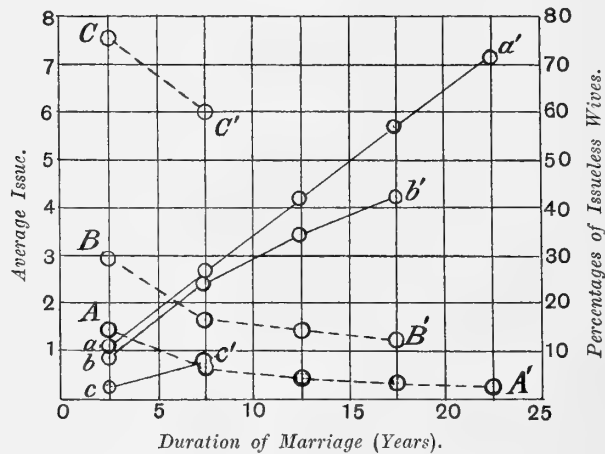
Age at Marriage, Under 25, Lines AA' and aa'.

,, ,, 25—35 ,, BB' and bb'.

,, ,, 35—45 ,, CC' and cc'.

prejudiced marriages in the younger group—and to this is also due the slightly larger average offspring in the $2\frac{1}{2}$ years' duration of marriage. Taking the six years 1897 to 1902 there were 9490 marriages and 3910 antenuptial conceptions, or 41·2 per cent. of prejudiced marriages in the age group under 20, whilst of the 26,453 marriages contracted between 20 and 25 there were 7899 antenuptial conceptions or 29·9 per cent. Nor does this represent the full difference, for some women who were aged 19 at marriage, were 20 at the birth of the child and consequently were included in the later group. If adjustment be made the result will of course intensify the difference. This excess of antenuptial conception in the lower age group must amply account for the slightly higher proportion of issueless wives in the group under 25, but the fact that the average offspring is greater in the latter group than in the former—notwithstanding the greater proportion of sterile women—proves conclusively that the maximum fertility— initial and continuous—lies somewhere between age at marriage 20 and 25. Of course the test of fertility is not in the number of offspring produced during the reproductive period—for that will be greatest at the earliest age at marriage—but the rapidity with which they are produced.

A favourite method amongst statisticians and others of comparing birthrates (at least amongst those who have been able to comprehend the fallacies in the

popular method of estimating the births per 1000 of the total population) has been to show the proportion of legitimate births per 1000 married women under 45 years of age. But it is evident from Table XXVII. that this method is liable to yield misleading comparisons, for the kinetic fertility is dependent not only upon the age of the woman but also upon the age at marriage. This objection of course also applies even when the more accurate measure of determining the movements in the birthrate is adopted, viz. when the rate is determined not in one group under age 45, but in quinquennial groups within that group. In New South Wales there has been a decline in each quinquennial group except at age under 20 since 1881, as will be seen from the following statement[*]:

Ages of Wives	1871 Birthrate per cent.	1881 Birthrate per cent.	1891 Birthrate per cent.	1901 Birthrate per cent.
15—19 years	50·10	51·60	47·91	56·28
20—24 ,,	44·15	45·79	41·63	39·70
25—29 ,,	40·75	40·52	35·37	29·87
30—34 ,,	33·67	33·86	29·23	22·68
35—39 ,,	27·04	27·36	23·63	17·25
40—44 ,,	13·41	12·89	11·84	8·81

The decline since 1881 is a large one and although there has been an undoubted rise in the age at marriage since that period, the decline cannot all be attributed to that rise—though probably much is so due. Prior to 1897 the ages at marriages were not tabulated—even if they had been it would not have helped us much since the marriages to which the births in 1871 and 1881 were due were for the most part contracted in Great Britain—the great majority of the adult population then being of British birth. But as has been said the decline is not wholly due to the advance in the age at marriage—it is chiefly due I think to the Law of Regression so ably formulated and demonstrated by Mr Francis Galton in "Natural Inheritance." Whilst the systems of free and assisted immigration were adopted in New South Wales the immigrants were of special physique and chiefly drawn from the two most fertile classes of the British community, viz. the agricultural and artisan. Medical examination was necessary before departure from Great Britain and thus a differentiated community was obtained. This immigration ceased about 1885, and this was followed by the smaller decline in 1891 and the larger in 1901—the standard of the fertility of the offspring of the immigrants regressing towards that of the population from which they originally sprang. What difference there will be in the average issue of women of past generations and the ultimate families of the present generation—the age at marriage being the same—it is of course impossible to say, but I think a very close estimate may be made. Take for instance age at marriage under 20. We see from Table XVII. the average family of women of past generations marrying at this age was 8·06. This is for women whose husbands may have predeceased them before the end of

the reproductive period or who may have been separated from them otherwise. For women of the present generation marrying at the same age and under the same conditions we may make use of the regression formula from Table XXIII.—limiting the reproductive period to age 46 and adopting the average age at marriage as 18·6 years (see Table XXVII.) we obtain 7·55. This is a reduction of about half a child per family. Adopting the same principle the averages for the present generation are for age at marriage *20—25*, 6·51, *25—30*, 5·12, *30—35*, 3·21, as against 6·90, 5·22 and 3·36 respectively for past generations. These differences are probably the extent of the regression.

I have calculated from the information contained in the English Registrar-General's Reports the average age of spinsters marrying at ages under 45 for the years 1861–62 and 1901–2. The mean age at the former period was 24·3 years; at the latter 25·1—an advance of nearly 1 year. This however hardly conveys an adequate idea of the change that has taken place in England and Wales during the last forty years in the age constitution at marriage. At the former period only 30·9 per cent. were married after what may be regarded as the critical age (so far as fertility is concerned) of 25, whilst at the latter period the percentage had increased to 39·5. Having in view the rapid fall in the initial and subsequent fertility of marriages contracted beyond age 25 it is unquestionable that some not inconsiderable portion of the decline in the birthrate in England and Wales is due to the increasing age of spinsters at marriage. In Australian age at marriage statistics distinction is not made between first and subsequent marriages.

In Victoria—which may be regarded as typical of Australia—the average age of all women marrying at ages under 45 was 24·1 years in 1875–6; in 1901–2 this had advanced to 25·6 years. At the former period 33·8 per cent. were over 25 years of age, whilst at the latter the percentage had increased to 47·3. It would thus appear that there is a general tendency to postpone marriage both in England and Australia (probably more pronounced in the latter) and thus not only diminishing the average family but as we have seen lengthening the intervals between the offspring.

So long as the average age of the female at marriage is in the vicinity of 25 and there are no further restrictions of the family than at present the community will not be a loss for natural increase. The average family will—at the present Australian rate—be between 5 and 6 children, and although this is the gross family it must be borne in mind that the infantile—as well as the subsequent—mortality is very light when compared with Great Britain and European countries. Indeed the deaths of infants form such a small percentage of the births—about 10 per cent.—that the birthrate is considerably diminished thereby. And here (although not directly connected with this section of the investigation) it may not be out of place to refer to the correlation between the birthrate and the infantile mortality—the latter being the deaths under 1 year of age per 100 births. It will readily be seen from the next table of birthrates and infantile mortalities in various countries that

there is such correlation, for it can scarcely be regarded as fortuitous that high birth-rates are associated with high mortalities and low birthrates with low mortalities. Although a higher rate of mortality might probably have been anticipated where the births are numerous, yet I think the high mortality is more the cause than the effect of the high birthrate. Now according to the English and Australian statistics of those infants who die under 12 months of age 50 per cent. die within 3 months of birth, and 70 per cent. within 6 months. If infants be reared in the natural way (which is a fairly general practice in Australia) an almost complete check is given to further conception until the completion of the period of lactation. Therefore where the mortality is heavy, even though the children have been naturally fed (falling as we have seen heaviest in the first few months of life) the impediment to further conception is removed. Where artificial feeding is resorted to of course there is no obstacle to immediate conception. Therefore in countries or communities in which artificial feeding may be the rule but where through ignorance or carelessness, or a like indifference to the production of life as to its subsequent preservation prevail, a high mortality and a high birthrate should be expected; as also in those countries where artificial feeding is the general practice. Thus we find that in Sweden, where it is stated* that almost every child is nursed by its own mother, both the infantile mortality and the birthrate are the lowest in Europe (except the birthrates in France and Ireland—both of which are abnormal). The same authority asserts that in Lower Bavaria where natural nursing has become the exception the infantile mortality reaches the extraordinary high proportion of 50 per cent.

The following table shows the birthrates and infantile mortalities in various countries so far as I have been able to obtain them. France and Ireland have been intentionally omitted—the former because the low birthrate (associated with high mortality) is well known to be due to restrictive measures, and the latter because the low birthrate (associated in this case with low mortality) is due to the constant emigration of adults of reproductive ages. Russia has also been omitted in consequence of the want of reliable figures, but it is generally understood that both the birthrate and infantile mortality are the highest in the civilized world:

Country	Birthrate	Infantile Mortality	Country	Birthrate	Infantile Mortality
Saxony ...	39·9	28·3	Japan	29·8	15·2
Hungary...	38·9	25·6	Scotland	29·8	12·2
Bavaria ...	37·0	27·9	England and Wales	29·0	15·7
Austria ...	37·0	24·7	Belgium	28·9	16·3
Prussia ...	36·4	20·8	Switzerland ...	28·6	15·9
Italy ...	33·5	19·0	Australia	27·1	11·0
Holland ...	32·0	17·5	Sweden	26·8	10·7
Norway ...	30·2	9·6	New Zealand ...	25·7	8·1
Denmark .	29·9	13·6			

* Hugh R. Jones, M.A., M.D., B.Sc., *Journal of the Royal Statistical Society*, March 1894.

From these figures the correlation between infantile mortality and birthrate was determined.   The general results were:

Mean Birthrate	...	...	...	...	31·79
„ Mortality	...	...	...	...	17·18
Standard Deviation { Birthrate	...	...			4·3328
Mortality	...	...			6·1795
Correlation ...	...	...	...	·93369 ± ·00210	
Coefficient of Regression	...	...	·6547		

Regression Straight Line (x = Infantile Mortality, y = Birthrate, Origin of x, Inf. Mort. = 0) $y = ·6547x + 20·5423$, the probable error of y being ± 1·046.

The next table shows the number of survivors of age 5 per 1000 births in various countries *:

Country	Survivors	Country	Survivors	Country	Survivors	Country	Survivors
New Zealand ...	889	Ireland	837	Belgium ...	756	Italy ...	632
New South Wales	850	Sweden	783	Denmark...	755	Austria	614
Victoria ...	844	Scotland	780	France ...	751	Hungary	598
Norway... ...	838	England and Wales	762	Switzerland	748	Spain...	571

So great is the mortality generally in high birthrate countries that the numerical advantage to the community almost entirely disappears by the end of the fifth year, reckoning per 1000 of the total population.   Reducing the birthrates in the proportions given in the last table, the net birthrate at the end of the fifth year is shown in the third columns of the following table:

Country	Birth-rate	Surviving 5th year	Country	Birth-rate	Surviving 5th year	Country	Birth-rate	Surviving 5th year
Hungary	38·9	23·3	Denmark	29·9	22·6	Sweden ...	26·8	20·9
Austria	37·0	22·7	England and Wales	29·2	22·1	New Zealand	25·7	22·8
Spain ...	34·8	19·9	Belgium	28·9	21·9	Victoria ...	25·7	21·7
Italy ...	33 5	22·2	Switzerland ...	28·6	21·4	France ...	22·0	16·5
Norway	30·2	25·3	New South Wales	27·4	23·3			

It may be urged that whilst the birthrate has fallen in England and Wales during the last 15 or 20 years the infantile mortality has practically remained unchanged.   Even if artificial limitation of families has become more generally the practice during that period an investigation of the birthrates and infantile mortalities of the various registration districts shows that the association between the two rates does exist and that those districts which include large cities and towns (where I should imagine the knowledge of restrictive measures would be

* I am responsible for the New Zealand, New South Wales and Victorian figures; the others have been obtained from Mulhall.

best known and the practice reach its zenith) still maintain a high birthrate associated with a high infantile mortality, whilst the essentially rural districts have generally a low birthrate and a low mortality.

These rates I have computed from the data in the Registrar-General's Report for 1902, and are shown in the following table:

Registration District	Birth-rate	Infantile Mortality	Registration District	Birth-rate	Infantile Mortality	Registration District	Birth-rate	Infantile Mortality
Durham	35·9	14·1	London	28·6	14·0	Cambridge	24·2	11·0
Monmouth	35·5	13·5	Cumberland	28·0	11·0	Huntingdon	24·1	9·4
Stafford	33·7	14·3	Worcester	27·8	11·7	Berkshire	23·9	10·1
South Wales	33·1	14·6	Cheshire	27·5	12·7	Surrey	23·7	11·2
Northumberland	32·7	13·1	Lincolnshire	26·7	12·3	Hertford	23·6	9·2
Nottingham	31·6	14·7	North Wales	26·6	12·3	Oxford	23·5	10·9
Warwick	30·3	14·1	Shropshire	26·6	10·4	Devonshire	23·4	12·4
Derbyshire	30·3	12·7	Suffolk	25·8	10·7	Hereford	23·4	10·8
Essex	29·9	11·9	Northampton	25·7	11·2	Bedford	23·3	10·7
Lancashire	29·8	15·0	Buckingham	25·6	10·4	Cornwall	23·2	13·9
North Riding, Yorkshire	29·5	13·5	Gloucester	25·6	12·1	Somerset	23·0	10·0
East Riding, "	29·2	12·9	Norfolk	25·5	12·8	Dorset	22·9	8·7
Middlesex	28·9	12·4	Kent	25·4	11·6	Sussex	22·1	9·8
Leicester	28·7	13·6	Hampshire	24·7	11·4	Westmoreland	21·9	8·2
West Riding, Yorkshire	28·5	14·4	Wiltshire	24·3	9·8	Rutland	21·9	7·1

Thus we see that notwithstanding a few irregularities birthrates and infantile mortalities are closely associated in the registration districts of England and Wales—the irregularities being probably due to the differing ages, conjugal habits, sex proportions and other conditions. Where these factors are alike I think the infantile mortality becomes the chief determinant of the birthrate. As we have seen from a previous table, although natural selection is responsible for a larger birthrate its further operation tends to restore the balance.

It might be inferred from the table showing the survivors at 5 years of age in various countries that by the operation of natural selection in countries where the mortality was heavy during that period the survivors could be regarded as specially selected lives, and that their subsequent expectation of life would be greater than in those countries where the mortality was comparatively light. But this is not always the case, as will be seen from the following table, which is extracted from life tables given by Mulhall from the experiences of the censuses of 1880 and 1881. I have added the results of New South Wales, which I have computed for the census of 1891.

Survivors at various ages of 1000 born.

Age	England	France	Prussia	Austria	Italy	Spain	Sweden	Norway	Belgium	Switzerland	New South Wales
20	706	685	616	532	554	496	711	760	699	689	798
40	597	566	514	423	462	412	621	644	589	578	696
60	405	408	351	267	328	292	473	494	419	393	500
80	96	104	65	44	65	69	139	161	101	69	116

From these figures may be derived the numbers of survivors to age 80 of every 1000 who attain age 5 and which are as follows :

England	126	Sweden	178
France	138	Norway	192
Prussia	95	Belgium	134
Austria	72	Switzerland	92
Italy	103	New South Wales	137
Spain	121		

From this it is seen that those countries with one exception—France—with heavy mortality under 5 years of age do not yield as favourable a result at age 80 even after such a severe weeding out, as those countries where the mortality under 5 is light.

E. *Reproductive Selection.*

Professor Pearson says* with regard to the question "Does Society ultimately recruit itself from above or below ?"

"(*a*) The fertility of one class must be shown to be sensibly greater than that of another class. Now the causes of fertility for two classes in the same community differ comparatively slightly. Hence the small but sensible effect of natural selection must be carefully estimated and allowed for, i.e., we must deal with net fertilities.

"(*b*) But the net fertility of one class may still be greater than that of a second and yet the second increase at a greater rate. We have to determine how far the greater fertility is merely potential. We must ascertain what proportion of the two classes remain unmated.

"(*c*) Not only may the net fertility be greater—i.e., the number of children per family who survive infancy, say live to 15 years, be greater—but also the percentage of mating may be greater in one class than another and yet the same class may yet have no reproductive advantage. We ought further to take into account the *adult* death rates in the two classes. Natural selection may not only be effective in modifying the gross fertility into a net fertility but also as an adult death rate."

The "Vital Statistics" of New South Wales for the years 1895 to 1899 inclusive throw much light upon this question of reproductive selection. In them tables will be found showing the occupations of married men who died during that period together with the total offspring—distinguishing the living and dead. The ages of the living offspring are not given, so that the net fertility must be regarded as the number of offspring living at the time of the death of the father and irrespective of their ages. Thus the conditions contained in (*a*) and (*c*) above except that the net fertility is as just stated are fairly complied with, and as the

* *The Chances of Death*, Vol I. pp. 96 and 97.

results can be modified in accordance with the rates of marriage in the various classes of occupations, condition (*b*) will also be complied with.

The occupations are divided into six classes, viz., I. Professional (mainly engaged in the government and defence of the country, and in satisfying the moral, intellectual and social wants of its inhabitants). II. Domestic (persons engaged in the supply of board and lodging and in rendering personal service for which remuneration is usually paid). III. Commercial (persons directly connected with the hire, sale, transfer, distribution, storage, and security of property and materials). IV. Industrial (persons principally engaged in various works of utility, &c., i.e., artisans). V. Pastoral, Agricultural, and Mining. VI. Indefinite. Table XXVIII. shows the number of deaths in the various classes during the years 1895 to 1899, together with the living and dead offspring, the average to each class, and the proportion of living to total offspring.

TABLE XXVIII.

Occupation	Number of Deaths	Offspring			Average Offspring			Percentage of Living to Total Offspring
		Living	Dead	Total	Living	Dead	Total	
I. Professional	886	2896	972	3868	3·269	1·097	4·366	74·9
II. Domestic	707	2429	847	3276	3·436	1·198	4·634	74·1
III. Commercial	3116	10929	3706	14635	3·507	1·189	4·696	74·7
IV. Industrial	5074	19176	7143	26319	3·779	1·408	5·187	72·9
V. Pastoral, Agricultural and Mining	5564	26730	7484	34214	4·804	1·345	6·149	78·1
VI. Indefinite	454	1821	727	2548	4·011	1·601	5·612	71·5
Total	15801	63981	20879	84860	4·049	1·321	5·370	75·4

Taking the results as they stand and disregarding the Indefinite class (the members of which should really have been distributed amongst the other classes) we find that the essentially Rural class—Pastoral, Agricultural, and Mining—yields the largest gross and largest net results and the Professional the lowest in both. The order of gross fertility which is also that of net fertility is Pastoral &c., Industrial, Commercial, Domestic, Professional. We also see with regard to the last four classes which comprise for the most part the urban occupations that the greatest mortality occurs in the largest families, with the slight exception in the cases of the Domestic and Commercial classes. This is in agreement with the results obtained by Professor Pearson in dealing with Rubin and Westergaard's Copenhagen statistics. But in this instance natural selection has not disturbed the order of gross fertility. But the rural class with by far the greatest fertility has also the least infantile and juvenile mortality, and it would seem likely that this class would ultimately recruit society. But the results shown in the sixth column of Table XXVIII. have yet to be modified by the marriage rates of each

class, and fortunately I have been able to obtain these with a great degree of accuracy. The total number of males over 20 years of age in each class was obtained at the census of 1901, and the occupations of the bridegrooms have also been obtained for the years 1901 and 1902, and which are published in the "Vital Statistics" of New South Wales for those years. Hence the marriage rates per 1000 adults of each can readily be ascertained. The statistics and results are shown in Table XXIX.

TABLE XXIX.

Marriage Rates of Various Classes of Occupations.

Class of Occupation	Number of Adults (20 upwards) at Census 1901	Number of Marriages Mean of Years 1901 and 1902	Marriage Rate per 1000 Adults	Index No. Mean = 1
Professional	23300	764·5	32·81	1·181
Domestic	17277	272	15·72	·566
Commercial	91939	2759·5	29·99	1·079
Industrial	102071	4130·5	40·46	1·456
Agricultural, Pastoral, and Mining	139947	2545·5	18·18	·654
Indefinite	3575	39·5		
Total	378109	10511·5	27·79	

By applying the index number shown in the last column to the net fertility shown in Table XXVIII. we obtain finally as the comparative net fertility :

1st, Industrial	Class	5·502,
2nd, Professional	„	3·861,
3rd, Commercial	„	3·784,
4th, Pastoral, Agricultural, &c.	„	3·142,
5th, Domestic	„	1·945.

Thus although natural selection failed to change the position of the classes in the net fertility from those occupied in the gross yet the class marriage rates have completely done so. The rural class with the highest gross and net fertilities, with the least mortality amongst the offspring, and also as we shall presently see with the lowest adult mortality, now falls to nearly the lowest position, whilst the industrial class with both these mortalities the highest occupies the highest position through the frequency of mating in that class. We have seen that the average gross fertility of the agricultural and pastoral class is 6·149, and the industrial 5·187 —merely a difference of one child per father; but this difference is probably not altogether due to a difference in actual fertility but to the heavier adult mortality in the latter class and consequently shortened life. This will be seen from the following

statement showing the adult deaths per 1000 of each class, and which have been calculated in the same way as the marriage rates:

Class of Occupation	Mortality Rate per 1000 Males over 20 years
Agricultural, Pastoral, &c	12·69
Commercial	13·29
Professional	15·73
Domestic	16·59
Industrial	22·72
Total	16·47

Thus notwithstanding an adult mortality far in excess of the other occupations and with the highest infantile and juvenile mortalities the frequency of marriage is also so much in excess that as Professor Pearson puts it, "after all society recruits itself from below," that is from the artisans—although as his statistics referred only to the City of Copenhagen he was not in possession of information regarding the rural occupations. This class it is seen with the highest fertility and the lowest mortality at every age absolutely fails to contribute its fair quota to the community in consequence of its low marriage rate.

When we examine the gross fertilities of the various classes (remembering that these are the results of all durations of marriage and of widowers, as well as of married men whose wives survive them) and compare with the Anglo-Saxon and Danish statistics, we again see that up to the present there is but little Malthusian restraint upon the population in New South Wales—what little there may be apparently being confined to the Professional, Domestic, and Commercial classes. The ages of the brides at marriage in these classes have not been tabulated, but it may be of interest to show the average age at marriage of the bridegrooms, which according to the marriages of the years 1901 and 1902 were as follows:

Class	Average Age at Marriage
Professional	30·22 years
Domestic	30·97 „
Commercial	28·88 „
Industrial	28·57 „
Agricultural, Pastoral, &c.	29·18 „

As the classes of occupations are rather comprehensive I append Table XXX. which shows similar information to Table XXVIII. for some of the sub-classes.

The agricultural class is unquestionably the most prolific in the community accompanied by an exceedingly low mortality among its offspring. The medical profession would appear to be conspicuously below the average of the whole community, whilst ministers of religion just reach that average. The comparatively heavy mortality amongst the offspring of the artisan classes is significant.

TABLE XXX.

Occupation		Deaths	Total Offspring			Average Offspring			Percentage of Living to Total Offspring
Class	Sub-Class		Living	Dead	Total	Living	Dead	Total	
I.	Religion*	68	264	101	365	3·882	1·485	5·367	72·3
I.	Health†	151	475	148	623	3·146	·980	4·126	76·2
IV.	Houses and Buildings‡	1173	4614	1822	6436	3·934	1·553	5·487	71·7
IV.	Labourers, &c. ...	1488	5527	1933	7460	3·715	1·299	5·014	74·1
V.	Agricultural ...	3149	16613	4377	20990	5·275	1·390	6·665	79·1
V.	Pastoral	1024	4712	1249	5970	4·610	1·220	5·830	79·1

TABLE XXXI.

Distribution of Fertility in Occupations. Deaths of Married Men, New South Wales,
1895—1902.

Number of Children in Family	Class of Occupation											
	Professional		Domestic		Commercial		Industrial		Agricultural, Pastoral, &c.		All§	
	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.
0	238	15·276	240	19·818	756	14·237	1136	12·900	899	10·103	3372	12·719
1	171	10·976	116	9·579	539	10·151	667	7·575	602	6·766	2151	8·113
2	161	10·334	110	9·083	529	9·962	819	9·300	606	6·811	2267	8·551
3	155	9·949	108	8·918	510	9·605	742	8·427	580	6·518	2163	8·159
4	138	8·858	92	7·597	501	9·435	734	8·335	648	7·283	2169	8·181
5	135	8·665	84	6·936	476	8·965	722	8·199	645	7·249	2103	7·932
6	110	7·060	94	7·762	384	7·232	720	8·176	631	7·091	1999	7·540
7	90	5·777	69	5·698	350	6·592	673	7·642	721	8·103	1953	7·366
8	96	6·162	70	5·781	336	6·328	666	7·563	750	8·429	1984	7·483
9	88	5·648	88	7·267	283	5·330	522	5·928	680	7·642	1720	6·488
10	73	4·685	41	3·386	212	3·992	497	5·644	684	7·687	1564	5·899
11	35	2·246	40	3·303	182	3·427	362	4·111	510	5·732	1159	4·372
12	22	1·412	22	1·817	105	1·977	235	2·669	421	4·731	830	3·131
13	19	1·220	20	1·652	71	1·335	144	1·635	228	2·562	488	1·841
14	11	·706	9	·743	34	·640	84	·954	139	1·562	278	1·049
15	6	·385	4	·330	13	·245	32	·363	61	·686	120	·452
16	2	·128	2	·165	14	·264	24	·273	49	·551	93	·351
17	3	·193	2	·165	8	·151	9	·102	10	·112	34	·128
18	2	·128	—	—	4	·075	8	·091	15	·169	30	·113
19	2	·128	—	—	1	·019	4	·046	11	·124	18	·068
20	1	·064	—	—	—	·000	1	·011	5	·056	7	·026
21	—	—	—	—	2	·038	3	·034	2	·022	7	·026
22	—	—	—	—	—	—	1	·011	—	—	1	·004
27	—	—	—	—	—	—	—	—	1	·011	1	·004
29	—	—	—	—	—	—	1	·011	—	—	1	·004
Totals	1558	100·000	1211	100·000	5310	100·000	8806	100·000	8898	100·000	26512	100·000

* Ministers of Religion, &c. † Doctors, Chemists, &c.
‡ Carpenters, Bricklayers, Masons, Plumbers, &c. § Including the Indefinite class not shown in Table.

In Table XXVIII. the results were obtained from the deaths during 1895–9 when the tabulation distinguished between the living and dead offspring. Since that date the offspring have not been so distinguished, but only the total or gross shown. As it may be of some value to show these for the longer period 1895–1902 and to show the range of families in each class I have compiled Table XXXI.

The mean family, median family, median children and percentage of married men producing half the children in each class are shown in Table XXXII.

TABLE XXXII.

Occupation	Mean Family	Median Family	Median Children	Percentage producing 50 per cent. of Children
Professional	4·55	3·39	7·09	22·58
Domestic	4·52	3·34	7·43	23·12
Commercial	4·70	3·64	7·08	23·30
Industrial	5·26	4·42	7·57	25·11
Agricultural, Pastoral, and Mining	6·21	5·74	8·56	27·37
All	5·39	4·54	7·85	25·07

It will be noticed that the averages—with one slight exception—are sensibly greater in each class for the period 1895–1902 than for the period 1895–9. This is not due to increasing fertility during the additional period but to the fact that New South Wales is annually increasing its proportion of old people which has not yet reached that of a normal community—in consequence of its comparatively recent settlement. For instance, the proportions of the population over 60 years of age in New South Wales were at the censuses of 1861, 1871, 1881, 1891, and 1901, 3·1, 4·0, 4·3, 4·4, and 5·6 per cent. respectively, whilst the proportion in England and Wales at the last census was 7·4 per cent. But as this increase in age is common to all sections of the community the comparison between the fertilities of the various occupations is in no way vitiated.

Tables XXXI. and XXXII. need but little comment. It is seen that Professor Pearson's conclusion that generally 25 per cent. of the married contribute 50 per cent. of the next generation is corroborated, though the percentage varies in the various classes, being largest in the rural and smallest in the professional classes. Further that his view that society is recruited from below is also sustained—that it is the comparatively short-lived artisan class, notwithstanding the highest mortality among its offspring, that is contributing the largest quota to the population.

CONTRIBUTIONS TO THE STUDY OF INTERRACIAL CORRELATION.

By E. TSCHEPOURKOWSKY, Moscow.

Many authors have tried to explain mechanically several of the distinctive characters of the races of man, especially the different forms of the skull. I may mention Nystrom's hypothesis: he asserted that the head-form may be the product of the work of muscles on the one hand, and of the inner pressure of blood on the other. We have also similar theories due to Ranke and Thomson, but the evidence on which they are based is very slight. I propose in this paper to state some considerations based on biometrical methods. In a paper read before the Congress of Naturalists and Physicians assembled in St Petersburg in 1900 I made the following statements based on the measurements of 700 skulls*.

(1) The form of the skull is connected with the form of its base, and the variation of the first produces a series of the variations in the second, some of which resemble embryonic growth. (2) We do not find between the lengths and breadths of skulls in the same race a relation which is in accordance with the principle spoken of as that of "compensational growth." If one increases, the other also increases. But if we pass from one race to another we find that with the growth of length there is a decrease of the breadth, and *vice versa*. This arises from the fact that the most brachycephalic races have the greatest breadths and the smallest lengths. The effect would be the same if, instead of different races, we took dolichocephalic and brachycephalic groups in the same race. (3) As the decrease of the skull length is connected with the decrease of the length of skull-base and of the face-length, we should expect that lower races with powerful jaws and small cranial capacity must have a tendency to dolichocephaly, and again that the decrease of the facial parts and the development of the brain are influences which are favourable to brachycephaly. (4) In the geographical distribution of the cephalic index we have two well-marked regions: generally speaking southern continents are dolichocephalic and northern are brachycephalic. We can perhaps distinguish on the earth two

* Extracted from the Report of the Congress.

centres isolated by the highest mountains of the globe, from which only the northern centre was favourable to the development of brachycephaly. The southern remained in a primitive state.

In the same report and in that read by me before the Anthropological Congress in Worms I made the following deductions, based on the measurements of 1,300 women and their new-born children* : (1) that the head-form of the mother is inherited by the new-born child; and (2) that not only is the form of the skull, but also the form of the base determined in this very early period of life. As the base is then relatively very small we can hardly speak about its influence on the form of the skull: both are equally inherited.

These last facts brought me more and more to the conclusion that sexual selection has played an important *rôle* in the variation of the cephalic index. Ripley denies this on the ground that it is not a prominent feature of the human body. But selection may be at work if the index be simply correlated with selected characters. In order to ascertain this, and throw light on many other problems which I hope shortly to treat in a special anthropological essay, I have collected material for the study of interracial correlation. This material is partly extracted from Ivanovsky's *Anthropological Constituents of the Population of Russia*, partly from Deniker's *Races of Man*, and as far as regards crania from different authors and from my own measurements. From the data given below, upon which the cross correlations can also be studied, the correlation tables (Appendix, Tables 1 to 28) were formed and the constants calculated, which are represented in the following table. Many conclusions can be drawn from this. I will mention some of the results.

1. About the correlation between stature and head form there exist the most contradictory opinions between anthropologists. Interracially for the races of Russia I found a very sensible *negative* correlation. The control table for other races of the earth was formed on Deniker's data, all Russian races of the preceding correlation table being excluded, and I obtained the same *negative* result. As the total number of races in both tables is 176, this result is most probably correct. I had very few data for women—only 28 cases—and they give me a very slight positive correlation. The material is not large enough to draw conclusions concerning the sexual difference. It follows from the masculine correlation that the head-form changes when stature is selected.

2. The correlation between the cephalic index and the shape of face, i.e. the relation of bizygomatic breadth to the total height of the face, measured from the top of the forehead to the chin, is positive and very sensible. It proves that the head-form can be influenced by sexual selection, because the shape of face is a prominent feature of racial "habitus." But as we pass to the upper face index, i.e. to the relation upper face height to bizygomatic breadth, which is calculated

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* "Ueber die Vererbung der Kopfform von Seiten der Mutter," *Correspondenzblatt der Deutschen Anthrop. Ges.* No. 12, 1903.

TABLE I. *Interracial Correlation and Variation.*

Tables		No. of Cases	First Character A	Second Character B	Standard Deviation A	Standard Deviation B	Mean A	Mean B	Correlation
1	Living Races	92	Stature	Cephalic Index	3·33	2·91	164·74	82·36	− ·184
2	,, ,, (from Deniker)	84	,,	,,	5·59	4·02	163·96	78·70	− ·224
3	,, ,, ,,	54	Cephalic Index	Face Index	2·97	2·56	82·87	77·59	+ ·262
4	Crania	47	,,	Upper Face Index	4·90	1·99	77·21	51·36	+ ·177
5	Living Races	65	,,	Nasal Index	3·10	6·73	82·78	64·35	+ ·050
6	,,	83	Stature	Head Length	3·43	2·77	164·98	185·89	+ ·096
7	,,	83	,,	Head Breadth	3·43	4·50	164·98	153·22	− ·042
8	,,	53	,,	Head Height	3·29	6·48	164·91	130·98	+ ·096
9	,,	58	,,	Bizygomatic Breadth	3·01	4·84	164·83	142·33	− ·172
10	,,	50	,,	Face Index	3·18	2·97	165·2	78·59	− ·164
11	Crania	43	Length	Breadth	4·68	6·49	182·12	137·65	− ·321
12	,,	43	,,	Height	4·73	4·49	181·93	135·86	+ ·615
13	,,	43	Breadth	Bizygomatic Breadth	6·49	4·49	137·65	135·86	− ·132
14	,,	32	Upper Face Height	Upper Face Height	3·60	4·28	68·03	131·66	+ ·641
15	,,	32	Length	,,	5·09	3·60	181·34	68·03	+ ·257
16	,,	32	Breadth	,,	6·59	3·60	137·62	68·03	+ ·504
17	,,	32	Height	Bizygomatic Breadth	4·43	3·60	135·06	68·03	+ ·074
18	,,	41	Length	,, ,,	4·89	4·12	182·19	131·95	+ ·489
19	,,	41	Breadth	,, ,,	6·07	4·12	137·15	131·95	+ ·257
20	,,	41	Height	Minimum Frontal Breadth	4·49	4·12	136·08	131·95	+ ·087
21	,,	38	Length	,, ,,	5·13	2·63	182·21	95·21	+ ·320
22	,,	38	Breadth	,, ,,	5·76	2·63	137·26	95·21	+ ·328
23	,,	38	Height	,, ,,	4·61	2·63	136·06	95·21	− ·135
24	,,	31	Skull Basis	Upper Face Height	3·34	3·73	100·55	68·22	+ ·124
25	,,	42	,,	Breadth	3·33	6·51	101·17	137·8	− ·262
26	,,	40	,,	Bizygomatic Breadth	3·34	4·15	101·30	132·02	+ ·255
27	,,	42	,,	Height	3·45	4·53	101·14	135·9	+ ·507
28	,,	42	,,	Length	3·34	4·90	101·14	182·31	+ ·688

Face Index on the living = 100 bizygomatic breadth/height from chin to top of forehead.

Upper Face Index on crania = 100 bizygomatic breadth/Upper face height.

Upper Face Height = distance from nasion to alveolar point.

Skull Basis = distance from nasion to basion.

Length and Breadth alone refer to maximum cranial length and breadth and Height alone to basio-bregmatic height.

mostly from my own measurements, we find a very interesting fact: the correlation of the cephalic index and upper face index is positive, i.e. the upper face height increases more rapidly with brachycephaly than the bizygomatic breadth, so that mongoloid races having very prominent jaws have, however, the upper face longer than negroid races. It depends, I think, upon the diminution of prognatism, and represents morphologically a more advanced state of phylogenetic growth. Perhaps it stands in connection with the more advanced development of the base in the case of brachycephals, which was first noticed by Schäffer. Intraracially I obtained the results represented in the Table II.

TABLE II.

Skulls ♂. *Intraracial Variation and Correlation of Cephalic Index and Upper Face Form.*

Race	No. of Cases	Means		Standard Deviations		Coefficient of Correlation
		Cephalic Index	Upper Face Index	Cephalic Index	Upper Face Index	
Australians	66	71·03	49·35	2·85	2·93	+ ·089
Negroes	77	74·34	51·27	3·60	3·95	+ ·182
Duke of York Islanders	53	70·90	48·70	2·91	3·22	− ·093
Malays	60	80·20	50·23	4·70	2·80	− ·185
Fijians	32	69·59	49·47	4·58	3·31	+ ·217
Papuans	39	72·36	49·51	4·41	3·40	− ·255
Polynesians	44	77·68	50·23	5·20	3·70	+ ·002
Alfourous	19	80·73	50·89	5·71	3·82	− ·302
Micronesians	32	74·41	51·15	3·69	3·71	− ·251
Copts	34	77·44	54·53	4·83	3·10	− ·147
Etruscans	47	78·15	54·15	2·93	2·75	− ·021
Europeans	80	80·24	53·95	3·80	3·44	− ·198
Ancient Thebans ...	152	75·05	53·52	3·17	3·54	− ·067

It follows from them that intraracially the correlation is mostly negative.

For nasal and cephalic indexes I found a slight positive correlation for the races of Russia.

3. The interracial correlation of the diameters of the head with stature shows that the height—measured from tragus to vertex—and the length slightly increase, but the breadth decreases. In accordance with these correlations we find that with the increase of stature not only the breadth of skull, but also the bizygomatic breadth of the face decreases. It is perhaps a general rule that horizontal diameters of the head decrease with increase of stature while vertical and longitudinal diameters increase.

4. Passing now to the cranial correlations we find interracially the negative correlation between length and breadth, which is in accordance with Dr Alice Lee's researches. The correlation of length with height is large and positive, of height

with breadth negative. In the diameters of the face we find a high correlation between upper face height and bizygomatic breadth. Very interesting is the fact that the upper face height is more closely correlated with breadth than with length. The bizygomatic breadth on the contrary is more correlated with length than with breadth. Both diameters of the face are very slightly correlated with the height. The minimum frontal breadth is positively correlated with length and breadth and negatively with height.

It follows from the values of the above constants that interracially with the increase of the length of the skull the breadth diminishes; the height and basal length on the contrary increase as well as the stature.

5. For interracial variation we have the chief results presented in the Table III. below. We conclude from these constants that interracially the length of the

TABLE III.

Coefficients of Interracial Variation.

Character	Coefficients of Variation	
	Living Subjects	Skulls
Stature	2·02	—
Cephalic Index	3·58	6·34
Face Index	3·29	—
Upper Face Index ...	—	3·87
Nasal Index	10·40	—
Cephalic Length	1·49	2·57
Cephalic Breadth	2·93	4·71
Cephalic Height	4·95	3·30
Bizygomatic Breadth ...	3·40	3·14
Minimum Frontal Breadth	—	2·76
Upper Face Height ...	—	5·29
Skull Basis	—	3·32

skull is less variable than the breadth. The height in the case of skulls is also less variable than the breadth. The variation constants for length and breadth of living subjects are in accordance with those for the skulls, but the height is not to be compared in the two cases: in living subjects it is measured as the distance between tragus and vertex, and its great variability is, I think, partly owing to the impossibility of perfect measurement, partly to inconstancy of tragus. Very interesting also is the great variability of the upper face height as compared with that of the bizygomatic breadth.

From the above given constants we can deduce, I think, the following statement: *The varieties of the human head-form are mostly owing to the variation of the breadth.*

In concluding this study I wish to state some theoretical considerations which, although perhaps far from the absolute truth may, I hope, yet show how important for anthropology the biometrician's work will be in the future when aided by extended anatomical studies. I call first the reader's attention to the fact that the several diameters of the head in new-born children do not stand in equal relation to those of adults. In other words they need not accomplish the same growth to reach the adult stage. The most complete table of these differences will be found in the work of Papillaut. From my own measurements of the base of the skulls of new-born infants I have reached, as a general result, the conclusion that the breadths of the skull and of its base are far less developed in the embryonic stage than the lengths when we compare their ratios with those of adults. I give here some numerical data extracted from the work of Papillaut.

Character	Upper Face Height	Bizygomatic Breadth	Breadth of Skull	Length of Skull	Minimum Frontal Breadth
Relation : $\dfrac{\text{New-Born}}{\text{Adult}}$	44°/₀	48°/₀	56·5°/₀	60°/₀	60°/₀
Coefficient of Interracial Variation	5·29	3·14	4·71	2·57	2·76

Compared with the coefficients of interracial variation they show parallelism with only one exception—that of bizygomatic breadth. I cannot affirm that racial differences in the skull are simply differences of growth, but the above-mentioned parallelism seems to me not without interest when compared with the other point which is also connected with growth. We have seen that interracially and partly intraracially—probably as far as we can eliminate the influence of mixture—the cephalic index diminishes with stature, because the length and height increase and the breadth decreases. I have also shown that the length of the basis is *positively* related with the length and height and *negatively* with the breadth of the skull. Now the length of the basis consists of two parts, which are very different embryologically. The distance between sutura spheno-basilaris and the basion is the end of the primordial skeleton-axis, i.e. of the chorda dorsualis. The second part, i.e. the distance between nasion and sutura, is connected with the development of the brain and partly influenced by sinus frontales. Manouvrier has shown that only the first part of the base-length depends on stature. It has much more to grow in order to reach the adult stage than the second, which is developed very early. The reason is the great development of the embryonic brain owing perhaps to its physiological faculties. It follows from these considerations that the negative correlation between stature and cephalic index can be explained by the growth of the length from basion to sutura basio-sphenoidalis, which increases much more rapidly with the stature than others do because it is a part of the primordial body-skeleton. With the increase of this diameter the length of the skull and the basio-bregmatic height increase also, but the breadth compensationally decreases. As it follows from the tables, all these correlations are

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observed only as averages, and separate races may present many exceptions. The cause of the negative correlation between stature and cephalic index is thus only one of many factors which can influence the head-form. We must, therefore, conclude that the diameters of the head and the cephalic index respectively depend on many causes. The anthropometric measurements are often artificial simplifications, and will, with the progress of anthropology, be made to approach more closely to those regions which are physiologically and embryologically individualized. The organic influences which exist in the individual, or within the limits of a race, can be masked, and correlations between numerical values of measurements inverted as we pass to interracial correlation because other factors come into play. The one of these factors is sexual selection, which is very powerful in the species *Homo*. Admit for instance that the general growth of an organism produces an elongation of the head. Then if the selection is connected with stature, and tall individuals survive, the new race thus formed must be long-headed. But if round-faced individuals become a type of racial beauty, the sexual selection works in the opposite direction, i.e. produces a round-headed tall race, so that in a particular case the organic influence can be masked.

The progress of the evolution of a race is the result of a number of organic correlations, and intellectual or instinctive impulses play an important part in it. Many correlations which are preserved interracially are organic, i.e. physiological correlations. Perhaps we may define the characteristics of a given race as its deviations from the most probable values, calculated from the coefficients of organic correlation. These deviations are the result of processes which were not the same in the evolution of all races, and which have been quite special for each individual race.

APPENDIX I. RACIAL MEAN CHARACTERS.

TABLE I. Races of or adjacent to the Russian Empire.*

Race or Group	Authority	Head Index	Stature	Head Length	Head Breadth	Height/Length	Height	Length of Face	Facial Index	Bizygomatic Breadth	Nasal Index	Capacity	Body Length (rel.)	Relative Capacity
Abazeck (Caucasian Tribe)	Erkert	81·6 (11)	1680 (23)	192 (11)	157	67·6 (11)	130	—	77·4	—	63·4 (11)	1466	—	—
Abazin (id.)	Sonnnier	82·5 (23)	1681 (18)	192 (23)	158	—	—	—	—	—	—	—	—	—
Afghans	Matziewsky	75·6 (18)	1638 (18)	189 (18)	143	74·4	149	190	73·2	—	50·9 (18)	1336	36·0 (18)	80
"	Maslowsky	84·0 (18)	—	—	—	—	—	—	—	—	—	—	—	—
Ainos	Koganei	77·3 (45)	1567 (91)	194 (35)	150	64·6	125	185	78·1	145	68·7 (87)	1383	—	88
Aisoren (Baku)	Arutinow	85·8 (12)	1562 (10)	184 (12)	160	75·3 (10)	139	182 (12)	78·7	143	60·0 (12)	1518	35·7 (10)	97
"	Pantuchow	87·9 (45)	1671 (43)	177 (45)	155	—	—	—	—	—	64·7 (34)	—	—	—
Arabs	Maslewsky	82·0 (17)	1637 (17)	183 (17)	150	—	—	189	72·5	137	—	—	—	—
"	Maziewsky	83·0 (50)	1611 (3)	186 (30)	154	—	—	184 (30)	—	—	56·9 (50)	—	36·4 (30)	—
Arbunsumun (Kulga, Kalmuks)	Ivanowsky	87·0 (160)	1672 (60)	181 (160)	157	72·8	132	184 (160)	78·3 (160)	144	63·5 (160)	1557	35·9 (160)	93
Armenians	Nasarow	83·2 (163)	1647 (63)	182 (163)	152	67·3 (193)	122	186 (160)	76·7 (160)	143	75·8 (160)	1308	30·4 (49)	79
Bashkirs	Talko-Grinzew	85·7 (535)	1631 (544)	187 (535)	160	—	—	192 (595)	79·9 (294)	153	57·0 (295)	—	35·7 (273)	—
Buriats	Diff. Authorities	85·9 (816)	1631 (825)	186 (815)	160	69·5 (100)	129	189 (484)	79·7 (484)	151	62·1 (556)	1443	35·7 (343)	88
Bow	Eichhold	81·0 (100)	1652 (100)	186 (100)	151	69·2 (100)	129	—	—	—	69·3 (100)	1380	—	84
White Russians, Smolensk	Rosdestwensky	—	1648 (57)	185 (57)	152	—	—	183 (57)	75·9 (57)	139	68·0 (57)	—	34·6 (54)	—
" Minsk	Shadrowizky	80·2 (445)	1675 (445)	190 (445)	149	60·5 (50)	—	187 (445)	76·3 (445)	143	69·4 (445)	—	—	—
" Elnin District	Galai	83·0 (200)	1660 (200)	179 (200)	153	70·6	126	183 (200)	74·6 (200)	137	69·7 (200)	1304	35·1 (200)	78
Great Russians, Twer	Wilga	81·8 (100)	1689 (200)	187 (100)	153	—	—	—	77·7 (100)	—	—	—	—	—
" Jaroslaw	Worobiew	81·5 (325)	1651 (325)	188 (325)	151	—	—	183 (323)	77·1 (323)	141	—	—	—	—
" Riazan	Lukine	81·7 (235)	1640 (235)	185 (235)	154	68·8 (235)	127	180 (235)	77·5 (235)	140	66·7 (235)	1358	34·7 (235)	83
" Kursk	Grigoriew	83·6 (63)	1643 (63)	184 (63)	152	71·1 (63)	131	182 (63)	76·6 (63)	139	71·8 (63)	1406	35·2 (63)	86
" Toula	Iwanowsky	82·3 (66)	1654 (60)	185 (66)	152	71·0 (60)	131	183 (60)	76·4 (60)	141	68·2 (60)	1398	35·0 (60)	85
" Eriwan	Zealand	81·3 (51)	1671 (51)	187 (51)	150	71·6 (51)	134	—	—	—	—	1435	—	86
" Sieminiechansk	T. Grinzewich	83·1 (175)	1669 (175)	—	—	—	—	—	—	—	—	—	—	—
Transbaikalia		—	—	183 (120)	155	—	—	—	—	—	—	—	—	—
Votjaks	Maliew	81·9 (120)	1619 (119)	182 (80)	143	—	—	177 (188)	78·7 (100)	139	63·4 (40)	—	—	—
"	Chantre	85·4 (118)	1646 (232)	188 (30)	151	—	—	181 (40)	79·9 (40)	145	51·8 (30)	—	—	—
Gruzins	Maziewsky	76·0 (50)	1659 (30)	—	—	71·9 (30)	135	199 (30)	73·7 (147)	136	62·0 (147)	1382	36·3 (30)	83
Dungans	Elkind	81·9 (200)	1610 (200)	184 (200)	156	67·1 (200)	124	184 (187)	—	—	—	1328	33·9 (100)	83
Jews, Warsaw		82·9 (438)	1625 (438)	—	—	—	—	—	—	—	—	—	—	—
" Kiew		81·7 (69)	1619 (68)	—	—	—	—	—	—	—	—	—	—	—
" Lithuania	Blechman	83·0 (52)	1623 (52)	188 (52)	154	—	—	179 (52)	76·8 (52)	137	—	—	—	—
" Kowno	Stieda	82·2 (67)	—	187 (67)	150	62·2 (67)	116	181 (67)	—	—	—	—	—	—
" Minsk	Jakowenko	81·7 (186)	1617 (139)	183 (139)	138	—	—	181 (186)	76·2 (148)	138	—	—	—	—
" Mohilew	Weissenberg	82·4 (100)	1648 (259)	183 (100)	151	—	—	—	—	—	59·9 (170)	—	—	—
South Russia	Iwanowsky	82·8 (50)	1626 (50)	183 (50)	151	66·1 (100)	121	183 (50)	76·1 (50)	139	62·4	1299	34·0 (50)	79
" Odessa	Pantuchow	87·5 (43)	—	184 (43)	143	66·7 (50)	122	184 (11)	76·9 (11)	142	62·7 (11)	1307	—	80
Transcaucasia		—	—	—	—	—	—	—	—	—	—	—	—	—

* The numbers in brackets refer to the number of individuals in the series.

TABLE I.—(*continued*).

Race or Group	Authority	Head Index	Stature	Head Length	Head Breadth	Height Length	Height	Length of Face	Facial Index	Bizygomatic Breadth	Nasal Index	Capacity	Body Length (rel.)	Relative Capacity
Eside	Horoshanko	77·9 (33)	1633 (24)	140 (33)	148	74·2 (29)	141	192 (98)	76·4 (98)	147	61·6	1480	—	91
Zirian	Nalimow	87·2 (98)	1628 (97)	177 (98)	154	76·6 (96)	136	186 (25)	78·4 (25)	146	64·7 (98)	1406	36·6 (70)	86
Ingoushi	Pantuchow	82·3 (55)	1713 (60)	190 (54)	157	—	137	186 (31)	74·6 (31)	139	67·9 (23)	—	—	—
Istigarzin	Danilow	80·4 (31)	1652 (31)	184 (31)	148	74·7 (31)	136	—	—	—	65·0 (37)	1370	36·8	83
Kabards	Wishgorod	83·7 (40)	1678 (40)	186 (40)	156	72·9 (40)	121	178 (39)	80·5 (37)	143	73·4 (97)	1475	—	87
Kalmuks, Astrakan	Korolew	81·1 (91)	1643 (93)	189 (90)	153	64·2 (92)	—	185 (95)	79·3 (95)	147	—	1343	35·5 (91)	82
” ”	Diff. Authorities	82·0 (212)	1643 (236)	189 (212)	155	62	130	—	—	—	54·8 (30)	—	—	—
” Chackari	Maziewsky	83·6 (30)	1622 (30)	186 (30)	156	69·8 (30)	127	184 (30)	—	—	58·5 (30)	1424	35·7 (30)	88
” Tourfan	”	85·8 (30)	1631 (30)	180 (30)	155	70·5 (30)	—	186 (30)	—	—	62·2	1357	34·4 (30)	83
Karaims	Weissenberg	84·8 (20)	1645 (30)	184 (20)	156	—	—	184 (30)	—	—	—	—	—	—
Kara-Kirghiz	”	87·3 (40)	1653 (40)	184 (40)	161	72·0 (40)	132	183 (40)	—	139	76·2 (40)	1464	36·0 (40)	88
Karachai	Zealand	84·4 (21)	1675 (20)	191 (21)	162	—	—	—	—	—	—	—	—	—
Karelians	Sommier	81·2 (27)	1720 (28)	186 (27)	151	—	—	189 (27)	73·7 (27)	—	—	—	—	—
”	Retzius	86·3 (145)	1629 (141)	187 (139)	162	72·6 (99)	131	184 (95)	80·4 (99)	148	71·8 (99)	1451	36·1 (99)	88
Kirghiz, Boukieew-orda	Charusin	89·1 (99)	1651 (99)	181 (99)	163	75·9 (30)	139	182 (99)	—	—	52·4 (30)	1452	36·0 (30)	89
” Sredni orda	Iwanowsky	87·9 (54)	1645 (54)	183 (54)	161	77·1 (30)	144	192	79·5 (53)	144	53·4 (30)	1465	35·8 (30)	87
” Sir-Daria	Maslowsky	83·2 (30)	1631 (30)	183 (30)	152	71·4 (104)	129	181 (53)	75·2 (300)	138	65·3 (107)	1393	35·4 (300)	85
” Kouldsa	Maziewsky	78·4 (30)	1670 (30)	186 (30)	146	70·1 (300)	133	183 (300)	—	—	62·3 (300)	1401	—	83
Chinese Koulsa	Maziewsky	87·0 (107)	1644 (128)	181 (107)	157	—	—	—	—	—	67·7 (140)	—	—	—
Koumik	Swidersky	77·6 (300)	1686 (300)	189 (300)	147	—	—	—	—	—	72·8 (35)	—	—	—
Kurds, Transcaucasia	Iwanowsky	78·5 (140)	1677 (60)	193 (140)	152	—	—	185 (60)	76·1 (27)	145	—	—	—	—
” Turkomans	Chantre	82·5 (35)	—	189 (35)	159	—	—	177 (89)	81·9 (89)	—	64·4 (27)	—	36·2 (18)	—
” Syrians	”	77·7 (19)	1642 (19)	189 (19)	147	—	—	184 (100)	—	—	63·8 (60)	—	—	—
” Persians	”	87·5 (27)	—	192 (27)	163	60·4 (60)	115	183 (60)	75·2 (70)	139	—	1297	36·7 (89)	76
Laze	Danilow	80·5 (27)	1705 (60)	190 (60)	153	75·8 (89)	139	165 (70)	—	—	72·9 (60)	1504	—	89
Lettish	Chantre	86·7 (59)	1680 (89)	183 (89)	159	62·1 (100)	118	181 (25)	—	—	—	1316	—	79
Lezgin	Waeber	79·9 (10)	1736 (100)	190 (100)	152	61·2 (60)	117	179 (24)	—	—	64·5 (200)	—	—	—
Livonians	Courdow	81·8 (60)	1662 (60)	191 (66)	156	—	—	176 (80)	80·4 (80)	142	69·3 (80)	1326	37·9 (20)	86
Lithuanians	Waldhauer	82·7 (70)	1656 (70)	184 (70)	152	66·7 (22)	122	181 (120)	77·8 (120)	141	71·6 (120)	1373	—	82
”	Brennsohn	84·0 (24)	1548 (44)	183 (24)	154	71·4 (200)	131	187 (102)	77·2 (102)	144	66·8 (48)	—	—	—
”	Baronass	84·5 (200)	1669 (200)	184 (200)	155	—	—	182 (48)	75·2 (48)	137	69·6	—	—	—
Lopar	Sommier	83·2 (80)	1666 (120)	182 (120)	156	70·3 (22)	133	177 (61)	79·6 (61)	141	—	1392	—	85
Little Russians, Kiew	Diebold	85·6 (120)	1632 (102)	187 (102)	152	77·1 (44)	146	179 (22)	74·7 (22)	134	63·5 (15)	1518	36·4 (22)	92
” Charkow	Krasnow	71·1 (102)	1657 (48)	186 (48)	149	66·0 (10)	130	189 (44)	78·6 (15)	140	59·1 (15)	1545	—	93
” Charnigow	Bielodied	79·8 (48)	1701 (61)	185 (61)	152	—	—	178 (15)	—	—	—	—	—	—
” Woroniege	Kosuehow	82·1 (61)	1643 (22)	190 (22)	145	—	—	—	—	—	—	—	—	—
” Wolin	”	76·5 (22)	1655 (45)	189 (44)	154	—	—	—	—	—	—	—	—	—
Kouban	Gilchanko	81·2 (44)	1647 (15)	197 (12)	162	—	—	—	—	—	—	—	—	—
”	Danilow	83·2 (12)	—	—	—	—	—	—	—	—	—	—	—	—
Mesleganz	Zograf	—	—	—	—	—	—	—	—	—	—	—	—	—
Meszeriaks	”	—	—	—	—	—	—	—	—	—	—	—	—	—
Mingrelians	Pantuchow	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE I.—(*continued*).

Race or Group	Authority	Head Index	Stature	Head Length	Head Breadth	Height / Length	Height	Length of Face	Facial Index	Bizygomatic Breadth	Nasal Index	Capacity	Body Length (rel.)	Relative Capacity
Moravia	Maïnow	83·2 (162)	1643 (162)	187 (162)	156	—	—	189 (162)	—	—	66·7 (162)	—	—	—
Orochons	Margaritow	82·8 (37)	1545 (37)	184 (37)	152	—	—	—	—	—	71 (37)	—	—	—
Ossetes	Riskine	81·5 (300)	1690 (300)	190 (300)	155	71·9 (120)	136	178 (120)	81·3 (120)	145	66·3 (300)	1492	33·6 (120)	88
Ostiaks	Chugunow	79·3 (100)	1595 (100)	192 (100)	152	71·0 (58)	136	181 (100)	79·6 (100)	144	74·1 (100)	1490	—	94
Permiaks	Maliew	82·4 (100)	1618 (100)	180 (100)	148	—	—	173 (100)	81·0 (100)	140	—	—	—	—
Persians	Diff. Authorities	78·7 (123)	1677 (142)	189 (123)	149	71·5 (62)	135	188 (111)	74·3 (105)	140	66·8 (14)	1432	36·1 (45)	85
Persians	Danilow	78·4 (46)	1654 (46)	188 (46)	147	72·2 (46)	136	185 (45)	73·9 (45)	137	63·3 (226)	1423	34·3 (223)	86
Poles, Warsaw	Elkind	80·8 (226)	1640 (191)	185 (204)	150	—	—	179 (207)	76·8 (207)	138	72·3 (100)	—	34·7 (100)	—
„ Lublin	Dzerzinsky	82·7 (116)	1655 (116)	186 (116)	154	—	—	186 (100)	75·3 (100)	140	—	—	—	—
„ Radom	Olecknowiz	83·0 (131)	1617 (131)	186 (131)	155	—	—	—	—	—	—	—	—	—
Samoyedi	Zograf	82·5 (22)	1579 (22)	185 (22)	153	72·1 (20)	134	192 (21)	—	—	—	1430	38·8 (22)	90
„	Sommier	84·4 (60)	1540 (62)	189 (68)	160	—	—	—	—	—	—	—	—	—
„	Matziewsky	83·9 (30)	1696 (30)	179 (30)	150	76·1 (30)	137	185 (30)	73·3 (45)	—	54·4 (30)	1398	35·4 (30)	82
Sartes		82·4 (30)	1668 (30)	184 (30)	152	74·2 (30)	137	199 (30)	—	—	51·8 (30)	1442	35·3 (30)	86
Sibo-Shibins		84·8 (356)	1691 (307)	184 (356)	156	72·1 (356)	129	193 (356)	73·3 (356)	142	66·5 (300)	1328	36·1 (60)	81
Tajiks	Maslowsky	86·9 (300)	1646 (307)	179 (300)	155	68·1 (204)	127	185 (300)	77·3 (300)	143	70·7 (199)	1377	—	84
Taranchi	„	82·1 (206)	1645 (206)	186 (207)	153	69·3 (30)	128	179 (207)	78·6 (205)	141	63·9 (30)	1368	—	83
Tatars, Kazan	Païssel	82·8 (30)	1642 (30)	184 (30)	152	—	—	—	—	—	—	—	—	—
Tatars, Kasim	Warushkine	80·9 (61)	1620 (61)	185 (61)	149	—	—	—	—	—	—	—	—	—
„ Ufa	Bensenger	74·5 (107)	1661 (280)	186 (85)	157	—	—	177 (85)	77·0 (50)	134	—	—	35·3 (50)	—
„ Crimea	Talko-Grinzewich	80·6 (50)	1665 (50)	186 (85)	152	—	—	174 (50)	75·9 (72)	140	63·2 (169)	1355	36·5 (34)	80
„	Diff. Authorities	77·6 (207)	1685 (172)	188 (50)	144	70·9 (631)	132	184 (75)	77·0 (50)	—	67·6 (38)	—	—	—
„	Lijin	78·6 (38)	1701 (21)	188 (38)	148	—	—	—	75·9 (72)	140	—	—	—	—
„ Azerbeïjan	Diff. Authorities	86·1 (210)	1636 (209)	186 (210)	160	69·9 (180)	130	188 (210)	80·3 (210)	151	69·1 (210)	1452	34·3 (174)	88
Tats	Chantre	84·5 (73)	1633 (178)	188 (73)	159	69·3 (73)	130	186 (73)	85·2 (73)	159	60·5 (73)	1452	35·0 (42)	89
Telenguts	Louzenko	84·5 (30)	1623 (30)	186 (30)	157	71·3 (30)	133	187 (30)	—	—	57·5 (30)	1448	35·0 (30)	89
Torguts, Tarbagatai	Iwanowsky	82·3 (87)	1631 (86)	192 (86)	158	65·2 (80)	125	185 (86)	81·5 (85)	151	56·9 (99)	1428	35·9 (21)	88
„ Koulja	Maziewsky	84·7 (40)	1660 (40)	183 (40)	155	—	—	—	—	—	56·5 (13)	—	—	—
Tungus	Maïnow	75·6 (58)	1696 (58)	193 (58)	146	68·6 (58)	132	185 (58)	—	—	56·9 (99)	1408	—	83
Turks	Pantuckow	86·9 (150)	1644 (149)	183 (150)	160	68·6 (58)	132	181 (150)	79·6 (150)	144	57·7	1460	33·0 (149)	89
Turkomans	Iaworsky	86·6 (196)	1678 (196)	182 (196)	158	73·1 (149)	134	190 (196)	77·4 (196)	146	—	—	—	—
Oudins	Arutinow	82·3 (24)	1679 (22)	186 (24)	153	—	—	176 (24)	79·1 (24)	140	—	—	—	—
Uzbegs	Maslowsky	80·4 (150)	1666 (150)	190 (150)	153	—	—	183 (150)	76·8 (150)	140	81·2 (150)	—	36·4 (35)	—
Finns, Tawastehus	Retzius	81·9 (30)	1611 (36)	189 (136)	155	—	—	197 (35)	76·6 (35)	151	57·1 (35)	—	—	—
„ Wiborg	Kolmogorow	79·0 (49)	1615 (48)	184 (48)	145	—	—	—	—	—	—	—	—	—
Chalchas	Talko-Grinzewich	81·8 (14)	1660 (14)	187 (14)	153	71·9 (14)	134	—	—	—	—	1439	—	87
Cheremiss	Maliew	79·3 (100)	1643 (100)	194 (100)	154	—	—	187 (100)	79·2 (207)	148	—	—	—	—
Chukchi	Olsufiew	82·3 (207)	1624 (207)	191 (207)	157	69·1 (207)	132	187 (207)	—	—	—	1478	—	91
Esthonians	Grube	82·3 (46)	1607 (46)	190 (46)	156	70·5 (44)	134	193 (46)	—	—	—	1473	—	91
Jakuts	Maïnow	—	—	—	—	—	—	—	—	—	—	—	—	—
„	Witachewsky	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE II.

Living Races from Deniker's Races of Mankind.

Race or Group	Stature	Head Index	Race or Group	Stature	Head Index
Aetas	146	85	Korumbas	156	77
Andamanese	148	81	Kulu-Lahulis	161	75
Annamese	157	83	Laotians	159	84
Arabs	166	76	Lapps	153	87
Australians	167	74	Malé	156	75
Badagas	166	72	Magyars	165	84
Baluchi	166	80	Mal-Paharias	158	76
Baltis	161	74	Mandingans	170	75
Batekes	164	74	Maricopas	172	83
Battas	160	81	Moquis	163	85
Bashilange	168	77	Mushikongos	166	72
Belgians	165	82	Nicobars	163	80
Bhumi (Bhils) ...	159	75	Omahas	173	82
Bilkula Indians ...	166	84	Pawnees	171	80
New Britain	162	77	Piedmontese	165	86
Burmese	165	83	Pimas	170	78
Caribs	157	80	Polynesians	173	80
Chakamas	160	84	Tahitians	173	85
Southern Chinese ...	161	81	Punjabis	168	74
Comanches	168	85	Rotti Islanders ...	160	77
Corsicans	163	77	Sardinians	162	77
Crow Indians	173	80	Santals	161	76
Dalmatians	171	87	Sicilians	163	79
Danakils	167	74	Singhalese	162	79
Dards	161	76	Siouans	173	80
Dravidians	163	74	Solorese	158	83
Eskimos, Alaska ...	163	79	Solomon Islanders ...	162	77
„ Greenland ...	162	77	Basques, Spanish ...	164	79
French	165	84	Spaniards	164	79
Basques	166	83	Sundanese	159	86
Fulahs	174	75	Tamils	165	76
Gypsies	165	80	„ S. India ...	167	77
Hindus	163	73	Tenggerese	160	80
„ Behar	163	76	Tipperahs	161	80
Iroquois	173	79	Toucouleurs	172	74
Italians	164	83	Trao Mois	158	77
Javanese	162	85	Uru-Kurubas	164	76
Jakuns	153	81	Ute Indians	166	79
Kharvars	162	76	Veddahs	155	75
Kothas	163	74	Wolofs	173	75
Kols	165	72	Zuñis	162	83
Kurmis	161	76	Zandehs, W.	172	78

TABLE III.

Cranial Indices.

Race or Group			Cephalic Index	Upper Face Index	Race or Group			Cephalic Index	Upper Face Index
Negroes	...	...	74·3	51·3	Hottentots	...	...	72·4	52·1
Australians	...	...	71·0	79·3	Tasmanians	...	...	75·8	46·3
Javanese	...	...	81·5	49·7	Torres Straits		...	68·6	52·6
Marquisans	...	...	75·8	51·6	Viti-Levu	...	...	66·8	48·2
Auvergnats	...	...	86·3	49·8	Maoris	...	...	74·0	49·4
Esthonians	...	...	78·6	53·1	New-Caledonians		...	71·6	48·6
Dawson Straits		...	74·2	49·5	Eskimos	...	...	71·8	51·7
Württembergians		...	82·6	53·0	„ Greenland		...	71·4	56·1
Bavarians	...	...	82·7	52·6	Veddahs	...	...	69·9	49·9
Chinese	...	...	79·0	54·0	Andamanese	...	...	81·5	48·7
Ainos	...	...	76·5	50·0	Malays	...	...	80·2	50·2
Kalmuks	...	...	81·6	53·2	Polynesians	...	...	77·7	50·2
Minusinsk	...	...	73·2	53·1	Alfurous	...	...	80·7	50·9
„	...	...	74·6	52·5	Micronesians	...	...	73·4	51·1
French	...	...	79·5	51·9	Papuans	...	...	72·4	49·5
Bashkirs	...	...	81·0	52·1	Czechs	...	...	83·2	51·4
Duke of York Islanders			70·4	48·7	Telenguts	...	...	86·1	51·0
Kanakas	...	...	78·8	49·8	Soiots	...	...	86·3	53·2
Celebes	...	...	82·1	51·8	Samoyeds	...	...	82·4	53·2
Naqada	...	...	72·9	53·8	Tarbagatai-Torguts		...	83·3	52·2
Thebans	...	...	75·1	53·5	Abyssinians	...	...	75·7	52·3
Copts	...	...	77·7	54·5	Senftenberg Crania		...	84·2	49·5
Etruscans	...	...	78·1	54·1	Rumanians	...	...	80·6	52·3
Kalmuks	...	...	81·2	55·2					

When a race is repeated, two series have been used.

TABLE IV. *Cranial Measurements of Various Races.*

Race or Group	Authority	Cephalic Index	Length	Breadth	Height	Upper Face Height	Length of Basis	Minimum Frontal Breadth	Upper Face Height	Bizygomatic Breadth
Negroes	Different Authorities	72·3	183·3	132·3	133·9	101·9	117·9	95·7	67·8	130·3
Australians	Smidt, Sandfroi, Flower, etc.	71·0	185·7	132·2	134·2	102·1	118·7	96·3	65·7	133·2
Javanese	Davis, Flower, etc.	81·5	174·2	141·9	145·4	99·3	121·2	93·9	67·2	124·3
Dawson Straits	Sergi	—	177·3	131·6	134·4	—	—	91·7	63·7	128·6
Württembergians	"	82·6	179·8	147·4	131·3	98·6	—	98·2	71·1	134·1
Torguts	Iwanowsky	81·6	179·0	146·0	129·0	96·0	—	96·0	73·0	138·0
Minousinsk	Goroshenko	73·2	186·2	138·3	135·7	101·8	—	102·1	74·8	136·8
Duke of York Islanders	Godefroi	—	180·1	132·0	137·0	100·1	118·1	92·0	68·5	133·4
Kanakas	Davis	—	181·9	142·8	141·8	103·8	124·9	94·2	67·7	134·5
Polynesians	Flower, Davis, Godefroi, etc.	—	183·6	140·7	138·0	104·5	125·1	94·5	68·3	136·5
Viti-Levu	Flower	66·8	190·9	127·6	140·1	102·2	115·1	94·3	64·8	131·2
Naqada	Fawcett	72·9	184·6	134·8	135·2	99·3	—	91·1	67·6	125·6
Thebans	"	75·1	181·9	136·6	136·0	100·6	—	93·8	68·8	128·3
Copts	Ranke	77·2	177·0	136·8	137·2	100·9	—	94·1	68·7	125·8
Bavarians	"	83·2	180·6	150·5	133·8	100·3	—	103·7	70·8	135·0
French	Koernef	—	180·4	144·1	131·0	99·7	—	96·2	68·2	131·3
Bashkirs	Nicolsky	81·0	174·4	140·8	129·3	97·0	—	95·4	68·8	132·0
Ainos	Koganei	76·5	183·8	141·2	139·5	105·4	—	96·2	69·8	137·3
Etruscans	Schmidt	79·8	182·8	142·8	139·9	105·0	122·3	—	70·9	131·2
Berbers	Crania Ethnica	73·9	184	136	138	101	—	96	—	129·0
Tahitians	"	75·1	185	139	143	106	—	94	—	133
Sungarians	"	78·2	179	140	143	102	—	92	—	131
Loyalty Islanders	"	69·8	189	132	139	104	—	96	—	136
Kioways	"	75·5	184	139	141	105	—	94	—	134
Andamanese	Flower	81·5	167·3	136·5	129·3	93·9	113·4	93·0	60·0	123·6
Veddahs	"	69·9	178·4	124·8	133·6	98·7	110·3	90·6	60·1	121·9
New Caledonians	Crania Ethnica	69·7	188	131	170	104	—	96	—	135
" " Kanala	"	71·3	185	132	139	104	—	97	—	137
Arabs	"	73·0	185	135	135	103	—	99	—	126
Tarbagataï	Iwanowsky	83·3	177	148	122	90	—	94	72	135
Chinese	Koganei	78·3	180·1	140·5	140·5	100·4	—	91·9	75·2	135·6
Eskimos	Crania Ethnica	71·8	188	135	139	106	—	95	—	135
Auvergnats	Broca	—	178·1	154·0	131·0	97·2	—	—	—	—
Basques	"	—	185·8	142·3	131·6	99·2	—	—	—	—
Hottentots	"	71·4	182·8	130·3	132·6	100·2	—	—	66·5	128·1
Papuans	Godefroi, etc.	—	179·6	121·7	134·3	99·9	120·9	93·7	65·1	133·8
Micronesians	Different Sources	—	180·0	133·8	136·6	105·8	123·6	—	67·8	128·2
Abyssinians	Koernef	—	181	137	136	102	—	95	68	130
Nordlander (Netherlands)	"	—	175	144·5	131	97	—	97	65	131
Tasmanians	Flower, etc.	—	185·3	139·3	133·0	100·5	121·6	95·1	60·6	131·3
Torres Straits	Thomas	68·3	190·6	130·0	135·4	103·4	—	98·3	71·5	135·2
Marquesans	R. College Skulls	—	185·9	141·1	137·9	104·5	125·3	95·0	70·0	136·8
Maoris	"	74·0	189·8	137·1	139·6	104·3	126·7	96·6	68·1	137·8

APPENDIX II.   CORRELATION TABLES.

TABLE I.   *Russian Races*, cf. Table II. p. 300.
Head Index.

Stature.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	Totals
154	—	—	—	—	—	—	—	1	1	—	—	—	—	—	2
155	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
157	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
159	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	1	—	—	3	1	—	—	—	—	—	—	5
162	—	—	—	—	—	1	4	1	2	—	—	—	—	—	8
163	—	—	1	—	—	—	1	1	1	—	3	1	—	—	8
164	—	—	1	1	—	2	2	2	1	1	1	2	1	1	14
165	—	—	1	—	1	2	2	3	—	1	—	2	—	1	13
166	1	—	—	—	1	1	2	1	1	1	—	—	—	—	8
167	—	—	1	—	2	1	1	1	1	—	1	1	—	—	9
168	1	—	2	1	—	—	2	—	1	—	—	2	—	—	9
169	—	—	1	—	—	1	—	—	—	1	—	—	—	—	3
170	1	—	—	1	1	—	1	—	1	—	—	—	—	—	5
171	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
172	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	3	1	7	5	6	9	20	11	10	4	6	8	1	1	92

TABLE III.
Head Index.

Face Index.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	Totals
72	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
73	—	—	—	—	—	—	—	1	1	—	—	—	—	—	2
74	—	—	1	—	—	1	1	—	—	—	—	—	—	—	3
75	1	—	1	—	2	—	—	3	—	—	—	—	—	—	7
76	—	—	1	—	1	—	1	—	—	—	—	2	—	—	5
77	—	—	—	—	1	2	3	1	1	—	—	3	—	—	11
78	—	1	—	—	—	—	1	—	—	—	1	1	—	—	4
79	—	—	—	—	—	1	4	1	—	—	1	1	—	—	8
80	—	—	—	1	—	—	1	—	1	1	2	1	—	1	8
81	—	—	—	—	—	1	1	—	—	—	—	—	—	—	2
82	—	—	—	—	—	1	—	—	—	—	—	1	—	—	2
83	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
84	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
85	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	1	1	3	1	4	6	13	5	4	2	4	9	—	1	54

TABLE IV.
Cranial Cephalic Index.

Upper Face Index.	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	Totals
46	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
49	—	—	—	—	2	2	—	1	1	—	—	—	—	—	1	—	—	1	—	—	8
50	—	—	—	1	—	—	—	1	—	1	—	1	1	1	1	—	—	—	—	1	7
51	—	—	—	—	—	1	1	—	—	—	—	—	—	—	1	—	1	—	—	1	5
52	—	—	1	—	—	2	—	1	2	—	—	1	—	—	2	1	1	—	—	—	11
53	—	—	—	—	—	—	1	—	1	—	—	1	1	—	—	2	2	—	—	1	8
54	—	—	—	—	—	1	—	—	—	—	—	2	1	—	—	—	—	—	—	—	4
55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
56	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	1	1	3	4	3	2	3	4	—	3	4	1	6	3	4	1	—	3	47

TABLE II. *Non-Russian Races.*
Stature.

Head Index (rows) × Stature (columns)

Head Index	146	147	148	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	Totals
72														1		1	2					1				3
73												1		3												2
74												2			1			2	1				1	1		9
75							2			1		2								1	1				1	8
76							1					1	1			1	1									8
77									1		1		4	1	1			1					1			11
78								1	1					1		1	1				1	1		1		2
79													1				1							3		6
80											1			2	1	1										9
81			1	1							1			1		1								1		3
82																1	1						1			2
83								1	1				1	1	1		1									8
84										1	1													1		4
85													1	1		1			1							5
86	1									1																2
87				1																		1				2
Totals	1	—	1	2	—	—	3	2	3	3	4	6	8	11	4	7	7	3	2	1	2	3	3	7	1	84

TABLE V.
Nasal Index.

Head Index (rows) × Nasal Index (columns)

Head Index	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	81	Totals
76	1	1																	1									2
77												2					1											1
78			1										1					2						1			1	6
79														1			1		1									2
80		1														1							1					3
81		1										1	1						1				2					4
82							2						1	1			1	2		1	1		1			1		12
83							1		1	1										1	1	1						10
84				1	1	1	1								1				1									6
85										1		1	1									1						3
86							1			1		1		1												1		6
87								1					2		2	1												8
88								1							1													1
89																						1						1
Totals	1	3	1	1	1	1	5	2	1	3	—	5	6	3	4	2	3	4	4	2	2	3	4	1	—	2	1	65

TABLE VI.

Length of the Head.

Stature	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	Totals
154	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
155	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
158	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
159	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	2	—	1	—	—	1	—	—	—	—	—	—	—	—	4
162	—	—	—	1	—	—	2	—	1	2	—	—	—	—	1	—	—	—	—	—	—	7
163	1	—	—	1	—	—	1	—	—	—	2	1	—	1	—	1	—	—	—	—	—	8
164	—	—	—	—	1	—	2	2	—	2	1	—	2	1	—	—	—	1	—	—	—	12
165	—	—	1	—	1	2	1	2	1	2	—	1	1	—	—	—	—	—	—	—	1	13
166	—	—	1	—	—	—	1	—	—	1	1	2	—	—	1	—	—	—	—	—	—	7
167	1	—	—	—	1	1	—	1	—	1	1	—	—	1	1	—	—	—	—	—	—	8
168	—	—	—	—	—	1	1	—	—	3	—	—	2	—	—	1	1	—	—	—	—	9
169	—	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	—	—	3
170	—	—	1	—	—	—	—	—	1	—	—	—	—	1	—	—	1	—	—	—	—	4
171	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
172	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	2	—	3	2	3	4	10	9	4	12	5	4	7	7	3	3	2	2	—	—	1	83

TABLE VII.

Head Breadth.

Stature	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	Totals
154	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
155	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
159	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	1	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	4
162	—	—	—	—	—	1	1	2	—	—	—	—	—	—	1	2	—	—	—	—	—	7
163	—	—	—	—	—	1	—	—	—	1	—	1	1	—	—	1	1	1	—	1	—	8
164	—	—	1	—	1	—	—	1	—	1	2	1	—	2	1	—	—	1	1	—	—	12
165	—	—	—	—	—	1	—	—	2	2	1	2	2	—	—	—	—	1	1	1	1	13
166	1	—	—	—	—	—	2	—	—	1	1	—	1	1	—	—	—	—	—	—	—	7
167	—	—	—	1	—	—	—	—	—	2	1	—	1	1	1	—	—	—	—	1	—	8
168	1	1	—	—	—	—	1	—	—	1	1	—	—	1	—	2	1	—	—	—	—	9
169	—	—	—	—	1	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	3
170	—	—	—	1	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	4
171	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
172	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	1	2	2	2	3	4	5	4	12	8	6	7	7	5	3	2	2	2	3	1	83

TABLE VIII.

Head Height.

Stature	115	116	117	118	—	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	—	144	145	146	Totals
155							1																								1
156										1																					—
157																			1												1
158																					1										1
159																															1
160									1								1														—
161							1			1					1			1			1										1
162												1		1	1											1					3
163							1					1	1	2	1	1	1	1			1			1							7
164															1	2															7
165																	1		2	1		1								1	11
166			1								1					1	1	1			1							1			3
167																				1	1	1		1	1						7
168	1																1														5
169																						1									1
170				1																											3
—																															—
174																															1
Totals	1	—	1	1	—	2	3	—	1	2	1	2	1	3	4	4	5	3	3	2	5	3	—	2	1	1	—	1	—	1	53

TABLE IX.

Bizygomatic Breadth.

Stature	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	—	151	152	—	159	Totals
156	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
157	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
159	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
160	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	1	—	1	1	1	—	—	—	—	—	—	—	—	—	1	—	—	—	2
162	—	—	—	1	1	1	1	—	—	—	—	—	—	—	1	—	—	—	—	—	5
163	—	—	—	—	—	1	—	—	—	—	1	—	—	1	—	—	2	1	—	1	7
164	1	—	—	1	1	1	1	1	—	—	2	—	—	1	—	—	1	—	—	—	10
165	—	—	—	1	—	2	2	2	—	2	—	1	—	—	1	—	—	—	—	—	11
166	1	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
167	—	—	—	—	—	—	1	1	—	1	1	—	—	—	—	—	—	—	—	—	4
168	—	—	—	—	—	—	3	—	—	1	—	1	1	—	—	—	—	—	—	—	6
169	—	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	3
170	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
171	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
172	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	—	1	5	3	7	8	5	1	5	5	4	2	2	2	—	4	1	—	1	58

TABLE X.

Face Index.

Stature	72	73	74	75	76	77	78	79	80	81	82	—	85	Totals
156	—	—	—	—	—	—	—	1	—	—	—	—	—	1
157	—	—	—	—	—	—	1	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—
159	—	—	—	—	—	—	—	—	1	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	1	—	—	1	—	—	—	—	—	—	—	2
162	—	—	—	—	1	—	—	2	—	1	—	—	—	4
163	—	—	—	—	1	—	—	—	1	1	—	—	1	4
164	1	1	—	1	—	—	—	3	2	—	—	—	—	8
165	—	—	—	2	1	3	—	1	2	—	—	—	—	9
166	—	—	—	3	—	1	—	—	—	—	—	—	—	4
167	—	—	—	—	1	1	2	—	—	—	—	—	—	4
168	—	—	1	—	1	1	—	1	1	—	1	—	—	6
169	—	1	—	1	—	—	—	—	—	1	—	—	—	3
170	—	—	—	—	—	—	—	—	1	—	—	—	—	1
171	—	—	—	—	—	—	1	—	—	—	—	—	—	1
172	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	1	2	3	7	5	7	4	8	8	3	1	—	1	50

TABLE XI.

Skull Length.

Skull Breadth	167	—	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	Totals
122	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
126	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
129	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	2
131	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
132	—	—	—	—	—	1	—	—	1	—	—	1	—	1	1	—	—	1	—	—	6
133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
134	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
135	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	3
136	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
137	—	—	—	—	—	1	—	—	—	1	1	—	—	—	—	—	—	—	1	—	4
138	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
139	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	3
140	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
141	—	—	1	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	—	—	4
142	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
143	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
144	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
146	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
147	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
148	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
150	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
154	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	2	1	—	3	2	2	6	2	2	3	3	5	5	—	2	1	1	2	43

TABLE XII.

Skull Length.

Skull Height	167	—	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	Totals
122	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
129	1	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
131	—	—	—	1	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	4
132	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
134	—	—	—	—	—	1	1	—	1	1	—	—	1	—	1	—	—	—	—	—	6
135	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	1	3
136	—	—	—	—	—	—	—	—	—	1	1	—	—	—	1	—	—	—	—	—	3
137	—	—	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	3
138	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	3
139	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	1	1	—	—	5
140	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	1	1	4
141	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
142	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	2
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
145	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	2	1	—	3	2	2	6	2	2	3	3	5	5	—	2	1	1	2	43

TABLE XIII.
Skull Breadth.

Skull Height (rows) × Skull Breadth (columns).

Skull Height \ Skull Breadth	122	—	125	—	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	—	154	Totals
122																														1
—																														
129																		1					1							3
130					1																				1					4
131			1															2												1
132									3			2				1			1		2			1			1			2
133							1																							6
134	1											1		2	1	1				1			1	1					1	3
135													1	1																3
136																	1													3
137											1							2		1										3
138									1														1							5
139								1	2							1	1	1												4
140																				1										1
141												1																		1
142									2										1											2
143																														
144					1													1												1
145																														
Totals	1		1		1		2	1	6		1	3	2	4	1	3	2	4	2	2	2		1	1	1		1		1	**43**

(For Table XIV. see p. 306.)

TABLE XVI.
Skull Breadth.

Upper Face Height (rows) × Skull Breadth (columns).

Upper Face Height \ Skull Breadth	—	122	—	125	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	Totals
60		1												1		1												2
61				1																								1
—							1		1		1	1							1	1	1							
64														2														1
65									1					2		1				1	1							3
66																								1				2
67					1										1													1
68																	1	2										9
69									1														1				1	3
70																												3
71																									1			2
72												1			1				1						1			4
73							1											1										1
74																	1											1
75																												
Totals	1	1		1			2		4	1	1	1	1	4	1	1	1	4	1	2	2		1	1	1	1		**32**

TABLE XIV.

Upper Face Height.

	60	61	—	64	65	66	67	68	69	70	71	72	73	74	75	Totals
122	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	2
125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	3
129	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
130	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
131	—	1	—	—	2	—	—	1	—	—	1	—	—	—	—	5
132	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
133	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
134	—	—	—	—	1	—	—	1	—	—	1	—	—	—	—	3
135	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	3
136	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	2
137	—	—	—	—	—	—	—	—	—	2	—	—	—	—	1	3
138	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	2
Totals	2	1	—	1	3	2	1	9	3	2	4	1	1	—	2	32

Bizygomatic Breadth.

TABLE XV.

Skull Length.

	167	—	174	175	176	177	178	179	180	181	182	183	184	185	186	—	190	191	Totals
60	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
65	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	1	3
66	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
67	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
68	—	—	—	—	—	—	—	—	3	1	1	1	1	1	—	—	1	—	9
69	—	—	1	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	2
71	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	1	4
72	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
73	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	2
Totals	1	—	2	1	—	3	1	1	6	2	2	3	1	2	4	—	1	2	32

Upper Face Height.

TABLE XVII.

Skull Height.

	122	—	129	130	131	132	133	134	135	136	137	138	139	140	141	142	—	145	Totals
60	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
65	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	—	3
66	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
67	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
68	—	—	—	—	1	—	—	1	1	1	2	1	—	1	—	1	—	—	9
69	—	—	1	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	2
71	—	—	—	—	1	—	—	1	1	—	—	—	1	—	—	—	—	—	4
72	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
73	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
Totals	1	—	3	—	3	—	2	6	2	3	3	2	2	3	—	1	—	1	32

Upper Face Height.

TABLE XVIII.

Skull Length.

	167	—	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	Totals
122	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	—	—	—	—	—	1	—	—	—	—	—	—	—	2	—	—	—	—	—	—	3
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	3
129	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
130	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
131	—	—	—	1	—	—	—	1	1	—	—	1	—	1	—	—	—	—	—	1	6
132	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	3
134	—	—	—	—	—	—	—	—	2	—	1	—	1	—	—	—	—	—	—	—	4
135	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	2	—	—	1	5
136	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	1	—	—	3
137	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	—	—	—	—	—	4
138	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	2
Totals	1	—	2	1	—	3	1	2	6	2	2	3	3	5	4	—	2	1	1	2	41

Bizygomatic Breadth.

39—2

TABLE XIX.
Skull Breadth.

Bizygomatic Breadth	122	125	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	Totals
122	1																									1
123											1															1
124																										—
125		1								2		1														3
128									1																	1
129																										—
130			1																2							2
131							1					1		1	1			1								6
132							1		1													1				1
133														1		1									1	3
134					1		2							1				1					1			4
135																										5
136	1									1			1													3
137					1	1									1	1					1					4
138							1					1				2										2
Totals	1	—	1	—	2	1	6	—	1	3	2	4	1	3	2	4	1	2	2	—	1	1	1	—	1	41

TABLE XX.
Skull Height.

Bizygomatic Breadth	122	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	Totals
122										1									1
123		1								1								1	3
124																			—
125				2						1									3
126																			—
127																			—
128						1			1						1				3
129							1		1										2
130							1		1										2
131							1	1			1	1	1					1	6
132											1								1
133							1			1		1							3
134	1					1		1		1	1								6
135								2					1						3
136							1			1	1								3
137											1	1	1						3
138												2			1				2
Totals	1	3	—	3	—	2	6	3	3	3	3	5	4	1	1	2	—	1	41

TABLE XXI.

Minimum Frontal Breadth.

Skull Length	91	92	93	94	95	96	97	98	99	—	102	103	104	Totals
167	—	—	1	—	—	—	—	—	—	—	—	—	—	1
—														
174	—	—	—	1	1	—	—	—	—	—	—	—	—	2
175	—	—	—	—	—	1	—	—	—	—	—	—	—	1
176	—	—	—	—	—	—	—	—	—	—	—	—	—	
177	—	1	—	2	—	—	—	—	—	—	—	—	—	3
178	1	—	—	—	—	—	—	—	—	—	—	—	—	1
179	—	1	—	—	—	1	—	—	—	—	—	—	—	2
180	—	2	—	1	—	1	—	1	—	—	—	—	—	5
181	—	—	—	—	1	—	—	—	—	—	—	—	1	2
182	—	—	—	2	—	—	—	—	—	—	—	—	—	2
183	—	—	—	—	—	1	—	—	—	—	—	—	—	1
184	—	—	—	2	—	1	—	—	—	—	—	—	—	3
185	1	—	—	1	1	—	1	—	1	—	—	—	—	5
186	—	—	—	—	1	2	—	—	—	—	1	—	—	4
187	—	—	—	—	—	—	—	—	—	—	—	—	—	—
188	—	—	—	—	1	1	—	—	—	—	—	—	—	2
189	—	—	—	—	—	1	—	—	—	—	—	—	—	1
190	—	—	—	—	—	—	1	—	—	—	—	—	—	1
191	—	—	—	1	—	—	—	1	—	—	—	—	—	2
Totals	2	4	1	10	5	8	3	2	1	—	1	—	1	38

TABLE XXII.

Minimum Frontal Breadth.

Skull Breadth	91	92	93	94	95	96	97	98	99	—	102	103	104	Totals
122	—	—	—	1	—	—	—	—	—	—	—	—	—	1
—														
125	1	—	—	—	—	—	—	—	—	—	—	—	—	1
—														
128	—	—	—	1	—	—	—	—	—	—	—	—	—	1
129	—	—	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	1	—	—	—	—	—	1
131	—	—	—	—	—	1	—	—	—	—	—	—	—	1
132	—	2	—	—	—	3	1	—	—	—	—	—	—	6
—														
135	1	—	—	—	1	—	—	—	1	—	—	—	—	3
136	—	—	1	—	—	1	—	—	—	—	—	—	—	2
137	—	—	—	2	1	—	1	—	—	—	—	—	—	4
138	—	—	—	—	—	—	—	—	—	—	1	—	—	1
139	—	—	—	2	1	—	—	—	—	—	—	—	—	3
140	—	2	—	—	—	—	—	—	—	—	—	—	—	2
141	—	—	—	1	2	1	—	—	—	—	—	—	—	4
142	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	—	—	1	—	—	—	—	—	—	—	—	—	1
144	—	—	—	—	—	1	1	—	—	—	—	—	—	2
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—
146	—	—	—	—	—	1	—	—	—	—	—	—	—	1
147	—	—	—	—	—	—	—	1	—	—	—	—	—	1
148	—	—	—	1	—	—	—	—	—	—	—	—	—	1
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—
150	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	4	1	10	5	8	3	2	1	—	1	—	1	38

TABLE XXIII.

Minimum Frontal Breadth.

Skull Height.	91	92	93	94	95	96	97	98	99	—	102	103	104	Totals
122	—	—	—	1	—	—	—	—	—	—	—	—	—	1
---	—	—	—	—	—	—	—	—	—	—	—	—	—	—
129	—	—	1	—	1	1	—	—	—	—	—	—	—	3
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—
131	—	—	—	—	—	1	1	1	—	—	—	—	—	3
132	—	—	—	—	—	—	—	—	—	—	—	—	—	—
133	—	—	—	—	1	—	—	—	—	—	—	—	—	1
134	1	1	—	1	—	2	—	—	—	—	—	—	1	6
135	1	—	—	—	—	—	—	1	1	—	—	—	—	3
136	—	—	—	1	1	—	—	—	—	—	1	—	—	3
137	—	1	—	1	—	—	—	—	—	—	—	—	—	2
138	—	—	—	1	1	1	—	—	—	—	—	—	—	3
139	—	—	—	—	1	2	1	—	—	—	—	—	—	4
140	—	1	—	1	—	1	1	—	—	—	—	—	—	4
141	—	—	—	1	—	—	—	—	—	—	—	—	—	1
142	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	1	—	1	—	—	—	—	—	—	—	—	—	2
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—
145	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2	4	1	10	5	8	3	2	1	—	1	—	1	38

TABLE XXIV.

Skull Basis.

Naso-Alveolar Height.	90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
60	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	3
66	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
67	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
68	—	—	—	—	—	—	—	1	2	—	1	—	3	—	1	8
69	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	3
71	—	—	—	—	—	—	—	1	1	—	—	1	—	1	—	4
72	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
73	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
Totals	1	—	1	—	1	2	—	4	7	2	5	1	4	2	1	31

TABLE XXV.

Skull Basis.

Skull Breadth	90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
122	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
129	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
131	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
132	—	—	—	—	—	—	—	—	1	—	2	—	2	—	—	5
133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
134	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
135	—	—	—	—	—	—	—	1	—	—	—	1	—	—	1	3
136	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	2
137	—	—	—	—	—	—	—	—	—	2	1	—	1	—	—	4
138	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
139	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	3
140	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
141	—	—	—	—	—	1	—	—	—	—	—	—	2	1	—	4
142	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
143	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2
144	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
146	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
147	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
148	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
150	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
154	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	—	1	—	1	3	—	5	6	4	6	2	7	3	3	42

TABLE XXVI.

Skull Basis.

Bizygomatic Breadth	90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
122	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	—	—	—	—	—	—	—	1	—	1	—	1	—	—	—	3
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	3
129	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
130	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	2
131	—	—	—	—	—	1	—	—	2	—	2	—	—	1	—	6
132	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	3
134	—	—	—	—	—	—	—	1	1	—	—	—	1	1	—	4
135	1	—	—	—	—	—	—	—	1	—	—	1	1	—	1	5
136	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	3
137	—	—	—	—	—	—	—	—	—	—	1	—	2	1	—	4
138	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	2
Totals	1	—	1	—	1	2	—	4	7	3	6	2	7	3	3	40

TABLE XXVII.

Skull Basis.

Skull Height.		90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
	122	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
	—																—
	129	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	3
	130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	131	—	—	—	—	—	2	—	1	1	—	—	—	—	—	—	4
	132	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
	133	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
	134	—	—	—	—	—	—	—	1	2	—	2	—	—	—	—	5
	135	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	3
	136	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
	137	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	3
	138	—	—	—	—	—	—	—	—	—	1	—	—	2	—	—	3
	139	—	—	—	—	—	—	—	—	—	—	—	—	2	2	1	5
	140	—	—	—	—	—	—	—	—	1	—	1	—	2	—	—	4
	141	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
	142	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
	143	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2
	144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	145	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
	Totals	1	—	1	—	1	3	—	5	7	3	6	2	7	3	3	42

TABLE XXVIII.

Skull Basis.

Skull Length.		90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
	167	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
	—																—
	174	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
	175	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
	176	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	177	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
	178	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
	179	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	2
	180	—	—	—	—	—	—	—	1	4	—	—	—	—	—	1	6
	181	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
	182	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	2
	183	—	—	—	—	—	—	—	—	1	—	1	—	—	1	—	3
	184	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	3
	185	—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	5
	186	—	—	—	—	—	—	—	1	—	—	2	—	1	1	—	5
	187	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	188	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2
	189	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
	190	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
	191	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	2
	Totals	1	—	1	—	1	3	—	5	7	3	6	2	7	3	3	42

STATISTICAL STUDIES IN IMMUNITY.

SMALLPOX AND VACCINATION.

By JOHN BROWNLEE, M.A., M.D., Glasgow.

In writing this paper on smallpox and vaccination I have no intention of entering into any controversy upon the subject. The fact that vaccination protects against smallpox is assumed, and consideration is alone given here to the relationship between vaccination, revaccination, and smallpox, in so far as the facts throw light upon the growth, decline, or establishment of immunity in persons who have passed through one or other of these infections. I would rather have chosen for this investigation some other disease, but as an inquiry of this kind is essentially statistical there was no other series of figures available. The disadvantage of choosing vaccination and smallpox arises from the fact that the former is to a considerable extent different from the latter, and though there seems good reason for believing that vaccination is a disease caused by the parasite of smallpox so modified as to have lost one stage in its life history, yet the fact that it protects chiefly against the second stage of smallpox causes the immunity relationship to be of a more complex character than is desirable.

Before proceeding, however, to consider the relationship of smallpox to vaccination one fact demands special preliminary notice: that is the change which has taken place in the age at which vaccinated persons are attacked by smallpox. All over the country from the earliest period of last century for which statistics are available there has been a steady increase in the mean age at which such persons have been attacked by smallpox.

This is quite distinct from the rise in the mean age at death, shown by the combined mortality statistics of the vaccinated and unvaccinated which is commonly and correctly looked upon as largely due to the protective influence of vaccination upon the younger members of the community. What I refer to is a process which applies specially to the vaccinated, and which does not seem to admit of any complete explanation from the point of view of alteration in the character of the population. In London, for instance, between the years 1836

and 1851, of 3,094 cases 2,719 were under the age of thirty, or more than seven vaccinated persons under thirty were affected with smallpox for each individual above that age. In Dr Gayton's statistics relating to the period 1870 to 1883 the corresponding proportion is $5\frac{1}{2}$ to 1. In Dr Sweeting's referring to 1880 to 1885 the proportion is 3 to 1, while in the epidemics of the last ten years two

TABLE I.

Age of Smallpox Attack in Case of Vaccination.

Age Periods	London, 1836–51 Cases	Deaths	London, 1870–72 Cases	Deaths	London, 1880–85 Cases	Deaths	London, 1892–95 Cases	Deaths	London, 1902 Cases	Deaths
0—5	7	2	195	38	26	—	15	—	16	—
5—10	56	7	786	60	108	2	101	—	98	2
10—15	206	10	4504	238	256	8	188	2	265	3
15—20	866	49			405	10	533	5	654	17
20—25	1058	93	3761	410	632	59	639	16	1049	45
25—30	526	55					454	16	1017	70
30—35	210	21	1297	257			370	14	827	91
35—40	102	20					234	14	616	111
40—50	61	10	435	110	455	68	293	8	726	154
50—60	2	1	134	30			89	8	254	52
60—70	—	—	62	12			24	7	105	27
70—							10	1	36	6
0—30	2719	216	9246	756	1427	79	1950	39	3099	137
30—	375	52	1928	389	455	68	1020	52	2564	441
Mean Age	22·6	24·5	21·9	26·4			26·4	33·4	28·6	38·2

persons under thirty years only are affected for each one over that age. In every town where there are statistics for a series of outbreaks of smallpox during the last thirty years exactly the same fact is observed. To mention Glasgow; in 1872 this proportion was 6 to 1, in 1892 to 1895 $2\frac{1}{2}$ to 1, and during the epidemic of 1901 the ratio reached unity. Now there can be no doubt that a part of this is due to the gradual disappearance of adult persons protected by previous attacks of smallpox. In addition during the years immediately succeeding Jenner's discovery vaccination was performed promiscuously on young and old alike, and persons vaccinated in adult years must have constituted during the first sixty years of the century a very considerable proportion of the total vaccinated persons at higher ages, as it can easily be seen that any person in 1836 whose age was greater than thirty-seven must have been vaccinated at the earliest at an age corresponding to the difference of the actual age and of thirty-seven years. By 1870, however, both these influences must have to a considerable extent disappeared, and yet we see that the change referred to has been going on just as markedly since that date. Against these two factors must be set one which undoubtedly acts in the opposite direction, and that is the gradual better enforcement of vaccination during infancy, which, by continually providing a larger number of vaccinated persons at lower ages, ought if no other change were taking place to supply a greater number of cases at these ages. The influence of this should at least be sufficient, at any rate since 1870, to neutralise that due

to the factors before mentioned. Unfortunately there is a complete absence of statistics referring to this matter, so that an examination of the relative influences of these different factors is impossible. The effect of revaccination on the age incidence of persons suffering from smallpox can be more readily estimated. When performed in the absence of an epidemic, or when slight outbreaks only of this disease are present, there is not a sufficient amount done to influence statistics much one way or another. In Sheffield, for instance, when the great epidemic of 1887 occurred, about five to seven per cent. of the population at each age period above fifteen years were found to be revaccinated. When, however, as in this epidemic in Sheffield and in the late epidemics in Glasgow revaccination is performed on a large scale, it is found that a much larger proportion of persons between ten and twenty years of age avail themselves of this means of protection than at higher ages. The effects of this on later epidemics are obviously, (1), if the second epidemic occurs within a few years of the first to raise the mean age; and (2), if the later epidemic occur about fifteen years thereafter to lower the mean age of attack. Revaccination, however, as a rule only begins to have an important modifying effect on the epidemic concerned when it is naturally approaching its end.

The change in the age incidence of smallpox just discussed is one which not only applies to the vaccinated as a class, but equally to the different groups comprehended in this class. Persons suffering from smallpox are for the purpose of classification commonly divided into groups as they present on their arms one, two, three, and four or more vaccinal cicatrices. In each of these groups the mean age has steadily risen. Now with regard to the group of those who present one cicatrix this is to be expected. Since 1870 persons vaccinated in this manner have been becoming progressively fewer and fewer, and consequently in London the bulk of those presenting one vaccinal cicatrix must consist of persons at the higher ages. Exactly the same change in the mean age, however, is found to be among the members of the group presenting one cicatrix in Glasgow, although no such change in the method of vaccination has taken place there. With regard to the group, however, of persons with four or more scars, year by year since 1870 a larger and larger number of such have come into existence at lower ages and consequently it would be expected that the mean age among this group would be at any rate stationary if not lower. This is not found to be the case. The annexed table exhibits the change in the mean age which has taken place in London since 1870. The rise noted in the mean age of persons with four marks

TABLE II.

Mean Age of Cases and Deaths in the Epidemics in London, 1870—1902.

	One Scar		Two Scars		Three Scars		Four Scars	
	Cases	Deaths	Cases	Deaths	Cases	Deaths	Cases	Deaths
1870–83	21·8	26·2	21·0	25·1	18·1	24·0	17·0	24·8
1892–95	26·8	35·3	29·5	43·9	26·5	31·7	23·2	35·7
1902	34·2	39·6	34·0	41·3	31·0	37·7	27·2	34·5

in view of the fact just mentioned seems to establish that this phenomenon is not due to alteration in the character of the population, but presents a biological as distinct from a statistical change. The explanations which can be offered from the biological point of view are three. Firstly, there may be some secular change taking place in the susceptibility of the population towards smallpox itself; secondly, there may be a like change taking place in the smallpox organism; and thirdly, it is possible that the passage of the organism of vaccinia through several thousand generations in the human subject has gradually increased the protective value of lymph; here again, however, there may be secular change. This increase in protective value, while it cannot be held proved, is in striking contradiction to what has been hitherto considered as probable. Plants propagated by cuttings, by tubers, and such like means, are well known to deteriorate in quality, and frequent recourse must be had to propagation by seeds if the strength of the species is to be maintained. To this rule, however, there are certain exceptions. Though it cannot be distinctly averred, however, that the vaccinal lymph has increased in potency, it can with all certainty be maintained that there is no evidence in favour of its deterioration. The evidence on which the argument regarding the deterioration of lymph was based cannot be regarded as complete. The chief points were, firstly, that as the century went on it became increasingly more possible to revaccinate successfully; and secondly, that in making attenuated forms of sheep-pox corresponding, as was thought, to vaccination, it was found that after a few generations these could not be relied on to give protection. In view of our fuller knowledge of the probable life history of the smallpox organism and its relations to vaccinia this latter argument ceases to present a reasonable analogy, while the former is more explicable on the view of the increasing potency of the lymph than on the other hypothesis.

The change just described is one which affects vaccinated persons as a whole and bears no special relationship to the question of the growth and decay of immunity. It is then to be noted that in all further discussions of smallpox statistics throughout the course of this paper the comparisons which are drawn, as far as possible concern persons in the same epidemic in the same place, because with different age constants at different places and at different times it can be seen what contradictory inferences could be made.

The first fact with regard to the immunity conferred by vaccination is one which seems unexpected. It is found that persons who are well protected have for all epidemics at all places a lower mean age than those who are badly protected. Exactly the same remarks apply to the groups which are based on the size or character of the cicatrix. Thus in each epidemic persons with four good marks have a lower mean age than those with four indifferent marks, a relationship which also applies to the group of persons presenting one, two, or three cicatrices. Taking the statistics of the large epidemics in Glasgow, of which more than half the cases consist of persons with one mark, it is found that the mean age in each of the three outbreaks falls steadily as the size of the mark

increases; that this is not due merely to the fact that larger marks are now put on than were formerly is shown moderately conclusively when it is considered further in detail. The degree of foveation of a cicatrix has been regarded as an indication of its protective value. Foveation, as a result of vaccination, no

TABLE III.

Mean Age of Persons with one Scar of different sizes, Glasgow 1901, 1902, and 1904.

Size of Scar	1901		1902*	1904
	Foveated	Not Foveated		
0—·25 sq. in.	32·7	33·7	38·3	37·3
·25—·5 ,,	31·6	32·1	34·8	33·3
·5—1 ,,	28·1	29·3	33·4	32·0
1— ,,	26·1	27·5	28·3	32·9

physician can foretell. The number, and, to a certain extent, the size of the cicatrices, is in the hands of the operator, but as to whether the scar will turn out to be foveated or not depends upon a number of factors beyond his control, and yet the mean age of those with unfoveated scars is for each group higher than that of those in which the scars are foveated. This relationship also applies uniformly as far as I have been able to observe, and in the large epidemics where there is a sufficient number of cases to allow of finer gradation the same relationship holds. For instance, taking those cases in the epidemic in London in 1902 in which there are four cicatrices of collective area greater than $\frac{1}{2}$ square inch it is found that among those in which more than half of this area is foveated the mean age is lower than that of those in whom less than half is foveated, and that the mean age of the latter is in defect of that of those patients in whom there is no foveation, the cicatrix being glazed or puckered. I do not think that this can be explained as may be suggested on the ground that foveation is a feature of the scar which tends to disappear with age, for it must be noted that the relationship just described bears a constantly present inverse proportion to that observed to exist between the same subdivisions and the mortality: that is to say, many marks are a better protection against death than few, larger than smaller, and foveated than non-foveated, and this to a greater extent than the mere difference of age incidence would imply.

The susceptibility to smallpox of those with different numbers of marks is unfortunately a matter which cannot be determined directly, and consequently its full significance cannot be expressed in figures. It however can be investigated for London to a certain extent. At intervals of ten years since 1870 large outbreaks of smallpox have occurred in London. The number of persons affected at each age period with the number of their vaccinal scars is therefore known at four epochs separated by almost equal intervals of time, and consequently the way

* The incidence of mean age is here due to the fact that the revaccination in 1901 was performed more largely in the earlier than in the later ages.

in which persons are protected by different kinds of vaccination can be judged by a comparison of the proportions in each group at the same age period at the different epochs. Persons thus of the age five to ten years in 1872 will be fifteen to twenty in 1882, and will be thirty-five to forty-five in 1902. These proportions are recorded in the accompanying table. From this it may be seen that in general after forty years all vaccinated persons are apparently equally liable to smallpox. The proportions of persons attacked with smallpox who were from forty to sixty in 1872, sixty to eighty in 1892, and above seventy in 1902 with one, two, three, and four scars respectively, are sensibly constant considering the small number of

TABLE IV.

Table showing the Percentage of Persons attacked by Smallpox, with the different numbers of Vaccinal Cicatrices at each age period, in the Epidemics 1870—72, 1880—85, 1892—95, and 1902 in London.

Group I.   Persons 0—10 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	0—10	24·6	25·5	22·6	27·3
1880—85	10—20	20·7	25·7	31·3	22·2
1892—95	20—30	14·3	23·7	28·3	35·1
1902	30—40	7·9	14·2	24·8	51·1

Group II.   Persons 10—20 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	10—20	22·2	36·8	23·3	17·5
1880—85	20—30	21·3	32·3	26·0	19·9
1892—95	30—40	20	27·0	29·0	23
1902	40—50	18·4	28·4	27·1	25·9

Group III.   Persons 20—30 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	20—30	24·6	39·6	19·4	16·7
1880—85	30—40	—	—	—	—
1892—95	40—50	28·6	28·4	15	22·8
1902	50—60	16·7	35·6	23	22·5

Group IV.   Persons 30—40 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	30—40	29·8	41·6	17·8	16·6
1880—85	40—50	—	—	—	—
1892—95	50—60	24·7	30·3	21·3	23·6
1902	60—70	26·4	31·8	18·7	23·1

Group V.   Persons 40—60 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	40—60	32·7	39·1	16·6	10·4
1880—85	50—70	—	—	—	—
1892—95	60—80	50	32·3	5·9	11·8
1902	70—	39·9	37·1	11·2	11·2

attacks at these ages. Persons with four marks are relatively well protected under five years, which is to a certain extent explained on the ground that the bulk of the patients at that age period belong to the class with very imperfect marks. Between twenty and forty years the advantage seems greatest, and after forty it has apparently entirely disappeared. Exactly the opposite would appear to be the case with persons with one mark. The cause of this in the early years, however, has just been explained, and the apparent decrease in later years is accounted for largely by the fact that a steady equalisation of susceptibility among the four groups takes place, and the number of persons with several cicatrices increases in proportion. It is to be noted that it is apparently between twenty and forty that persons with four scars have most advantage over persons with one. It may be added that the group of those with three marks resembles in general that with four, and the group with two that with one. This question of relative susceptibility at different ages is closely allied to that just discussed regarding the lowering of the mean age in those groups where vaccination is most perfectly performed. It certainly cannot be inferred from the latter that the more incomplete the vaccinal protection the more likely is the attack of smallpox to be delayed, provided such an attack is going to come. It appears much more likely that the decay of protective power is longer delayed, and that consequently while for the first twenty years or so any kind of fair mark indicates much the same protective power, there is a period in the middle ages of life when those who have good cicatrices are much better protected than those with inferior, and again after this period of relative protection has decayed all again become equally susceptible *. The only statistics which are available to check this conclusion are those of Warrington in the epidemic 1892–3, where a census as to the state of

* How this produces a higher mean age among those badly vaccinated than among those well vaccinated can be seen if the figures of Table V. are examined, though it is perhaps not immediately apparent. Here the divisions do not lend themselves to any accurate calculation of mean ages. In the approximations however the mean age of all those found to be badly vaccinated is in much the same excess over that of all found to be well vaccinated, as the mean age of those in the former class who were attacked is in excess of the mean age of those in the latter. But this high mean age among the badly vaccinated attacked by smallpox is not necessarily a consequence of this grouping of the population in the houses invaded by smallpox. As high ages are more susceptible among the badly vaccinated the population of the houses invaded will necessarily contain more old persons as there must be some correlation of age among persons living in the same houses, young families being of course more common among young parents. The high mean age referred to is independent of this, as will be seen if the percentage attacks in both are calculated. These show that in equal populations the mean age of those badly vaccinated attacked by smallpox would be in considerable excess of that of the well vaccinated.

Proportions Attacked.

Age Periods	Well Vaccinated	Badly Vaccinated
0—10	3·7	6·8
10—20	22·6	19·5
20—30	48·3	47·1
30—40	27·5	54·9
40—50	16·2	19·7
50—	13·4	19·5

vaccination was taken for the different age periods of all those in infected houses. Unfortunately the vaccinal scars are simply divided into two classes according as they exhibit good or imperfect characters. It will be seen that under ten years those well vaccinated have a certain measure of protection probably for the reason

TABLE V.

Correlation between Exposed and Attacked, Vaccinated Well and Badly, Warrington, 1893.

Age Periods	Well Vaccinated		Badly Vaccinated		Correlation
	Not Attacked	Attacked	Not Attacked	Attacked	
0—10	466	18	124	9	+ ·21
10—20	346	114	136	32	− ·13
20—30	144	135	74	66	− ·03
30—40	95	36	46	56	+ ·43
40—50	98	19	61	15	+ ·09
50—	45	7	68	14	+ ·14

already indicated, from ten to thirty years there is practically no difference between the classes. Between the ages of thirty and forty the advantage however is very definitely in favour of those with good marks, and after the age of forty this is still present, though not nearly so marked. These figures so far as they go confirm the explanation just given.

Before proceeding further to discuss the character of the immunity conferred by vaccination it is necessary to have as far as possible an accurate knowledge of the susceptibility to smallpox exhibited at different ages among persons who have no acquired protection at all either from vaccination or previous attack of the disease. With regard to the age susceptibility to smallpox in an unvaccinated community not a great deal can be said. In prevaccination days statistics were rarely kept with any accuracy, and though records of death in age periods exist relating to considerable numbers of persons and considerable periods of time, yet these afford a quite inadequate basis on which to compute the susceptibility at different ages. The most complete tables calculated from the number of deaths are those published by Duvillard with the authority of the French Academy of Sciences in 1806. He deduces the result that persons at all ages are about equally susceptible to smallpox, but that the maximum susceptibility is between the ages of ten and eleven years, though the excess is small. His conclusions lead him, however, into the difficulty of requiring that a much larger number of persons at a higher age should be attacked by smallpox than is actually observed. He explains this on the ground that the great majority of these attacks are so mild as to escape notice. Now of this there was no evidence at the time at which Duvillard wrote, nor is there any at the present date, when the diagnosis of modified smallpox has been brought to as great a pitch of perfection as any clinical diagnosis can. Consequently his conclusions afford an example of the untrustworthiness of calculating the susceptibility at different ages from the number of deaths alone without any knowledge of the actual case mortalities. For the determination of the susceptibility at different ages among persons who

are unvaccinated at the present day certain statistics exist. The chief of these are contained in the account of the epidemic in Sheffield in 1887 presented in Dr Barry's report to the Local Government Board. In this case a census was taken of nearly the whole population. The facts are tabulated so as to give the number of cases at each period who were unvaccinated, vaccinated, or revaccinated, and also those who had previously had smallpox. The state as regards the vaccination of the inhabitants of those houses actually invaded by smallpox is also tabulated separately. On account of the size of the epidemic this census must be held as the most important group of statistics at our disposal. It seems better to consider fully in the first instance the conclusions which may be based on these figures, and afterwards to compare the results with the statistics affecting naturally much smaller numbers which have been published with regard to other towns.

Considering, in the first place, the alterations in susceptibility to smallpox among the unvaccinated which takes place as the age increases, and expressing the relationship in terms of the correlation coefficients between those exposed to infection who escape, and those exposed who are attacked for each two succeeding age periods, it is found that these coefficients vary in the following

TABLE VI.

Correlation between those who were Attacked, and those who Escaped in Vaccinated and Unvaccinated respectively, at each age and that immediately succeeding.—Sheffield, 1887—8.

Age Periods	Vaccinated	Unvaccinated	Age Periods	Vaccinated	Age Periods	Unvaccinated
0— 1 & 1— 5	− ·10	− ·28	—	—	0— 1 & 1— 5	− ·28
1— 5 & 5—10	− ·21	− ·17	—	—	1— 5 & 5—10	− ·47
5—10 & 10—15	− ·34	− ·53	—	—	5—10 & 10—15	− ·84
10—15 & 15—20	− ·29	·72	—	—	10—15 & 15—20	− ·20
15—20 & 20—25	·04	·04	15—20 & 20—25	·04	15—20 & 20—25	− ·18
20—25 & 25—30	·01	·38	20—25 & 25—30	·04	20—25 & 25—30	·24
25—30 & 30—35	·09	·14	25—30 & 30—35	·13	25—30 & 30—35	·34
30—35 & 35—40	·26	·24	30—35 & 35—40	·31	30—35 & 35—40	·54
35—40 & 40—45	·47	·75	35—40 & 40—45	·53	35—40 & 40—45	·73
40—45 & 45—50	·48	− ·25	40—45 & 45—50	·67	40—45 & 45—50	·47
45—50 & 50—55	·01	·80	45—50 & 50—55	·68	45—50 & 50—55	·95
50—55 & 55—60	·15	− ·17	50—55 & 55—60	·77	50—55 & 55—60	·93
55—60 & 60—	·02	·04	55—60 & 60—	·79	55—60 & 60—	·92

way. From birth till between ten and fifteen years they are negative, indicating that the susceptibility steadily increases to this age, as was before found by Duvillard, and that thereafter each succeeding coefficient is positive of greater or less amount. If the susceptibility under one year is chosen for comparison with that of each succeeding age period it is seen that the strength of resistance which was less marked between ten and fifteen years has between twenty and thirty again increased so as to be equivalent to that in the first year of life, and thereafter steadily becomes greater. The period of maximum susceptibility between ten and fifteen years is one which is well marked in a large number of epidemics, although incapable of any very accurate measurement. The number

of children successfully vaccinated is published as a Government Return for every year since 1872, so that for epidemics occurring since 1890 for any definite place

TABLE VII.

Table showing the rate per Million of Attack by Smallpox among the Unvaccinated calculated as described in the text with the number of Attacks in an epidemic of Smallpox in 1726 at Aynho in Northamptonshire for comparison.

Age Periods	London, 1902	Leicester, 1892–93	Gloucester, 1895–96	Aynho, 1726
0— 5	—	255	727	13
5—10	315	607	909	15
10—15	543	700	955	33
15—20	584	554	774	14
20—25	339	396	434	16
25—30	225	386	446	9
30—35	150	77	351	32*

a rough approximation can be made of the respective number of vaccinated and unvaccinated, it being always borne in mind that the number of unvaccinated thus calculated is a maximum, and that many persons included have probably been vaccinated at some period subsequently during the existence of smallpox scares. The occurrence of such vaccination obviously becomes more probable the older the patient is, so that if a maximum is found between ten and fifteen years it is probably, as thus measured, to be in defect of the reality. The relative insusceptibility of old persons is also in statistics of epidemics an almost constant feature but the small numbers of unvaccinated persons attacked with smallpox from twenty years upwards makes the tracing of the development of insusceptibility in many cases less definite than in the case of Sheffield. This represents the main course of the increase and the decrease of susceptibility towards smallpox presented by the unvaccinated person. It is in general agreement with the few figures which exist regarding the age incidence of smallpox in the eighteenth century. This age incidence of course depends in a very large measure in any locality on the frequency with which epidemics occur. If epidemics break out every three or four years, as is known to have been the case in many places, then it is obvious that the most frequent age of attack will be that between two and three years, a characteristic well shown for example in the statistics of some German towns. The ages of two to three years indeed seem to constitute a period of greater relative susceptibility, as is already indicated in the figures for Sheffield, where the correlation coefficient between the age periods 0–1 and 1–5 years is greater than that between the periods of 1–5 and 5–10, though the numbers are too small to make this absolute. If, however, epidemics occur at greater intervals the natural period of greatest susceptibility will tend to become more evident. A maximum between ten and fifteen years is shown markedly in the statistics of Aynho in Northamptonshire, given by Dr Jurin, and, though in his record there is no statement to indicate the intervals between the succeeding epidemics, it is probable that this represents an epidemic outbreak of smallpox among a comparatively virgin community.

* This is number of cases above 30.

Still confining our consideration to the Sheffield figures we proceed to investigate the susceptibility among the vaccinated. Among these, as we pass from age period to age period an increase of susceptibility occurs from the period of vaccination till between the ages of fifteen to twenty years. At this age the period of greatest susceptibility is reached, and thereafter immunity is gradually regained, increasing with succeeding years, but never actually reaching that existing immediately after the operation was performed. It is to be noticed that the growth of immunity with old age proceeds from the age of twenty upwards in practically the same manner as among the unvaccinated. The curve is much more continuous, as is to be expected from the larger numbers which are available for its construction. The correlation coefficients fall slightly short of the similar ones for the unvaccinated, which is to be expected, as the old age protection starts originally from a higher level, the acquired immunity not being wholly lost at the age chosen for comparison. It is a matter of great interest from the point of view of the theory of immunity that with this large mass of statistics suitable for the calculation of the effect which one form of immunity produces upon another, namely, the effect of the presence of acquired immunity upon the growth of natural immunity, it is found that the latter develops exactly as if the former were completely absent. This is a result which could not have been anticipated, and its meaning will be further discussed in another paper.

The relative susceptibilities of the vaccinated and unvaccinated now falls to be examined, and here the Sheffield census is exceptionally valuable, as it allows the relative susceptibilities of vaccinated and unvaccinated to be determined and

TABLE VIII.

Table showing Correlation between Vaccinated and Unvaccinated Persons as regards Attack by Smallpox and escape from Attack when the numbers are considered of all persons (a) in the town itself, (b) in infected houses.

Age Periods	(a) All persons in the town considered Sheffield, 1887–88	(b) Those in the infected houses alone considered Sheffield, 1887–88	Dewsbury, 1892*	Leicester, 1892–93
0— 1	·84	·94	·90	1
1— 5	·63	·94	·93	1
5—10	·63	·96	·84	·93
10—15	·67	·99	·56	·77
15—20	·45	·72	·73	
20—25	·34	·74 ⎱	·66	·50
25—30	·12	·40 ⎰		
30—35	·17	·35 ⎱	—	
35—40	·16	·39 ⎰		·17
40—45	·11	·28 ⎱	·30	
45—50	·19	·49 ⎰		
50—55	— ·05	— ·11 ⎱		
55—60	·05	·07 ⎰	— ·03	
60—	·11	·05 ⎰		

* In this instance the number of revaccinated persons and of those who have had previous attacks of smallpox are excluded in the statistics, so that the correlation coefficients are as free from error due to the admittance of such persons as is possible. The whole numbers affected are however small.

compared, not only when the risk of infection is comparatively slight, as in the general population, but when it is much more considerable, as in infected houses. In the annexed table are given in parallel columns a series of coefficients expressing the relative susceptibilities under each of these conditions for all age periods. It will be seen that the protection which vaccination affords against attack is much less strong, considered in relation to the total numbers of such persons in the town generally, than to the numbers in houses actually infected. In the former it ceases to be considerable after twenty-five years of age, and is practically negligible after the age of fifty. In infected houses, however, this decline is not nearly so marked, but after the age of fifty it also becomes practically negligible. This is an example of a property which at times seems to ensure a greater value to action on a mass than on a small section of that mass which differs from the mass in being apparently more vulnerable. It is not a case of the poison acting equally on the two classes as may be taken for granted is likely to be the case in invaded houses, but that the relative sparseness of unvaccinated persons affords them a protection beyond what random selection would give. It is conceivable that some such factor plays an adjuvant part in natural selection. There are no other statistics which enable comparison to be made between those in infected houses and those of the general population, but the relative susceptibility between the two classes of vaccinated and unvaccinated in infected houses has been calculated for some other epidemics, Leicester, etc., which show local differences, but exhibit on the whole the same kind of relation from age period to age period which has been noted in the case of Sheffield. All the figures agree in making a rapid fall in the protective power of vaccination after the age of fifteen years. It is, however, to be noticed that the protection against a severe attack of smallpox or death afforded by vaccination is of a much more permanent nature than that against an attack of the disease. This is to be expected when the clinical resemblance between vaccination and the second stage of the course of smallpox both as regards local appearances and the course of the pyrexia is recalled, and when it is noted that the clinical course of smallpox suggests the presence of two very different classes of toxins in the two different stages of the disease, so that the immunity against the first stage is much more readily lost than that against the second. That this is the explanation will be apparent when the close relationship between the protection against severe attack, that is an attack with a severe second stage of the fever, and a fatal attack is noted. There are no very ample data to determine this comparison, but in the annexed table will be seen in parallel columns the correlation betwixt recoveries and deaths, and mild cases and severe for the Gloucester epidemic of 1896, and it will be seen that the correspondence, considering the small numbers involved, is sufficiently close. The figures which Macdonell has already given are quoted for comparison, when it will be readily seen that the deaths give a very fair measure indeed of the severity of the disease, closer than might be expected when it is considered how large a factor the personal equation plays in the tabulation of degrees of severity. This is important, as the great majority of statistics

as published refer only to cases and deaths, and give no detail of the character of the illness.

TABLE IX.

Correlation, Vaccinated and Unvaccinated. Recoveries and Deaths, and Cases Mild and Severe, for Epidemic in Gloucester, 1896.

Age Periods	Recoveries and Deaths	Cases Mild and Severe
0— 5	—	—
5—10	—	·94
10—15	·94	·93
15—20	·77	·75
20—30	·63	·88
30—40	·73 ⎫ ·63	·68
40—50	·43 ⎬	·63
50—60	⎫ ·63	·60
60—	⎬	—

TABLE IX *bis.*

Correlation between Cases Mild and Severe and Recoveries and Deaths. Vaccinated and Unvaccinated (Macdonell).

	Recoveries and Deaths	Mild and Severe
Sheffield	·77	·79˙
Glasgow, 1892—5 ...	·78	·91
Glasgow, 1901 ...	·63	·51

The way in which the protection against death from smallpox afforded by vaccination is gradually lost is of considerable interest. As shown in the appended table it gradually decreases from the date of vaccination up to the age of twenty-

TABLE X.

Correlation, Vaccinated and Unvaccinated, Recoveries and Deaths in Various Epidemics.

Age Periods	London 1856–61	London 1870–72	Sheffield 1887	Gloucester 1896	London 1902
0— 5	—	—	1	1	1
5—10	·34	·74	·79	1	·77
10—15	·65	⎫ ·60	·78	·94	·93
15—20	·59	⎬	·76	·77	·74
20—25	·66	⎫ ·59	·67	⎫ ·63	·61
25—30	·65	⎬	·49	⎬	·67
30—35	⎫ ·70	⎫ ·46	·55	⎫	·51
35—40	⎬	⎬	·53	⎬ ·63	·43
40—45	⎫ ·76	⎫ ·31	·38	⎬	⎫ ·74
45—50	⎬	⎬	·64	⎬	⎬
50—55		⎫ ·42	·48	⎫	⎫ ·65
55—60		⎬		⎬	⎬
60—65			⎫ ·49	⎫ ·62	⎫ ·46
65—70			⎬	⎬	⎬

five, after which it is moderately constant. One point, however, of interest becomes manifest here, and that is the way in which the correlation coefficients for the different ages between vaccinated and unvaccinated tend to assume higher values, at the ages of 10 to 20 years[*], as the century has gone on. Taking the first set of statistics which exist, namely those of Dr Marson, passing from these to the great epidemic of 1870–72 in London up to the last epidemic of 1902, we see the tendency for those coefficients to become larger continuously manifest, and the corresponding figures for the other epidemics for which they have been calculated seem to fit into place much as might be expected from their date of occurrence. This affords some confirmation of what has been suggested earlier in this paper in connection with the rise in age at which vaccinated persons are attacked with smallpox, namely, that vaccinal lymph has gradually gained potency through being passed for many generations through the human subject [†].

The question now arises as to whether in different epidemics the extra protection which good scars are known to afford to their possessors compared with bad scars assumes greater relative importance in mild or severe epidemics. There seems, however, to be little difference. In the accompanying table will be seen

TABLE XI.

Correlation between Recoveries and Deaths in the Epidemics 1902 and 1892—95, and 1902 and 1870—83 for each Vaccinal Group.

Nature of Group	1902 & 1892–95	1902 & 1870–83
One Scar	·22	·43
Two Scars	·18	·37
Three Scars	·24	·43
Four Scars	·21	·52

the correlation between cases and deaths for the epidemic in 1902 and those in 1892–95 and 1870–83. These coefficients indicate that there is no great difference in a severe epidemic or in a mild between the manner in which each different group is affected in comparison with the same group in a severe or in a mild epidemic. The figures, it is true, show some little difference, but not much when it is considered that the change in the age distribution of the different groups has been proceeding not by any means equally in them all.

Another point which may be noted is the manner in which the relative protection against death between those with good and those with bad marks varies. It is found to differ not very considerably below the age of twenty, to

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[*] The whole correlation coefficients are given in the table, but only the early years are referred to in the text, as adult primary vaccination was very common in the first half of the century.

[†] It is this change in the correlation coefficients which serves to differentiate the two interpretations which one puts on the rise of the mean age: (1) that due to the lymph gaining potency which is the cause of the rise of the general mean age of vaccinated persons attacked by smallpox, here the correlation between vaccinated and unvaccinated has increased as the century has gone on: and (2) the higher mean age of indifferently vaccinated persons taking smallpox than of well vaccinated, here the correlation between recoveries and deaths is in favour of the well vaccinated. It is thus seen that the mean age may be raised in two different ways. (Cf. footnote, p. 319.)

be at a maximum between twenty and thirty years, and then to gradually disappear towards old age. The figures here, however, are not large enough to base any definite conclusions.

It now falls to be considered at what rate immunity develops after vaccination, that is to say, how long must vaccination be performed prior to the onset of smallpox to prevent the attack? The incubation period of smallpox is from twelve to fourteen days, and it is found to be practically an absolute rule that no person develops smallpox more than fourteen days after a successful vaccination has been performed. If vaccination be performed during incubation period of smallpox the attack is seen often to be much modified. Here again the statistics refer to small numbers. In the first place, taking all ages together and estimating the correlation between the attacks and deaths in unvaccinated persons and in

TABLE XII.

Correlation between Recoveries and Deaths in Unvaccinated Persons, and those Vaccinated at various Dates prior to onset of illness.

Time before onset Days	Gloucester (all ages) 1896	London (all ages) 1902	London (ages above 1 year) 1902
0— 3	·13	·06	·40
3— 6	·16	·12	·40
6— 9	·65	·47	·72
9—14	·14	·12	·45
14—	1·00	1·00	1·00

those vaccinated after infection, we find that even among those vaccinated within three days of the onset of the illness there is a distinct positive correlation in favour of the process. Among those vaccinated from three to six days before the onset of attack the correlation is slightly higher, from six to nine days it is very considerable, from nine to fourteen days, however, it is again very much less, apparently due to the fact, as will be shown later, that only a few persons who are really very susceptible develop the disease after nine days. After fourteen days the protection against death is absolute. If children under one year be eliminated from the numbers considered the protection is seen to be considerably higher, but still to exhibit a maximum at the period of six to nine days.

Though the subtraction of those cases under one year makes such a difference in the correlation coefficients, yet when those who have been vaccinated more than six days before the onset of the illness are compared for the varying age

TABLE XIII.

Correlation between Recoveries and Deaths of those Unvaccinated, and those Vaccinated 6—14 days prior to onset of illness. London, 1902.

Age Periods
0— 1	·61
1— 5	·63
5—10	·73 ⎱ ·60
10—15	·38 ⎰
15—20	·49
20—30	·61

periods with those who are unvaccinated at the same age period it is found that the correlation is practically constant from birth onwards, and that the smaller correlation coefficients obtained when all ages are grouped together is brought about by the fact that a large proportion of the total cases are under one year, and that among these there is so special a mortality as to vitiate seriously the value of the correlation coefficients of the statistics when all ages are combined.

As to the actual susceptibility of persons vaccinated after exposure to small-pox there are no actual figures in existence. Some, however, can be inferred in the Gloucester epidemic; out of 653 persons vaccinated after exposure only 85 developed the disease as compared with 604 out of 832 among the unvaccinated who were exposed to infection and were not vaccinated. Out of the 85, 23 were vaccinated on the day of onset of the disease or on the day after, at a time when, therefore, the process could have no preventive action. The distribution of the remaining 62 persons is as follows: The number of those vaccinated from 0–3 days before the onset of attack was 19, from 3–6 days also 19, from 6–9 days 18, while for 9 days and upwards only 6. Now the great majority of persons exposed to infection are vaccinated within four days of the exposure, so that it becomes evident that if the operation be performed up to the ninth day before that on which onset of the attack could be expected there is a very high proba-bility that no further developments will take place. This is a question on which statistics cannot be compiled, because, though a person may be exposed to infec-tion for three to four days there is nothing to indicate on which day he may be infected, if at all, unless he subsequently develops the disease, so that the above figures are probably as satisfactory as any can be. The similar relation which exists between persons revaccinated more than six days before the onset of the

TABLE XIV.

Correlation between Recoveries and Deaths of those Vaccinated in Infancy only, and those Revaccinated 6—14 days prior to onset of illness.

Age Periods	London, 1902
20—40	·46
40—50	·35

illness shows a correlation of a similar kind, but not so great as in the former class, which is to be expected as before remarked, as the level of the protection from which the improvement is made is originally much higher. The smallness of the figures does not permit it to be given for more than two age groups.

The effect which an attack of smallpox exercises in establishing immunity is much more difficult to ascertain than that produced by vaccination. In general the great bulk of persons who are vaccinated have been vaccinated in infancy. With regard, however, to those who have had smallpox there are no data as to the ages at which they were attacked. In London between 1836 and 1851 out of 2654 unvaccinated persons suffering from smallpox treated in hospital only

690 were under the age of ten years; between 1870 and 1883 out of 2169 like persons 1197 were under the age of ten years. In both of these sets of statistics the distribution of the numbers from birth till ten years is fairly uniform. So that it may be taken for granted that towards the end of last century among persons over twenty years of age who had had smallpox previously one half was the greatest proportion of these who had the attack during the first ten years of life. In the accompanying table I have collected the details of fifty-one cases

TABLE XV.

Table of Second Attack of Smallpox.

Age at Second Attack.

		15—20	20—25	25—30	30—35	35—40	40—45	45—50	50—55	55—60	60—65	65—70	70—75	75—80	—90	Totals
	0— 5	4	5	2	4	4	1	2	2	1	1	2	1	—	1	30
	5—10	—	4	—	1	—	—	2	1	—	1	—	—	—	—	9
	10—15	—	—	1	1	2	—	1	1	—	—	—	—	—	—	6
	15—20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	20—25	—	—	—	1	—	1	—	—	—	—	1	—	—	—	3
	25—30	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	40—45	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
	Totals	4	9	3	7	6	3	5	6	1	2	3	1	—	1	51

(Row heading, vertical, at left: Age at First Attack.)

where persons have had a second attack of smallpox between the years 1887 and 1902, and of these fifty-one it is seen at once that thirty suffered from their first attack of smallpox below the age of five years, nine between the age of five and ten years, leaving a total of only twelve who had their first attack of smallpox above this age. This would seem to show that the protection afforded by an attack of smallpox in childhood is much less complete than that afforded by an attack after ten years. A study of the figures in the table seems to show that this is not due simply to the natural lessening in numbers which occurs in the population at each age period from childhood onwards. A certain amount of support is lent by this to the suggestion advanced earlier in this paper that vaccination performed in early adult life afforded a longer period of protection than when performed in infancy, and in both instances it may be surmised that as the natural susceptibility to smallpox increases from birth onwards till about twelve years there may be a greater loss of acquired immunity during the same period of life than is likely to occur when the natural protection against smallpox has become more definitely established. With regard to the protection which an attack of smallpox provides against death even less can be said. Of these fifty-one cases seven died. Five of these deaths, however, occurred in one epidemic out of twenty-three cases, and, although one of these five died from apoplexy, yet even four deaths out of twenty-three cases represents a mortality so much in

excess of what might be expected as to make it probable that these figures are not sufficiently large to be taken as trustworthy. The actual protection, however, which one attack of smallpox affords against a second is much larger than that conferred by vaccination; this protection, judging from the Sheffield census, is very high, the correlation coefficient being ·6, and this high correlation of protection against attack suggests the probability of some error in the death rate as given above.

The protection given by revaccination is still higher than that given by a previous attack of smallpox and by recent revaccination practically absolute. Of the 400,000 persons revaccinated in Glasgow in 1901 six were admitted during the epidemic of 1904; three of these stated that there had been no local reaction, they were therefore not effectively revaccinated; one stated that there had been a local reaction, which, however, left no scar. In the other two the condition of the arm indicated successful revaccination. That is out of 400,000 persons three at the most were affected with smallpox after a lapse of three years, while of the other 250,000 inhabitants of Glasgow more than 800 succumbed. In the statistics published in the London epidemic of 1902 the mean period which has elapsed between revaccination and the attack of smallpox (23·9 years) falls considerably short of the mean age of those attacked by smallpox in the same epidemic (28·6 years) with the protection alone of infantile vaccination. These persons, however, must represent a more susceptible class, for, judging from the Sheffield epidemic, the correlation between the revaccinated who escape and those who were attacked and the like classes of the vaccinated is very considerably in favour of the former.

TABLE XVI.

Table of Revaccinated Persons Attacked by Smallpox. London, 1902.

Age at Attack.

Age at Revaccination	10—15	15—20	20—25	25—30	30—35	35—40	40—45	45—50	50—55	55—60	60—65	65—70	70—75	Totals
0— 5	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5—10	1	—	—	1	1	1	—	4	1	—	—	—	—	9
10—15	1	—	2	5	2	4	7	5	—	—	1	—	—	27
15—20	—	—	1	—	6	2	10	3	2	2	—	1	—	27
20—25	—	—	—	—	1	2	5	1	2	—	—	1	—	12
25—30	—	—	—	—	1	—	1	2	—	1	—	—	—	5
30—35	—	—	—	—	—	—	1	3	2	—	—	1	—	7
35—40	—	—	—	—	—	—	—	—	1	—	—	—	—	1
40—45	—	—	—	—	—	—	—	—	1	—	—	—	—	1
45—50	—	—	—	—	—	—	—	—	—	1	—	—	—	1
50—55	—	—	—	—	—	—	—	—	1	—	—	—	—	1
55—60	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60—65	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	—	3	6	11	9	24	18	10	4	1	3	1	92

In the accompanying table those cases of revaccinated persons attacked by smallpox during the London epidemic of 1902 are given; the number is ninety-two in all, and among these two deaths occurred, in both of which the period of the latest vaccination had been at least twenty-four years previously. Few inferences can be drawn from this table; it is, however, to be noted that more than a third of the cases occur in persons revaccinated before the age of fifteen years, who, as far as the statistics of this hospital go, constitute a very much smaller proportion of the total revaccinated persons, indicating again that the operation performed at this age affords less permanent protection than when done later.

In conclusion, it is necessary to state that all the correlation coefficients in this paper were calculated by the short method given at the end of Prof. Pearson's memoir on the correlation among attributes not quantitatively measurable * by means of the formula

$$r = \sin \frac{\pi}{2} \left\{ \frac{1}{\sqrt{1 + \kappa^2}} \right\}, \text{ where } \kappa^2 = \frac{4abcdN^2}{(ad - bc)^2 (a + d)(b + c)}.$$

If the values of r corresponding to $\log \kappa^2$ from $\bar{1}$ to 4 be first calculated, as is easily done with the help of a table of Gaussian addition logarithms, then it is a matter of only a few minutes to obtain a correlation coefficient. Prof. Pearson has found this formula true in most instances within the probable error; I myself have checked it with about twenty examples and only once found it give a result outside that error, so for the accuracy required in this paper it may be considered sufficient. When the numbers are in nearly all cases small the probable error is large, and little stress can be placed on the accuracy of an individual coefficient. A far better test in a subject like this, when very large numbers of coefficients have to be calculated, is their general concordance. In many cases they form part of a series which is à priori continuous, and consequently each affords a mutual check on the other of much greater value considering the number of cases involved than the calculation of the probable error. Considering the series tabulated in this paper, many instances will be seen where the coefficients fall so out of line with their immediate neighbours that it is evident that some large error is present.

REFERENCES TO ORIGINAL SOURCES OF THE FIGURES ON WHICH THIS PAPER IS BASED.

Duvillard.   Analyse et tableaux de l'influence de la petite vérole sur la mortalité à chaque âge. Paris, 1806.

Macdonell.   Biometrika, Vol. i. p. 375, Vol. ii. p. 135.

General Board of Health.   History and Practice of Vaccination, 1851.

Local Government Board.   Epidemic of Smallpox at Sheffield, 1887–8.

Royal Commission on Vaccination : Appendices on Epidemics in Warrington, Dewsbury, Gloucester and Leicester.

Report of the Metropolitan Asylums Board Hospitals, 1870–1902.

* *Phil. Trans.* Vol. 195, A, p. 16.

ON TESTS FOR LINEARITY OF REGRESSION IN FREQUENCY DISTRIBUTIONS.

By JOHN BLAKEMAN, M.Sc., B.A.

In considering any frequency distribution of two characters, say x and y, one of the things to be determined is whether the regression curve of y on x is linear or non-linear.

In his memoir " On the general theory of skew correlation and non-linear regression*" Karl Pearson has given the test for linearity of regression expressed in terms of the correlation coefficient r and the correlation ratio η.

It is there shown that, while η is by definition an essentially positive quantity r may be either positive or negative; that η^2 and r^2 are both quantities which can only vary between the limits 0 and 1 and moreover η^2 is never less than r^2; finally, that the necessary and sufficient condition for linear regression is $\eta^2 - r^2 = 0$.

It has been remarked that r may be positive or negative, but it is obvious that whenever r works out negative we can, by changing the direction of either one of the axes of reference, make r positive. Thus, in all frequency distributions, the axes may be so drawn that r shall be positive. There is therefore no loss of generality in assuming r positive and, for simplicity of statement, in what follows r will always be taken as positive.

With this convention we see that, if we write $\zeta = \eta^2 - r^2$, $\vartheta = \eta - r$, $\varpi = \log_e \frac{\eta}{r}$, the test for linear regression may be written indiscriminately as $\zeta = 0$, or $\vartheta = 0$, or $\varpi = 0$, and in many other equivalent ways.

Now, in any actual distribution, we shall not expect to find the quantities ζ, ϑ, ϖ absolutely zero but, even when the regression of y on x is linear, we shall expect the values of ζ, ϑ, ϖ to be influenced in the usual way by the fact that our material is only a random sample of the general population.

Thus, as in all statistical tests, having found ζ, or ϑ, or ϖ, to have certain positive values, we want to know if these values are such as might arise from

* *Drapers' Company Research Memoirs*. Biometric Series II. Dulau and Co.

errors due to random sampling from a general population whose regression is linear, or whether the values are such as to indicate significant non-linearity.

For this purpose we compare the calculated value of ζ, or ϑ, or ϖ with its probable error, where, if the standard deviations of ζ, ϑ, ϖ are written Σ_ζ, Σ_ϑ, Σ_ϖ the probable errors, say E_ζ, E_ϑ, E_ϖ, are defined by the equation $E = \cdot 67449\Sigma$.

While Karl Pearson has given in his memoir a formula for Σ_η, and the formula for Σ_r is well-known, no formula has yet been given for calculating any such quantity as Σ_ζ, Σ_ϑ, Σ_ϖ for the testing of linear regression.

It is the object of the present investigation to determine such formulae; the obvious thing to do would be to determine Σ_ζ, as ζ is the quantity occurring naturally in Karl Pearson's work; but the formula for Σ_ϖ is so very much simpler that it has been made the basis of this investigation; it is then shown how to determine Σ_ζ and Σ_ϑ from Σ_ϖ and other known quantities.

Thus it is at the worker's choice to test linearity of regression by comparing any one of the quantities ζ, ϑ, ϖ with its probable error.

Following Karl Pearson's notation n_{x_ρ} will denote the number of individuals in the x_ρ-array of y's, n_{y_s} the number of individuals in the y_s-array of x's and $n_{x_\rho y_s}$ the number of individuals in the sub-group common to these two arrays. N is the total number of individuals in the table; $\bar{x}$, $\bar{y}$ are the means of the two characters, while σ_x, σ_y are their standard deviations; y_{x_ρ} is the mean of the x_ρ-array of y's while $\sigma_{n_{x_\rho}}$ is the standard deviation of that array and $p_{uv}, \lambda_m, \sigma_M, \eta, r$ are quantities defined by the following equations, S' denoting a double summation:

$$N p_{uv} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})^u (y_s - \bar{y})^v\},$$

$$N\lambda_m = S\{n_{x_\rho}(y_{x_\rho} - \bar{y})^m\},$$

$$N\sigma_M{}^2 = S\{n_{x_\rho}(y_{x_\rho} - \bar{y})^2\} \quad \{\text{i.e. } \sigma_M{}^2 = \lambda_2\},$$

$$\eta = \frac{\sigma_M}{\sigma_y}, \qquad r = \frac{p_{11}}{\sigma_x \sigma_y},$$

and further we shall define a quantity λ_{uv} by the equation

$$N\lambda_{uv} = S'\{n_{x_\rho}(x_\rho - \bar{x})^u (y_{x_\rho} - \bar{y})^v\}.$$

The fundamental results on which our work is based may be stated as follows:—

Let n, n' be the number of individuals falling within any two mutually exclusive groups, then if Σ_n, $\Sigma_{n'}$, be the standard deviations of n, n' and $R_{nn'}$ the correlation between deviations in n and n' due to random sampling, we have

$$\Sigma_n{}^2 = n\left(1 - \frac{n}{N}\right), \qquad \Sigma_{n'}{}^2 = n'\left(1 - \frac{n'}{N}\right) \quad \dots\dots\dots\dots(i),$$

$$\Sigma_n\Sigma_{n'}R_{nn'} = -\frac{nn'}{N} \quad \dots\dots\dots\dots\dots\dots\dots(ii).$$

General Theorem. Let n_1, n_2, ... be the number of individuals in any number of mutually exclusive groups and let quantities X, Y be defined by the equations

$$X = A_1 n_1 + A_2 n_2 + A_3 n_3 + ...,$$
$$Y = B_1 n_1 + B_2 n_2 + B_3 n_3 + ...,$$

where the A's and B's are quantities which remain constant as the frequency distribution is changed. Then

$$\delta X = A_1 \delta n_1 + A_2 \delta n_2 + A_3 \delta n_3 + ...,$$
$$\delta Y = B_1 \delta n_1 + B_2 \delta n_2 + B_3 \delta n_3 + ...$$

Multiply these two expressions together, sum for all random samples and divide by the number of such samples; then using (i) and (ii) we get immediately

$$\Sigma_X \Sigma_Y R_{XY} = -\frac{XY}{N} + A_1 B_1 n_1 + A_2 B_2 n_2 + A_3 B_3 n_3 + ... \quad \text{............(iii).}$$

Problem I. To determine the correlation between the deviations due to random sampling in the values of $n_{x_\rho y_s}$ and $y_{x_{\rho'}}$.

We have $\qquad\qquad n_{x_{\rho'}} y_{x_{\rho'}} = S\{n_{x_{\rho'} y_s} y_s\}.$

Hence $\qquad\qquad n_{x_{\rho'}} \delta y_{x_{\rho'}} \pm S\{\delta n_{x_{\rho'} y_s} y_s\} - \delta n_{x_{\rho'}} y_{x_{\rho'}},$

$$\delta n_{x_\rho y_s} = \delta n_{x_\rho y_s}.$$

Multiply these two expressions together, sum for all random samples and divide by the number of such samples; then

(α) ρ and ρ' different

$$n_{x_{\rho'}} \Sigma_{n_{x_\rho y_s}} \Sigma_{y_{x_{\rho'}}} R_{n_{x_\rho y_s} y_{x_{\rho'}}} = -\frac{n_{x_\rho y_s} n_{x_{\rho'}} y_{x_{\rho'}}}{N} + \frac{n_{x_\rho y_s} n_{x_{\rho'}} y_{x_{\rho'}}}{N} = 0 \quad \text{.........(iv).}$$

(β) ρ equal to ρ'

$$n_{x_\rho} \Sigma_{n_{x_\rho y_s}} \Sigma_{y_{x_{\rho'}}} R_{n_{x_\rho y_s} y_{x_\rho}} = n_{x_\rho y_s} y_s - n_{x_\rho y_s} y_{x_\rho} = n_{x_\rho y_s}(y_s - y_{x_{\rho'}}) \quad \text{......(v).}$$

We shall need, besides, the results proved by Karl Pearson as Propositions III, IV, VIII in the memoir previously cited, which we will state here for reference.

Proposition III.

$$R_{y_{x_\rho} n_{x_\rho}} = 0 \quad \text{.....................................(vi).}$$

Proposition IV.

$$R_{y_{x_\rho} n_{x_{\rho'}}} = 0 \ (\rho' \text{ not equal to } \rho) \quad \text{............ (vii).}$$

Proposition VIII.

$$\Sigma_{\sigma_M}{}^2 = \frac{\lambda_4 - \lambda_2{}^2}{4N\lambda_2} + \frac{\chi_1 \sigma_y{}^2 (1 - \eta^2)}{N} \quad \text{.................(viii),}$$

where χ_1 is a quantity defined by the equation

$$S\{n_{x_\rho} \sigma_{n_{x_\rho}}{}^2 (y_{x_\rho} - \overline{y})^2\} = N\sigma_y{}^2 (1 - \eta^2) \sigma_M{}^2 \times \chi_1,$$

and is obviously such that, in the case of normal distributions when $\sigma_{n_{x_\rho}}$ is constant and $r = \eta$, we get $\chi_1 = 1$.

Problem II. To find the correlation between the deviations due to random sampling in the values of σ_x and σ_M.

We have
$$N\sigma_x^2 = S\{n_{x_\rho}(x_\rho - \bar{x})^2\},$$

$$\therefore\ 2N\sigma_x\delta\sigma_x = S\{\delta n_{x_\rho}(x_\rho - \bar{x})^2\} - 2\delta\bar{x}S\{n_{x_\rho}(x_\rho - \bar{x})\}.$$

But
$$S\{n_{x_\rho}(x_\rho - \bar{x})\} = 0.$$

Hence
$$2N\sigma_x\delta\sigma_x = S\{\delta n_{x_\rho}(x_\rho - \bar{x})^2\}.$$

Again
$$2N\sigma_M\delta\sigma_M = S\{\delta n_{x_\rho}(y_{x_\rho} - \bar{y})^2\} + 2S\{\delta y_{x_\rho}n_{x_\rho}(y_{x_\rho} - \bar{y})\}.$$

Multiply these two expressions together, sum for all random samples, and divide by the number of such samples; then using (iii), (vi) and (vii) we find

$$4N^2\sigma_x\sigma_M\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M} = -\frac{N^2\sigma_x^2\sigma_M^2}{N} + S\{n_{x_\rho}(x_\rho - \bar{x})^2(y_{x_\rho} - \bar{y})^2\}$$

$$= -N\sigma_x^2\sigma_M^2 + N\lambda_{22}.$$

Hence
$$4\sigma_x\sigma_M\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M} = \frac{\lambda_{22} - \sigma_x^2\lambda_2}{N} \quad\dots\dots\dots\dots\dots\dots\dots\text{(ix).}$$

Problem III. To find the correlation between the deviations due to random sampling in the values of p_{11} and σ_M.

We have
$$Np_{11} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})\},$$

$$N\delta p_{11} = S'\{\delta n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})\} - \delta\bar{x}S'\{n_{x_\rho y_s}(y_s - \bar{y})\} - \delta\bar{y}S'\{n_{x_\rho y_s}(x_\rho - \bar{x})\}.$$

Hence
$$N\delta p_{11} = S'\{\delta n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})\}.$$

But
$$2N\sigma_M\delta\sigma_M = S\{\delta n_{x_\rho}(y_{x_\rho} - \bar{y})^2\} + 2S\{\delta y_{x_\rho}n_{x_\rho}(y_{x_\rho} - \bar{y})\}.$$

Multiply these two expressions together, sum for all random samples and divide by the number of such samples ; then using (iii), (iv) and (v) we find

$$2N^2\sigma_M\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M} = -\frac{N^2 p_{11}\sigma_M^2}{N} + S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_{x_\rho} - \bar{y})^2\}$$

$$+ 2S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})\}.$$

But
$$S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_{x_\rho} - \bar{y})^2\} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})^2\}$$

$$+ S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_{x_\rho} - \bar{y})^3\},$$

while
$$S\{n_{x_\rho y_s}(y_s - y_{x_\rho})\} = 0.$$

Summing with respect to s and keeping ρ constant. Hence

$$S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_{x_\rho} - \bar{y})^2\} = N\lambda_{13}.$$

Again
$$S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})\} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - y_{x_\rho})^2(y_{x_\rho} - \bar{y})\}$$

$$+ S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})^2\}.$$

Sum with respect to s, keeping ρ constant. Since

$$S\{n_{x_\rho y_s}(y_s - y_{x_\rho})\} = 0,$$

and
$$S\{n_{x_\rho y_s}(y_s - y_{x_\rho})^2\} = n_{x_\rho}\sigma_{n_{x_\rho}}^2,$$

we get
$$S'\left\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_s - y_{x_\rho})(y_\rho - \bar{y})\right\} = S\left\{n_{x_\rho}\sigma_{n_{x_\rho}}^2(x_\rho{'} - \bar{x})(y_{x_\rho} - \bar{y})\right\}.$$

We may write
$$S\left\{n_{x_\rho}\sigma_{n_{x_\rho}}^2(x_\rho - \bar{x})(y_{x_\rho} - \bar{y})\right\} = N\sigma_y^2(1 - \eta^2)p_{11} \times \chi_3,$$

where χ_3 is a purely numerical constant, which is obviously equal to unity in the case of normal correlation, when $\sigma_{n_{x_\rho}}^2 = \sigma_y^2(1 - r^2)$ for all values of ρ and $\eta = r$.

Hence finally
$$2\sigma_M\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M} = \frac{1}{N}\left\{\lambda_{13} - p_{11}\sigma_M^2 + 2\rho_{11}\sigma_y^2(1 - \eta^2)\chi_3\right\} \quad \ldots\ldots\ldots(\text{x}).$$

Problem IV. To find the standard deviation of the values of ϖ defined by the equation
$$\varpi = \log_e\frac{\eta}{r}.$$

We have
$$\varpi = \log_e\frac{\eta}{r}.$$

Hence
$$\delta\varpi = \frac{\delta\eta}{\eta} - \frac{\delta r}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xi}).$$

But
$$\eta = \frac{\sigma_M}{\sigma_y}.$$

Hence
$$\frac{\delta\eta}{\eta} = \frac{\delta\sigma_M}{\sigma_M} - \frac{\delta\sigma_y}{\sigma_y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xii}),$$

and
$$r = \frac{p_{11}}{\sigma_x\sigma_y},$$

$$\therefore \quad \frac{\delta r}{r} = \frac{\delta p_{11}}{p_{11}} - \frac{\delta\sigma_x}{\sigma_x} - \frac{\delta\sigma_y}{\sigma_y} \quad \ldots\ldots\ldots\ldots\ldots\ldots(\text{xiii}).$$

Squaring equations (xi), (xii), and (xiii), and forming the product of equations (xii) and (xiii), then on summing for all random samples and dividing by the number of such samples we get in succession

$$\Sigma_{\varpi}^2 = \frac{\Sigma_\eta^2}{\eta^2} + \frac{\Sigma_r^2}{r^2} - \frac{2\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xiv}),$$

$$\frac{\Sigma_\eta^2}{\eta^2} = \frac{\Sigma_{\sigma_M}^2}{\sigma_M^2} + \frac{\Sigma_{\sigma_y}^2}{\sigma_y^2} - \frac{2\Sigma_{\sigma_M}\Sigma_{\sigma_y}R_{\sigma_M\sigma_y}}{\sigma_M\sigma_y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (\text{xv}),$$

$$\frac{\Sigma_r^2}{r^2} = \frac{\Sigma_{p_{11}}^2}{p_{11}^2} + \frac{\Sigma_{\sigma_x}^2}{\sigma_x^2} + \frac{\Sigma_{\sigma_y}^2}{\sigma_y^2} - \frac{2\Sigma_{p_{11}}\Sigma_{\sigma_x}R_{p_{11}\sigma_x}}{p_{11}\sigma_x}$$
$$- \frac{2\Sigma_{p_{11}}\Sigma_{\sigma_y}R_{p_{11}\sigma_y}}{p_{11}\sigma_y} + \frac{2\Sigma_{\sigma_x}\Sigma_{\sigma_y}R_{\sigma_x\sigma_y}}{\sigma_x\sigma_y} \quad \ldots\ldots\ldots\ldots(\text{xvi}),$$

$$\frac{\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r} = \frac{\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M}}{p_{11}\sigma_M} - \frac{\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M}}{\sigma_x\sigma_M} - \frac{\Sigma_{\sigma_y}\Sigma_{\sigma_M}R_{\sigma_y\sigma_M}}{\sigma_y\sigma_M}$$
$$- \frac{\Sigma_{\sigma_y}\Sigma_{p_{11}}R_{\sigma_y p_{11}}}{\sigma_y p_{11}} + \frac{\Sigma_{\sigma_x}\Sigma_{\sigma_y}R_{\sigma_x\sigma_y}}{\sigma_x\sigma_y} + \frac{\Sigma_{\sigma_y}^2}{\sigma_y^2} \quad \ldots\ldots\ldots\ldots(\text{xvii}).$$

Adding together equations (xiv), (xv), (xvi) and (xvii) multiplied by -2, we get

$$\Sigma_{\varpi}{}^2 = \frac{\Sigma_{\sigma_M}{}^2}{\sigma_M{}^2} + \frac{\Sigma_{p_{11}}{}^2}{p_{11}{}^2} + \frac{\Sigma_{\sigma_x}{}^2}{\sigma_x{}^2} - \frac{2\Sigma_{p_{11}}\Sigma_{\sigma_x}R_{p_{11}\sigma_x}}{p_{11}\sigma_x}$$
$$- \frac{2\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M}}{p_{11}\sigma_M} + \frac{2\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M}}{\sigma_x\sigma_M} \quad\dots\dots\dots\text{(xviii)}.$$

But we have as well-known results

$$\Sigma_{p_{11}}{}^2 = \frac{p_{22} - p_{11}{}^2}{N}, \qquad \Sigma_{\sigma_x}{}^2 = \frac{p_{40} - p_{20}{}^2}{4p_{20}N} \quad\dots\dots\dots\dots\text{(xix)},$$

$$2\Sigma_{\sigma_x}\Sigma_{p_{11}}R_{\sigma_x p_{11}} = \frac{p_{31} - p_{20}p_{11}}{N\sigma_x} \quad\dots\dots\dots\dots\dots\text{(xx)}.$$

Hence, substituting in equation (xviii) the values given by equations (viii), (ix), (x), (xix) and (xx), we get

$$\Sigma_{\varpi}{}^2 = \frac{\lambda_4 - \lambda_2{}^2}{4N\lambda_2{}^2} + \chi_1\frac{1 - \eta^2}{N\eta^2} + \frac{p_{22} - p_{11}{}^2}{Np_{11}{}^2} + \frac{p_{40} - p_{20}{}^2}{4Np_{20}{}^2} - \frac{p_{31} - p_{20}p_{11}}{Np_{20}p_{11}} + \frac{\lambda_{22} - \sigma_x{}^2\lambda_2}{2N\sigma_x{}^2\lambda_2}$$
$$- \frac{1}{Np_{11}\lambda_2}\{\lambda_{13} - p_{11}\lambda_2 + 2p_{11}\sigma_y{}^2(1 - \eta^2)\chi_3\} \quad\dots\dots\text{(xxi)},$$

which may obviously be re-written

$$\Sigma_{\varpi}{}^2 = \frac{1}{N}\left(\frac{1}{r^2} - \frac{1}{\eta^2}\right) + \frac{p_{40} - 3p_{20}{}^2}{4Np_{20}{}^2} - \frac{p_{31} - 3p_{20}p_{11}}{Np_{20}p_{11}} + \left\{\frac{p_{22} - 3p_{11}{}^2}{Np_{11}{}^2} - \frac{1 - r^2}{Nr^2}\right\} + \frac{\lambda_4 - 3\lambda_2{}^2}{4N\lambda_2{}^2}$$
$$- \frac{\lambda_{13} - 3\lambda_{11}\lambda_2}{N\lambda_{11}\lambda_2} + \frac{\lambda_{22} - 3\sigma_x{}^2\lambda_2}{2N\sigma_x{}^2\lambda_2} + \frac{1 - \eta^2}{N\eta^2}\{(\chi_1 - 1) - 2(\chi_3 - 1)\} \quad\dots\dots\text{(xxii)}.$$

This is the complete formula for Σ_{ϖ} to obtain which has been the first object of the present investigation. The formula has been thrown into the form given as equation (xxii) because each term as there exhibited vanishes for a normal distribution, since for normal surfaces we have

$$p_{40} = 3p_{20}{}^2, \qquad p_{31} = 3p_{20}p_{11}, \qquad \frac{p_{22} - 3p_{11}{}^2}{p_{11}{}^2} = \frac{1 - r^2}{r^2}.$$

Again for a normal distribution

$$y_{x_\rho} - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x_\rho - \bar{x}).$$

Hence
$$N\lambda_4 = S\{n_{x_\rho}(y_{x_\rho} - \bar{y})^4\} = \frac{r^4\sigma_y{}^4}{\sigma_x{}^4}.S\{n_{x_\rho}(x_\rho - \bar{x})^4\}$$
$$= \frac{r^4\sigma_y{}^4}{\sigma_x{}^4}3N\sigma_x{}^4 = 3N\lambda_2{}^2,$$
$$N\lambda_{13} = S\{n_{x_\rho}(x_\rho - \bar{x})(y_{x_\rho} - \bar{y})^3\} = \frac{r^3\sigma_y{}^3}{\sigma_x{}^3}.3N\sigma_x{}^4$$
$$= 3N.r^2\sigma_y{}^2 \times r\sigma_x\sigma_y = 3N.\lambda_2.\lambda_{11},$$
$$N\lambda_{22} = S\{n_{x_\rho}(x_\rho - \bar{x})^2(y_{x_\rho} - \bar{y})^2\} = \frac{r^2\sigma_y{}^2}{\sigma_x{}^2}.3N\sigma_x{}^4$$
$$= 3Nr^2\sigma_y{}^2\sigma_x{}^2 = 3N\lambda_2.\sigma_x{}^2.$$

Further $\chi_1 = \chi_3 = 1$, while $\eta = r$.

Now the formula for Σ_r^2 consists of a term $\dfrac{(1-r^2)^2}{N}$ involving r only together with a number of terms involving product moments which are just such terms as the product moment terms of equation (xxii)*. Again the formula for Σ_η^2 consists of a term $\dfrac{(1-\eta^2)^2}{N}$ involving η only together with a number of terms of exactly the same nature as these moment terms†. In practical statistical work the simple formulae $\Sigma_r^2 = \dfrac{(1-r^2)^2}{N}$, $\Sigma_\eta^2 = \dfrac{(1-\eta^2)^2}{N}$ are used and the nature of the justification for these simple formulae can only be appreciated by a reference to the numerical work of Karl Pearson's memoir where he shows that, as a matter of experience, in actual frequency distributions which are very far from normal, although the separate product terms occurring in the formula for Σ_η^2 may be significant, yet, these product terms occur in such a manner in the formula as to be insignificant in the aggregate. This work suggests that we should consider if anything of a like nature occurs in the case of the formula for Σ_ϖ. Equation (xxii) gives Σ_ϖ^2 as the sum of a term $\dfrac{1}{N}\left(\dfrac{1}{r^2}-\dfrac{1}{\eta^2}\right)$ involving η and r only and a number of product moment terms of exactly the same type as those occurring in the formulae for Σ_r^2, Σ_η^2. I suggest therefore the formula

$$\Sigma_\varpi^2 = \frac{1}{N}\left\{\frac{1}{r^2}-\frac{1}{\eta^2}\right\} \quad\dots\dots\dots\dots\dots\dots\dots\text{(xxiii)},$$

as being exactly analogous to the simple formulae for Σ_r^2, Σ_η^2; the complete justification for the formula (xxiii) will be found in the practical statistical work given later in the paper.

Before proceeding to the numerical work I will give some formulae which enable us to determine other quantities of importance when Σ_ϖ^2 is known.

Problem V. To find the correlation between the deviations due to random sampling in η and r.

We have
$$\Sigma_\varpi^2 = \frac{\Sigma_\eta^2}{\eta^2}+\frac{\Sigma_r^2}{r^2}-\frac{2\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r}$$

$$= \left(\frac{\Sigma_\eta}{\eta}-\frac{\Sigma_r}{r}\right)^2+\frac{2\Sigma_\eta\Sigma_r}{\eta r}(1-R_{\eta r}),$$

or
$$\frac{2\Sigma_\eta\Sigma_r}{\eta r}(1-R_{\eta r}) = \Sigma_\varpi^2-\left(\frac{\Sigma_\eta}{\eta}-\frac{\Sigma_r}{r}\right)^2 \quad\dots\dots\dots\dots\text{(xxiv)}.$$

This equation gives the numerical value of $R_{\eta r}$ as soon as r, η, Σ_r, Σ_η, Σ_ϖ have been determined, no matter in what way.

Assuming the simple formulae for Σ_r, Σ_η and the formula (xxiii) for Σ_ϖ, we get a corresponding formula

$$2\eta r\,(1-\eta^2)(1-r^2)(1-R_{\eta r}) = \eta^2-r^2-(\eta-r)^2(1+\eta r)^2 \dots\dots\text{(xxv)}.$$

* *Phil. Trans.* A, Vol. 191, p. 245.　　　　　　† *Loc. cit.* p. 20.

Problem VI. To find the standard deviation of the values of ζ defined by the equation

$$\zeta = \eta^2 - r^2.$$

We have $\qquad \frac{1}{2}\delta\zeta = \eta\delta\eta - r\delta r.$

Hence $\qquad \frac{1}{4}\Sigma_\zeta{}^2 = \eta^2\Sigma_\eta{}^2 + r^2\Sigma_r{}^2 - 2\eta r\Sigma_\eta\Sigma_r R_{\eta r}.$

But $\qquad \Sigma_\varpi = \dfrac{\Sigma_\eta{}^2}{\eta^2} + \dfrac{\Sigma_r{}^2}{r^2} - \dfrac{2\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r},$

$$\therefore \quad \eta^2 r^2 \Sigma_\varpi{}^2 = r^2\Sigma_\eta{}^2 + \eta^2\Sigma_r{}^2 - 2\eta r\Sigma_\eta\Sigma_r R_{\eta r}.$$

Hence $\qquad \frac{1}{4}\Sigma_\zeta{}^2 = (\eta^2 - r^2)(\Sigma_\eta{}^2 - \Sigma_r{}^2) + \eta^2 r^2 \Sigma_\varpi{}^2$ ....(xxvi).

As in equation (xxiv) this is a formula always applicable. The formula for Σ_ζ which corresponds to equation (xxiii) for Σ_ϖ is therefore

$$\frac{1}{4}\Sigma_\zeta{}^2 = \frac{\eta^2 - r^2}{N}\left\{(1-\eta^2)^2 - (1-r^2)^2 + 1\right\}................(xxvii).$$

Problem VII. To find the standard deviation of the values of ϑ defined by the equation

$$\vartheta = \eta - r.$$

Proceeding exactly as before, we get

$$\Sigma_\vartheta{}^2 = \Sigma_\eta{}^2 + \Sigma_r{}^2 - 2\Sigma_\eta\Sigma_r R_{\eta r},$$

giving $\qquad \eta r\Sigma_\vartheta{}^2 = (\eta - r)\left\{r\Sigma_\eta{}^2 - \eta\Sigma_r{}^2\right\} + \eta^2 r^2\Sigma_\varpi{}^2$ ....(xxviii),

applicable in all cases.

The corresponding simple formula is

$$\eta r\Sigma_\vartheta{}^2 = \frac{\eta - r}{N}\left\{r(1-\eta^2)^2 - \eta(1-r^2)^2 + \eta + r\right\} \quad ....(xxix).$$

Statistical Illustrations.

I proceed to discuss, by appeals to actual statistical experience, the degree of closeness to which the simple formulae (xxiii), (xxvii) and (xxix), may be used instead of the complete formulae (xxii), (xxvi), and (xxviii).

If, as regards any frequency distribution, we have tabulated for each x-array of y's the n_{x_ρ}, y_{x_ρ} and $\sigma_{n_{x_\rho}}$, we may proceed to calculate Σ_ϖ from the formula (xxii) without further reference to the sub-groups of the correlation table. This follows at once since we have, in general,

$$Np_{m1} = S\left\{n_{x_\rho y_s}(x_\rho - \bar{x})^m(y_s - \bar{y})\right\}$$

$$= S\left\{n_{x_\rho y_s}(x_\rho - \bar{x})^m(y_s - y_{x_\rho})\right\} + S\left\{n_{x_\rho y_s}(x_\rho - \bar{x})^m(y_{x_\rho} - \bar{y})\right\}$$

$$= N\lambda_{m1},$$

[since $S\left\{n_{x_\rho y_s}(y_s - y_{x_\rho})\right\} = 0$], hence

$$p_{m,1} = \lambda_{m1} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(xxx).$$

Also we have

$$Np_{m2} = S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_s - \bar{y})^2\}$$

$$= S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_s - y_{x_\rho} + y_{x_\rho} - \bar{y})^2\}$$

$$= S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_s - y_{x_\rho})^2\} + S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_{x_\rho} - \bar{y})^2\}$$

$$+ 2S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_{x_\rho} - \bar{y})(y_s - y_{x_\rho})\}$$

$$= S\{n_{x_\rho}\sigma_{n_{x_\rho}}^2 (x_\rho - \bar{x})^m\} + N\lambda_{m2},$$

hence

$$p_{m2} = \lambda_{m2} + \frac{1}{N} S\{n_{x_\rho}\sigma_{n_{x_\rho}}^2 (x_\rho - \bar{x})^m\} \quad \dots\dots\dots\dots\dots\dots\dots\dots(\text{xxxi}).$$

Thus the formula for Σ_{ϖ} contains only λ's and χ_1, χ_3 which, by their definition, are calculable when n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$ are known for every array, together with product moments of the form p_{m1}, p_{m2} which are also calculable if we use equations (xxx) and (xxxi).

In his memoir Karl Pearson discusses four actual frequency distributions which exhibit skew correlation and which he names illustrations A, B, C, D. For these distributions he has given the numerical values of many of the statistical constants and has also tabled the n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$, for each array, to four decimal places in the cases of illustrations A, B, C, and to three decimal places in the case of illustration D. It is thus relatively easy to calculate Σ_{ϖ} from the formula (xxii) for these four cases. Starting with the tabled values of n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$, I recalculated all the constants necessary, keeping six decimal places throughout the arithmetical processes and using Sheppard's corrections or not, according as Karl Pearson did or did not use them in his work. I have tabled all the constants to four decimal places, but, whenever any constant was used to obtain a new quantity, its value as obtained by me to six decimal places was used.

Although Karl Pearson obtained his constants by a different arithmetical process, and his results are necessarily more accurate, since he has kept six decimal places throughout, yet, wherever the values of constants given by my work may be compared with his, they agree to at least four decimal places excepting in the case of illustration C, in which the general agreement is only as far as the third decimal figure. This illustration has given throughout more irregular results than the others, due probably to the disturbing influence of the ecdyses.

Having obtained the necessary constants I proceeded to calculate Σ_{ϖ} for each case from the formula (xxii).

It remained to test the closeness to which the simple formulae (xxvii), (xxix) might be used instead of the formulae (xxvi), (xxviii) for Σ_ζ, $\Sigma_\searrow$. For this purpose Karl Pearson's four illustrations are again extremely convenient since these are the only published frequency distributions for which Σ_η has been calculated from the complex formula. In each of the four cases I substituted in the formulae (xxvi) and (xxviii) for Σ_ϖ, Σ_η the values found from the complex formulae, and

for Σ_r the value found from the usual approximate formula $\Sigma_{r^2} = \dfrac{(1-r^2)^2}{N}$. Thus values for Σ_ϖ, Σ_ζ, $\Sigma_\backprime$ were found which are certainly true values excepting in so far as the formula for Σ_{r^2} is only approximate.

Writing E_ϖ, E_ζ, $E_\backprime$ for the probable errors thence deduced we have $E_\varpi = \cdot67449\Sigma_\varpi$, $E_\zeta = \cdot67449\Sigma_\zeta$, $E_\backprime = \cdot67449\Sigma_\backprime$.

The standard deviations and probable errors were now found from the simple formulae (xxiii), (xxvii) and (xxix) and the resulting values denoted by Σ'_ϖ, Σ'_ζ, $\Sigma'_\backprime$, $E'_\varpi, E'_\zeta, E'_\backprime$. Finally the ratios $\dfrac{\varpi}{E'_\varpi}$, $\dfrac{\zeta}{E'_\zeta}$, $\dfrac{\backprime}{E'_\backprime}$ were calculated and the significance or non-significance of the deviations from linearity judged by the values of these ratios.

Exhibiting the values of the constants thus deduced; we have

Illustration A. On the Skew Correlation between Number of Branches to the Whorl and Position of the Whorl on the Spray in the case of *Asperula odorata.*

The position of the whorl is the x-character and the number of branches to the whorl is the y-character. Taking the values of n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$ as given by Pearson, we have

	n_{x_ρ}	y_{x_ρ}	$\sigma_{n_{x_\rho}}$
x_1	150	6·7800	·8553
x_2	150	6·8133	·8437
x_3	150	6·8133	·9047
x_4	142	6·4859	·8780
x_5	87	6·1724	·8605

We find, the quantities being determined without the use of Sheppard's corrections,

$$N = 679,$$

$$\bar{x} = 2\cdot8027, \qquad\qquad \bar{y} = 6\cdot6554,$$
$$\sigma_x = 1\cdot3369, \qquad\qquad \sigma_y = \cdot8978,$$
$$\nu_2 = 1\cdot7873, \qquad\qquad \mu_2 = \cdot8061.$$
$$\nu_4 = 5\cdot8417,$$

$$\sigma_M{}^2 = \lambda_2 = \cdot0503, \qquad\qquad p_{11} = \cdot2492,$$
$$\lambda_4 = \cdot0075, \qquad\qquad p_{31} = \cdot8964,$$
$$\lambda_{13} = \cdot0342, \qquad\qquad p_{22} = 1\cdot4906.$$
$$\lambda_{22} = \cdot1678,$$

$$\sigma_M = \cdot2244, \qquad\qquad \eta = \cdot2499, \qquad\qquad r = \cdot2076,$$
$$\chi_1 = \cdot9909, \qquad\qquad \chi_3 = \cdot9771.$$

Substituting these values in formula (xxii) we may exhibit the result so as to show the numerical contribution of each term in the form

$$\Sigma_{\varpi}^{2} = \frac{1}{N}\{7{\cdot}1970 - {\cdot}2928 + {\cdot}9870 - 1{\cdot}1980$$
$$- {\cdot}0129 + {\cdot}2704 - {\cdot}5673$$
$$+ {\cdot}5492\}$$

$$= \frac{1}{N}\{7{\cdot}1970 - {\cdot}5038 - {\cdot}3099 + {\cdot}5492\}$$

$$= \frac{1}{N}\{7{\cdot}1970 - {\cdot}2544\} = \frac{1}{N}\{6{\cdot}9426\} \quad \ldots\ldots\ldots\ldots(\text{xxxii}).$$

This gives $\qquad\qquad \Sigma_{\varpi} = {\cdot}1011, \qquad E_{\varpi} = {\cdot}0682.$
Pearson gives $\qquad\qquad\qquad E_{\eta} = {\cdot}0242.$
Hence $\qquad\qquad\qquad\qquad \Sigma_{\eta} = {\cdot}0359.$

From the formula $\qquad\qquad \Sigma_{r}^{2} = \frac{(1-r^{2})^{2}}{N},$

$$\Sigma_{r}^{2} = {\cdot}0013.$$

Hence from formulae (xxvi) and (xxviii) we get
$$\Sigma_{\zeta} = {\cdot}0108, \qquad \Sigma_{\gimel} = {\cdot}0217.$$

Hence, finally $\quad E_{\varpi} = {\cdot}0682, \qquad\qquad E_{\zeta} = {\cdot}0073, \qquad\qquad E_{\gimel} = {\cdot}0147,$
while, from the simple formulae (xxiii), (xxvii), (xxix) we get
$$E'_{\varpi} = {\cdot}0694, \qquad\qquad E'_{\zeta} = {\cdot}0071, \qquad\qquad E'_{\gimel} = {\cdot}0150.$$

These give $\qquad \dfrac{\varpi}{E'_{\varpi}} = 2{\cdot}6728, \qquad \dfrac{\zeta}{E'_{\zeta}} = 2{\cdot}7389, \qquad \dfrac{\gimel}{E'_{\gimel}} = 2{\cdot}8236.$

Illustration B.   On the Correlation between Age and Head Height in Girls.
Age is taken as the x-character and height of head as the y-character.
We have in this case

x_{ρ}	$n_{x_{\rho}}$	$x_{x_{\rho}}$	$\sigma_{n_{x_{\rho}}}$
3— 4	1	115·2500	0·0000
4— 5	7	116·9643	5·7706
5— 6	18	117·4722	5·8552
6— 7	40	119·1000	5·9282
7— 8	76	120·3026	5·9764
8— 9	125	121·6340	5·2732
9—10	177	121·7246	6·7754
10—11	235	122·8160	5·9306
11—12	261	123·1427	6·4178
12—13	309	123·8908	6·4122
13—14	263	124·8622	6·7178
14—15	198	125·7146	7·1730
15—16	214	126·1565	6·9326
16—17	162	126·5340	7·7392
17—18	95	126·9132	6·3358
18—19	61	127·0205	6·2470
19—20	13	129·5577	9·6812
20—21	7	123·8214	5·0622
21—22	8	126·5000	8·2828
22—23	2	125·2500	1·9148

We find, using Sheppard's corrections throughout,

$$N = 2272,$$

$$
\begin{aligned}
\bar{x} &= 12\cdot7007, & \bar{y} &= 124\cdot0467, \\
\sigma_x &= 3\cdot0648, & \sigma_y &= 6\cdot9083, \\
\nu_2 &= 9\cdot3931, & \mu_2 &= 47\cdot7239. \\
\nu_4 &= 239\cdot1571,
\end{aligned}
$$

$$
\begin{aligned}
\sigma_M{}^2 = \lambda_2 &= 4\cdot3822, & p_{11} &= 6\cdot2274, \\
\lambda_4 &= 62\cdot3991, & p_{31} &= 148\cdot8952, \\
\lambda_{13} &= 80\cdot4155, & p_{22} &= 510\cdot1883. \\
\lambda_{22} &= 107\cdot3209,
\end{aligned}
$$

$$\sigma_M = 2\cdot0934, \qquad \eta = \cdot3030, \qquad r = \cdot2941,$$

$$\chi_1 = \cdot9993, \qquad \chi_3 = \cdot9953.$$

Hence we deduce from formula (xxii)

$$
\begin{aligned}
\Sigma_{\varpi}{}^2 &= \frac{1}{N}\{\cdot6688 - \cdot0724 + \cdot4546 - \cdot4035 \\
&\qquad + \cdot0623 + \cdot0533 - \cdot1964 \\
&\qquad + \cdot0867\} \\
&= \frac{1}{N}\{\cdot6688 - \cdot0213 - \cdot0808 + \cdot0867\} \\
&= \frac{1}{N}\{\cdot6688 - \cdot0154\} = \frac{1}{N}\{\cdot6534\} \quad\ldots\ldots\ldots\ldots\ldots\text{(xxxiii).}
\end{aligned}
$$

This gives $\qquad \Sigma_{\varpi} = \cdot0170, \qquad E_{\varpi} = \cdot0114.$

Pearson gives $\qquad\qquad\qquad E_{\eta} = \cdot0129.$

Hence $\qquad\qquad\qquad \Sigma_{\eta} = \cdot0191.$

From the formula $\qquad \Sigma_r{}^2 = \dfrac{(1 - r^2)^2}{N},$

$$\Sigma_r{}^2 = \cdot0004.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_{\zeta} = \cdot0030, \qquad \Sigma_{\backprime} = \cdot0051.$$

Hence, finally

$$E_{\varpi} = \cdot0114, \qquad E_{\zeta} = \cdot0020, \qquad E_{\backprime} = \cdot0034,$$

while, from the simple formulae (xxiii), (xxvii) and (xxix) we get

$$E'_{\varpi} = \cdot0116, \qquad E'_{\zeta} = \cdot0021, \qquad E'_{\backprime} = \cdot0034.$$

These give

$$\frac{\varpi}{E'_{\varpi}} = 2\cdot5752 \qquad \frac{\zeta}{E'_{\zeta}} = 2\cdot5879, \qquad \frac{\backprime}{E'_{\backprime}} = 2\cdot5973.$$

Illustration C.   On the Skew Correlation between Size of Cell and Size of Body in *Daphnia magna.*

Length of body is taken for the x-character and size of cell for the y-character. We have

x_ρ	n_{x_ρ}	y_{x_ρ}	$\sigma_{n_{x_\rho}}$
1	20	5·3000	0·9092
2	150	5·8333	1·6620
3	100	7·7900	1·6860
4	80	8·0500	2·1539
5	150	9·4733	2·6694
6	140	8·4357	1·7087
7	230	8·5957	1·9384
8	120	10·2667	1·9439
9	180	10·7611	2·4469
10	150	11·0267	2·6949
11	150	10·9533	2·4470
12	140	9·1000	1·7947
13	120	9·0000	1·8619
14	220	10·0364	1·8894
15	60	10·3167	1·5165

We find, using Sheppard's corrections for the cell sizes, but not for the body lengths,

$$N = 2010,$$

$$\bar{x} = 8\text{·}5025, \qquad\qquad \bar{y} = 9\text{·}2687,$$
$$\sigma_x = 3\text{·}8648, \qquad\qquad \sigma_y = 2\text{·}5417,$$
$$\nu_2 = 14\text{·}9366, \qquad\qquad \mu_2 = 6\text{·}4604.$$
$$\nu_4 = 432\text{·}7695,$$
$$\sigma_M{}^2 = \lambda_2 = 2\text{·}1136, \qquad\qquad p_{11} = 3\text{·}8929,$$
$$\lambda_4 = 15\text{·}1375, \qquad\qquad p_{31} = 127\text{·}3503,$$
$$\lambda_{13} = 27\text{·}8071, \qquad\qquad p_{22} = 108\text{·}1612.$$
$$\lambda_{22} = 56\text{·}3594,$$
$$\sigma_M = 1\text{·}4538, \qquad\qquad \eta = \text{·}5720, \qquad\qquad r = \text{·}3963,$$
$$\chi_1 = \text{·}8918, \qquad\qquad \chi_3 = \text{·}7574.$$

Substituting in formula (xxii) we get

$$\Sigma_{\varpi^2} = \frac{1}{N} \{3\text{·}3108 - \text{·}2651 + \text{·}8098 - 1\text{·}2303$$
$$+ \text{·}0971 - \text{·}3796 - \text{·}6074$$
$$+ \text{·}7752\}$$

$$= \frac{1}{N} \{3\text{·}3108 - \text{·}6855 - \text{·}8898 + \text{·}7752\}$$

$$= \frac{1}{N} \{3\text{·}3108 - \text{·}8001\} = \frac{1}{N} \{2\text{·}5106\} \quad \dots\dots\dots\dots(\text{xxxiv}).$$

This gives $\Sigma_{\varpi} = \text{·}0353, \qquad E_{\varpi} = \text{·}0238.$
Pearson gives $E_\eta = \text{·}0097.$
Hence $\Sigma_\eta = \text{·}0144.$

From the formula $\qquad \Sigma_{r^2} = \dfrac{(1 - r^2)^2}{N},$

$$\Sigma_{r^2} = \cdot 0004.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_\zeta = \cdot 0125, \qquad \Sigma_\backsim = \cdot 0138.$$

Hence, finally

$$E_\varpi = \cdot 0238, \qquad\qquad E_\zeta = \cdot 0084, \qquad\qquad E_\backsim = \cdot 0093,$$

while, from the simple formulae (xxiii), (xxvii), (xxix) we get

$$E'_\varpi = \cdot 0274, \qquad\qquad E'_\zeta = \cdot 0107, \qquad\qquad E'_\backsim = \cdot 0114.$$

These give

$$\frac{\varpi}{E'_\varpi} = 13 \cdot 4052, \qquad \frac{\zeta}{E'_\zeta} = 15 \cdot 8934, \qquad \frac{\backsim}{E'_\backsim} = 15 \cdot 3972.$$

Illustration D. On the Skew Correlation between Number of Branches to the Whorl and Position of the Whorl on the stem in *Equisetum arvense.*

Position of whorl is taken as the x-character and number of branches to the whorl as y-character. We have

x_ρ	n_{x_ρ}	y_{x_ρ}	$\sigma_{n_{x_\rho}}$
1	126	7·619	2·360
2	126	9·294	1·273
3	126	9·627	1·187
4	126	9·730	1·151
5	126	9·643	1·158
6	124	9·427	1·375
7	123	8·732	1·781
8	121	7·297	2·291
9	119	5·555	2·553
10	110	3·964	2·199
11	97	2·443	1·506
12	67	1·866	0·960
13	39	1·462	0·746
14	12	1·333	0·471
15	4	1·250	0·433
16	2	1·000	0·000

Not using Sheppard's corrections for either character, we find

$$N = 1448,$$

$$\begin{aligned}
\bar{x} &= 6 \cdot 4033, & y &= 7 \cdot 2169, \\
\sigma_x &= 3 \cdot 5426, & \sigma_y &= 3 \cdot 2785, \\
\nu_2 &= 12 \cdot 5500, & \mu_2 &= 10 \cdot 7486. \\
\nu_4 &= 319 \cdot 5158,
\end{aligned}$$

$$\begin{aligned}
\sigma_M{}^2 = \lambda_2 &= 7 \cdot 7838, & p_{11} &= 8 \cdot 2256, \\
\lambda_4 &= 140 \cdot 4409, & p_{31} &= 205 \cdot 0840, \\
\lambda_{13} &= 152 \cdot 2416, & p_{22} &= 205 \cdot 2746. \\
\lambda_{22} &= 171 \cdot 1280,
\end{aligned}$$

$$\sigma_M = 2 \cdot 7899, \qquad\qquad \eta = \cdot 8510, \qquad\qquad r = \cdot 7082,$$

$$\chi_1 = \cdot 6437, \qquad\qquad\qquad \chi_3 = \cdot 7992.$$

Hence from formula (xxii) we have

$$\Sigma_{\varpi}^2 = \frac{1}{N}\{ \cdot 6128 - \cdot 2428 + 1 \cdot 0134 - \cdot 9598$$
$$- \cdot 1705 + \cdot 9708 - \cdot 6241$$
$$+ \cdot 0173\}$$

$$= \frac{1}{N}\{ \cdot 6128 - \cdot 1893 + \cdot 1762 + \cdot 0173\}$$

$$= \frac{1}{N}\{ \cdot 6128 + \cdot 0042\} = \frac{1}{N}\{ \cdot 6170\} \quad \dots\dots\dots\dots\dots(\text{xxxv}).$$

This gives $\qquad\qquad \Sigma_{\varpi} = \cdot 0206, \qquad E_{\varpi} = \cdot 0139.$

Pearson gives $\qquad\qquad\qquad\qquad E_{\eta} = \cdot 0054.$

Hence $\qquad\qquad\qquad\qquad \Sigma_{\eta} = \cdot 0080.$

From the formula $\qquad\qquad \Sigma_{r}^2 = \frac{(1-r^2)^2}{N},$

$$\Sigma_{r}^2 = \cdot 0002.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_{\zeta} = \cdot 0229, \qquad \Sigma_{\vartheta} = \cdot 0153.$$

Hence, finally

$$E_{\varpi} = \cdot 0139, \qquad E_{\zeta} = \cdot 0154, \qquad E_{\vartheta} = \cdot 0103,$$

while, from the simple formulae (xxiii), (xxvii) and (xxix) we get

$$E'_{\varpi} = \cdot 0139, \qquad E'_{\zeta} = \cdot 0152, \qquad E'_{\vartheta} = \cdot 0102.$$

These give

$$\frac{\varpi}{E'_{\varpi}} = 13 \cdot 2341, \qquad \frac{\zeta}{E'_{\zeta}} = 14 \cdot 6291, \qquad \frac{\vartheta}{E'_{\vartheta}} = 13 \cdot 9785.$$

In order to appreciate the significance of the results in these four cases they are collected and tabulated as follows:

$$\left[\text{Note}: \; \varpi = \log_e \frac{\eta}{r} = \log_{10} \frac{\eta}{r} \times \log_e 10 \right]$$

	ϖ	ζ	ϑ	$\dfrac{\varpi}{E'_{\varpi}}$	$\dfrac{\zeta}{E'_{\zeta}}$	$\dfrac{\vartheta}{E'_{\vartheta}}$		E_{ϖ}	E_{ζ}	E_{ϑ}
A	·1856	·0194	·0423	2·6728	2·7389	2·8236	E'	·0694	·0071	·0150
							E	·0682	·0073	·0147
							$E' \sim E$	·0012	·0002	·0003
B	·0298	·0053	·0089	2·5752	2·5879	2·5973	E'	·0116	·0021	·0034
							E	·0114	·0020	·0034
							$E' \sim E$	·0002	·0001	·0000
C	·3669	·1701	·1757	13·4052	15·8934	15·3972	E'	·0238	·0107	·0114
							E	·0274	·0084	·0093
							$E' \sim E$	·0036	·0023	·0021
D	·1836	·2226	·1438	13·2341	14·6291	13·9785	E'	·0139	·0152	·0102
							E	·0139	·0154	·0103
							$E' \sim E$	·0000	·0002	·0001

We conclude from this table:

I. As regards the linearity of the regression in the statistical examples chosen, illustrations C and D are certainly skew. The skewness of these examples is obvious either from a glance at the diagrams accompanying Karl Pearson's memoir or from his discussion of the cubic and quartic regression curves for these cases. Illustrations A and B give $\dfrac{\varpi}{E_\varpi}$, $\dfrac{\zeta}{E_\zeta}$, $\dfrac{\vartheta}{E_\vartheta}$ values between 2 and 3. The limiting value of these ratios which should denote significance is a matter which can only be decided as a result of large statistical experience. When the ratio gets as low as 2 we are getting on the border line and for cases A and B we can only say that the skewness is probably significant. This is the kind of result we should expect from the corresponding diagrams of Karl Pearson's memoir. He finds, however, that he obtains so much better fits by curves of higher order that he concludes that the regression of A is parabolic and of B cubic.

II. If we refer to equations (xxxii), (xxxiii), (xxxiv), (xxxv) we see exhibited the numerical contribution of each term of formula (xxii) for $\Sigma_\varpi{}^2$. As regards the numerical values of the quantities which vanish for normal correlation we see that they by no means become zero in the examples chosen.

Also in these equations I have grouped together the three p terms and the three λ terms, but the numerical values of the separate terms are still considerable. When the whole expression for $\Sigma_\varpi{}^2$ is summed however we find that the last three terms of these numerical equations so nearly cancel that the difference between E_ϖ and E'_ϖ is at the most $\cdot004$, a quantity which is of no significance in the value of a probable error. This is exactly the kind of result Karl Pearson found in comparing the values of E_η as found from the complex formula and the simple formula. There is thus exactly the same justification for the use of formula (xxiii) as for the formula $\Sigma_\eta{}^2 = \dfrac{(1-\eta^2)^2}{N}$.

Comparing the values of E_ζ, E_ϑ with E'_ζ, E'_ϑ we see that the agreement is closer still, the difference being at the most of order $\cdot002$; and if we except illustration C the agreement in the values of E_ζ, E_ϑ is remarkably close, being of the order $\cdot0003$ at the most.

Thus our numerical work leads us to the conclusion that we have the same justification for the use of the simple formulae (xxiii), (xxvii), and (xxix) for E_ϖ, E_ζ, E_ϑ as for the use of the accepted simple formulae for E_r, E_η.

These simple formulae have been obtained by making approximations of a statistical nature, i.e. approximations suggested entirely by statistical experience apart from arithmetic. I proceed to examine the formulae thus obtained to see if any simplification can be made by arithmetical approximation.

Accepting the simple formula (xxiii) for Σ_{ϖ}, we get

$$\frac{\varpi^2}{N\Sigma_{\varpi}{}^2} = \frac{\left[\log_e \dfrac{\eta}{r}\right]^2}{\dfrac{1}{r^2} - \dfrac{1}{\eta^2}} = \frac{\left[\log_e \left(1 + \dfrac{\vartheta}{r}\right)\right]^2}{\dfrac{1}{r^2} - \dfrac{1}{(r+\vartheta)^2}}.$$

Assuming $\dfrac{\vartheta}{r} < 1$, we can expand this expression in a series of ascending powers of ϑ, and, neglecting cubes and higher powers, we get

$$\frac{\varpi^2}{N\Sigma_{\varpi}{}^2} = \frac{r\vartheta}{2} + \frac{\vartheta^2}{4} \dotfill \text{(xxxvi)}.$$

Proceeding in exactly the same way from equations (xxvii) and (xxix), we get

$$\frac{\zeta^2}{N\Sigma_{\zeta}{}^2} = \frac{r\vartheta}{2} + \frac{\vartheta^2}{4} + \frac{\vartheta^2}{4}.8r^2(1-r^2)\dotfill \text{(xxxvii)},$$

$$\frac{\vartheta^2}{N\Sigma_{\backprime}{}^2} = \frac{r\vartheta}{2} + \frac{\vartheta^2}{4} + \frac{\vartheta^2}{4}.(1+3r^2)(1-r^2) \dotfill \text{(xxxviii)}.$$

When $\dfrac{\vartheta}{r}$ is small these approximate formulae will be very nearly accurate, and they exhibit the arithmetical reason why $\dfrac{\varpi}{E'_{\varpi}}$, $\dfrac{\zeta}{E'_{\zeta}}$, $\dfrac{\vartheta}{E'_{\backprime}}$ work out nearly equal and, moreover, we see in what manner they will differ when they are not equal. In our statistical examples $\dfrac{\vartheta}{r}$ is small and we should expect from these second order approximations that $\dfrac{\zeta}{E'_{\zeta}}$, $\dfrac{\vartheta}{E'_{\backprime}}$ would be greater than $\dfrac{\varpi}{E'_{\varpi}}$, which will be seen to be the case on referring to the table of values. Again, $\dfrac{\vartheta}{E'_{\backprime}} \gtreqless \dfrac{\zeta}{E'_{\zeta}}$ according as $1 + 3r^2 \gtreqless 8r^2$, i.e. as $1 \gtreqless 5r^2$; if r^2 is nearly $\dfrac{1}{5}$, $\dfrac{\vartheta}{E'_{\backprime}}$, $\dfrac{\zeta}{E'_{\zeta}}$ will be nearly equal, but we cannot say which will be the greater as the third order terms are then likely to become important in determining the difference. These conclusions are seen to be borne out by the table of values, and other peculiarities in the values might be explained in the same way.

When $\dfrac{\vartheta}{r}$ is small, equations (xxxvi), (xxxvii), and (xxxviii), obviously suggest the arithmetical approximation

$$\frac{\varpi}{E_{\varpi}} = \frac{\zeta}{E_{\zeta}} = \frac{\vartheta}{E_{\backprime}} = \frac{\sqrt{N}}{\cdot 67449}.\tfrac{1}{2}\sqrt{\zeta}\dotfill \text{(xxxix)}$$

since

$$\frac{r\vartheta}{2} + \frac{\vartheta^2}{4} = \frac{\eta - r}{4}\{2r + \eta - r\} = \frac{\eta^2 - r^2}{4}.$$

I append the values given by (xxxix), for the four statistical illustrations:

	A	B	C	D
$\dfrac{\vartheta}{r}$	·2039	·0302	·4433	·2016
$\dfrac{\sqrt{N}}{·67449} \cdot \frac{1}{2}\sqrt{\zeta}$	2·6883	2·5711	13·7078	13·3086

Even when $\dfrac{\vartheta}{r}$ is not very small

$$\frac{\zeta}{E'_\zeta} = \frac{\sqrt{N}}{·67449} \cdot \tfrac{1}{2}\sqrt{\zeta} \cdot \frac{1}{\sqrt{1 - 2\,(\eta^2 - r^2) + \eta^4 - r^4}},$$

and it is obvious that if there be any doubt as to linearity of regression, when $\dfrac{\vartheta}{r}$ is not small, then r and η must be both small and $\dfrac{\zeta}{E'_\zeta} = \dfrac{\sqrt{N}}{·67449} \cdot \tfrac{1}{2}\sqrt{\zeta}$ will still be a good arithmetical approximation for equation (xxvii).

General Conclusion.

Thus to test whether the regression of any frequency distribution is linear or not we have three possible methods, viz.:—to compare the value of either ϖ, ζ, or ϑ, with its probable error. As regards the best quantity to choose for this we may first say that ϖ has no advantages whatever. In favour of the use of ϑ we may say that the terms of ϑ are just η and r, which are the quantities in terms of which we naturally judge our distribution; we must be careful however in using formula (xxix) for Σ_ϑ to remember that r is supposed positive; i.e. if r is negative for any case we must change its sign before substituting in the formula and we then get the probable error of the difference between η and the absolute magnitude of r.

ζ however is the term occurring naturally in the work and is the one quantity of the three with a direct physical meaning; i.e. ζ gives the mean square deviation of the distribution from the regression line. This will probably be sufficient to assure that tests for linearity will be conducted in terms of ζ.

Finally,

(i) A simple test for linearity of regression which will be sufficient in very many cases is

$$\frac{\sqrt{N}}{·67449} \cdot \tfrac{1}{2}\sqrt{\zeta} < 2·5.$$

(ii)　For more exact work $\dfrac{\zeta}{E_\zeta}$, $\dfrac{\vartheta}{E_\vartheta}$ should be calculated from the formulae

$$\frac{\zeta}{E_\zeta} = \frac{\sqrt{N}}{\cdot 67449} \cdot \tfrac{1}{2}\sqrt{\zeta} \cdot \frac{1}{\sqrt{1 + (1 - \eta^2)^2 - (1 - r^2)^2}},$$

$$\frac{\vartheta}{E_\vartheta} = \frac{\sqrt{N}}{\cdot 67449} \cdot \tfrac{1}{2}\sqrt{\zeta} \cdot \frac{\sqrt{4\eta r}}{\eta + r} \cdot \frac{1}{\sqrt{1 + \dfrac{r(1 - \eta^2)^2 - \eta(1 - r^2)^2}{\eta + r}}}.$$

Each of these results gives an approximate formula for the same physical quantity (i.e. the number to be looked up in the table of the probability integral if we want the probability in favour of the distribution being linear).

If these ratios work out sufficiently nearly equal this constitutes some justification for the statistical approximations made in obtaining the formulae, but if on comparing the values of the ratios the agreement is unsatisfactory, and no mistake in the arithmetic can be detected, then the probable errors of E_ζ, E_ϑ must be calculated from the complete formulae (xxii), (xxvi), (xxviii) in terms of the subsidiary quantity ϖ and substituting for Σ_η^2 its value as determined by the complete formula given by Karl Pearson.

THE LONG BARROW AND ROUND BARROW SKULLS IN THE COLLECTION OF THE DEPARTMENT OF COMPARATIVE ANATOMY, THE MUSEUM, OXFORD.

By E. H. J. SCHUSTER, M.A., New College, Oxford.

THE following work was undertaken at the desire of Professor Weldon. It is the first part of a descriptive catalogue of the British skulls in the collection of the Department of Comparative Anatomy, Oxford, and is an attempt to lay before the public in an accessible and partially digested form the material in this collection. Owing to the small number of the measurements of each group which it has been possible to take, the statistical constants deduced from them are not of very much value as they stand, but it is hoped that similar catalogues of other collections may be published which will enable one to deal with larger groups.

Professor Weldon and Professor Karl Pearson have written a descriptive catalogue of the skulls, which has been based to a certain extent on the manuscript catalogue prepared some years ago for the Department of Comparative Anatomy by Mr Hatchett Jackson. The account of the anatomical peculiarities was most kindly provided by Professor Thane.

Professor Thane's determinations of the sex have also been adhered to throughout.

The following measurements were taken to such extent as was possible in each individual case.

A. CRANIAL.

L'. Horizontal length. This was measured with instruments described in Miss Fawcett's memoir (*Biometrika*, Vol. I. p. 408).

L. Greatest length measured from the *glabella* to *occiput*.

L". Greatest length measured from the *nasion*.

F. Ophryo-occipital length. Flower. Greatest length measured from the middle point between the frontal prominences.

B. Greatest breadth measured outside the squamosals.

B″. Auricular breadth of Virchow—distance between the upper borders of the auditory canal.

B′. Smallest frontal breadth.

H′. Height. From the anterior border of the foramen magnum to the point on the surface of the skull, in a line vertical to the horizontal plane.

H. Auxiliary height. From the anterior border of the foramen magnum to the bregma, i.e. the juncture of the sagittal and coronary sutures.

OH. Auricular height, measured with the vertical scale and sliding rod of the craniophor (*Biometrika*, Vol. I. p. 414).

LB. Basicranial length, measured from the anterior border of the foramen magnum to the nasion.

fml. Greatest length of *foramen magnum.*

fmb. Greatest breadth of *foramen magnum.*

U. Horizontal circumference, measured with the steel band measure over the superciliary ridges in front and the most projecting part of the skull behind.

S. Sagittal circumference, measured from the nasofrontal suture to the posterior border of the foramen magnum over the sagittal suture.

Q. Vertical transverse circumference, measured from the upper border of the ear opening of one side to that of the other, in a plane vertical to the horizontal plane.

The foregoing measurements are described in the *Archiv für Anthropologie,* Band XV., Braunschweig, 1884.

B. Facial Measurements.

G′H. Upper face height, measured from the nasion to the middle of the central process of the upper jaw between the middle incisor teeth, (i.e. the alveolar point).

GB. Face breadth, being the distance from one zygomatic maxillary suture to the other. The measurement must be taken from the lower end of these sutures, from the lower front rim of one cheek-bone to that of the other.

J. Bizygomatic breadth, from the outermost point of one zygomatic arch to the like point on the other.

NH. Nasal height, measured from the nasion to the lowest edge of the pyriform aperture.

NB. Nasal breadth, greatest breadth of the nasal aperture, wherever it may be.

O_1. *Breadth of orbit* for both left (L) and right (R) eyes, measured from the "point lacrymal" on the inner side to the inner margin of the orbit on the other side, along a line parallel to the general slope of the orbit.
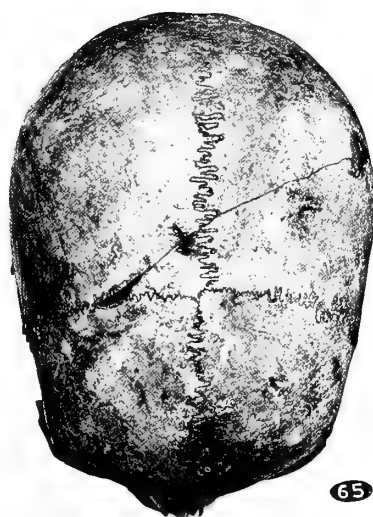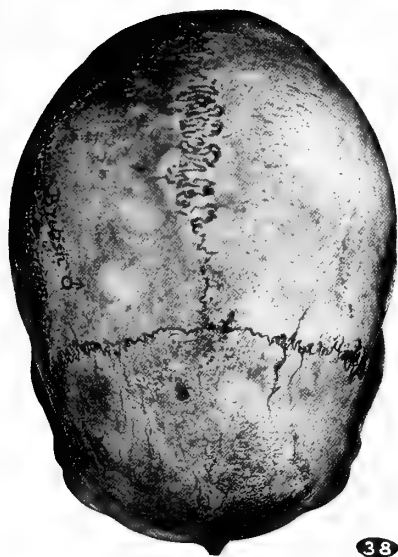
Long Barrow Crania.

N. verticalis.

Round Barrow Crania.
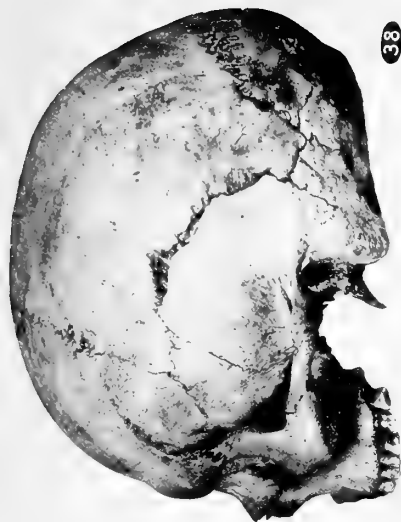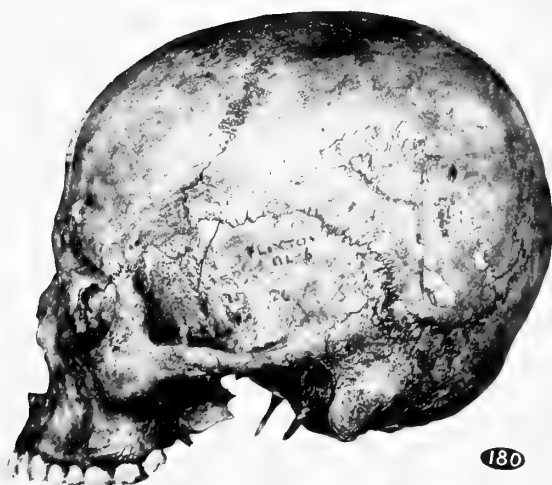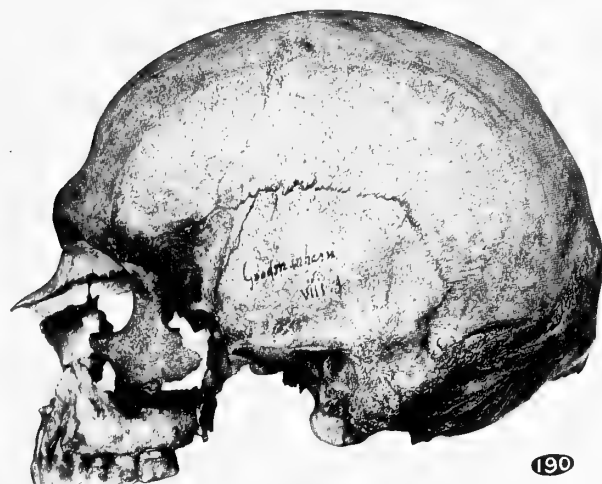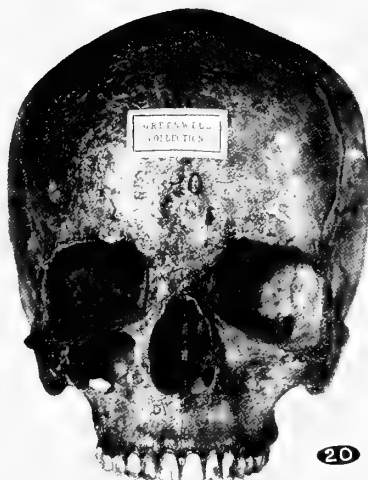
N. verticalis.

Long Barrow Crania.
N. lateralis

Round Barrow Crania.

N. lateralis.
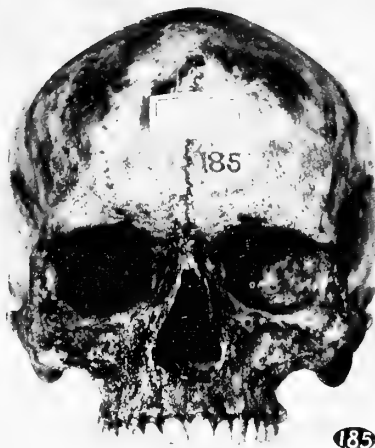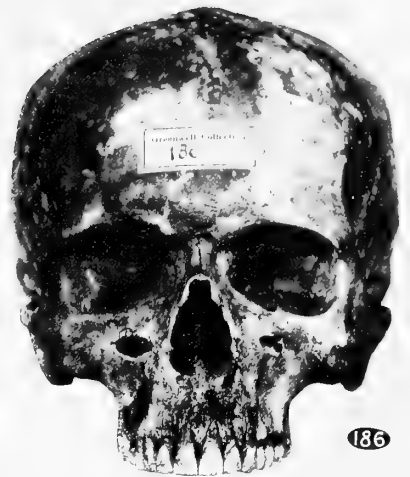
Round Barrow Crania.

N. facialis.
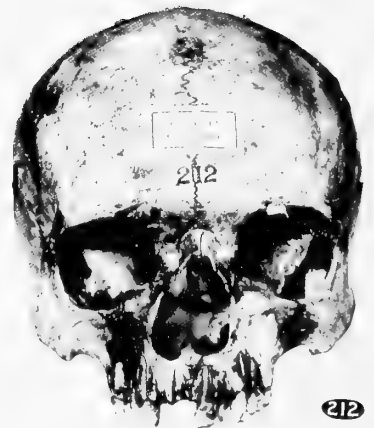
Round Barrow Crania.
N. facialis.

"Point lacrymal" is the point at which the hinder edge of the lacrymal groove meets the lacrymo-frontal suture.

O_2. *Greatest height of orbit,* for both (L) and (R) eyes, taken as closely as possible perpendicular to O_1.

G_1. *Length of palate,* measured from the point of the *Spina nasalis posterior* to the inner wall of the alveolar rim between the middle incisors.

G_2. *Breadth of palate* between the alveolar walls at the second molars.

GL. *Profile length* from the basion to the alveolar point.

W_1. *Condylar width.* Greatest width of mandibles at condyles, from the outside of one condyle to the outside of the other.

W_2. *Angle width.* Greatest width of mandibles at angles, from the outside of one angle to the outside of the other.

h_1. *Greatest height of mandible,* measured from the lowest median projection to the top of process between middle incisors.

f. *Distance between foramina mentalia.*

C. Indices.

B/L'. *Cephalic index* for horizontal length.

B/L. *Cephalic index* for greatest length measured from the glabella.

H'/L'. *The ratio of height to length,* for horizontal length and the whole height.

H/L. *The ratio of height to length,* for the greatest length measured from the glabella and for the auxiliary height.

$G'H/GB$. The upper face height to face breadth index.

NB/NH. The nasal breadth to nasal height index.

$O_2/O_1 (R)$, $O_2/O_1 (L)$. The orbital height to orbital breadth index for both right and left eyes.

G_2/G_1. The palate breadth to palate length index.

P. The profile angle found with Ranke's Goniometer.

The description of the facial measurements, and of the indices calculated, is taken from Miss Fawcett's paper on the Naqada Crania (*Biometrika,* Vol. i.), and the lettering used in the tables of that memoir to denote these various measurements and indices has been retained. But it will be noticed that certain of the measurements have been omitted, and others slightly modified. Thus owing to the imperfect state of the teeth, in many cases, the face height was not measured. Also the orbital breadths have been measured from a different point on the inner side as it was found more convenient to do them in this way.

Owing to the few cases in which these determinations could have been made, it was not considered worth while to calculate the angles $A \angle$, $N \angle$, $B \angle$, θ_1, θ_2.

The measurement of the capacity was not attempted owing to the fragmentary nature of the majority of the skulls*.

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* The following six were found fit for the measurement of the capacity which is given in cm.^{3}:

24, 1606. 26, 1496. 38, 1592. 83, 1667. 173, 1437. 180, 1502.

Table I. shews the means of the measurements thus taken, with the number of measurements in each case. Tables II. to IV. give the actual measurements. Plates I. to VI. give characteristic specimens of Long and Round Barrow Crania with a few abnormal cases.

TABLE I. *Showing means for each group of skulls and each measurement, with number of observations in each case.*

Measurement	Long Barrow				Round Barrow			
	Mean ♂	Number of observations ♂	Mean ♀	Number of observations ♀	Mean ♂	Number of observations ♂	Mean ♀	Number of observations ♀
L'	191·88	8	185·33	3	185·75	8	179·45	·11
L	190·56	16	182·61	13	188·61	36	177·97	34
L''	188·44	16	181·50	12	185·03	32	175·88	30
F	187·15	17	183·96	12	187·01	37	178·63	35
B	142·44	18	138·62	12	144·77	31	136·56	34
B''	121·37	16	116·94	9	121·68	25	113·64	22
B'	98·94	16	94·09	11	98·27	32	95·98	29
H'	138·44	9	138·83	3	138	3	136·00	2
H	137·83	12	135·11	9	135·94	16	130·80	15
OH	120·67	9	118·00	3	123·62	8	115·91	11
LB	101·95	11	96·81	8	103·94	16	99·33	12
fml	35·68	11	34·50	6	36·97	15	35·44	9
fmb	27·73	11	30·25	6	31·00	15	28·79	7
U	534·87	16	518·71	7	537·85	27	506·43	30
S	384·77	13	382·00	8	376·53	15	365·30	15
Q	321·78	9	312·00	3	333·38	8	313·54	11
$G''H$	69·92	13	66·75	4	70·44	16	66·20	10
GB	95·88	12	92·75	4	97·42	12	90·29	7
J	134·00	3	132·5	1	133·50	3	—	—
NH	49·43	15	47·00	7	50·18	14	48·22	9
NB	24·07	15	22·79	7	24·61	14	23·50	9
$O_1 \begin{cases} L \\ R \end{cases}$	38·12 / 38·28	8 / 9	36·87 / 37·87	4 / 4	39·03 / 40·00	15 / 21	38·78 / 39·35	9 / 10
$O_2 \begin{cases} L \\ R \end{cases}$	31·81 / 31·28	8 / 9	30·62 / 30·87	4 / 4	32·46 / 31·79	14 / 21	32·67 / 32·19	9 / 8
GL	95·33	9	92·62	4	96·00	8	93·43	7
G_1	55·31	8	57·00	1	55·59	11	50·69	8
G_2	43·70	5	36·00	1	40·73	15	39·11	9
W_1	121·21	7	113·83	3	123·38	13	117·35	10
W_2	104·90	10	92·92	6	105·04	24	94·85	13
h_1	34·13	15	31·80	5	33·83	21	30·89	17
f	45·67	15	85·5	3	45·23	22	43·69	16
B/L'	74·38	8	74·33	3	77·88	8	76·36	11
B/L	74·93	16	76·33	12	76·70	30	77·73	33
H'/L'	72·25	8	74·66	3	73·67	3	74·50	2
H/L	72·73	11	74·00	8	72·81	16	74·85	13
$G''H/GB$	71·44	9	81·00	1	72·83	12	73·50	6
NB/NH	49·00	15	49·10	6	49·14	14	48·89	9
$O_2/O_1 : L$	83·37	8	81·66	3	82·87	15	84·22	9
$O_2/O_1 : R$	81·89	9	80·66	3	79·60	20	82·12	8
G_2/G_1	78·40	5	—	—	73·82	11	76·75	8
P	83·00°	5	—	—	84·14°	7	84·00°	6

TABLE II.

G'_1	G_2	W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
—	—	—	—	33	43·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	33	45	—	69	—	—	—	—	—	—	—	—
—	—	—	115·5	35	46·5	—	—	—	—	—	—	—	—	—	—
55	—	—	—	41	46·5	—	69	—	—	—	44	—	—	—	—
—	—	—	—	—	—	71	71	72	72	—	43	88	89	—	—
53	—	—	—	—	—	72	72	71	71	65	45	82	83	—	86°
—	—	—	—	—	—	—	—	—	—	70	49	80	78	—	84°
—	—	123·5	—	38	42	73	72	71	70	—	45	89	87	—	—
56·5	41	—	—	—	—	—	—	—	—	—	45	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	48	—	93	73	—
—	—	111	105	37·5	45	—	75	—	—	78	45	84	—	—	—
55·5	—	—	99·5	31	46	—	66	—	—	—	—	—	—	—	—
—	—	—	—	33	45	73	73	69	69	76	41	85	87	—	81°
—	—	—	—	—	—	—	—	—	—	—	—	—	66	—	—
58	44	125·5	112	30·5	46·5	72	72	72	73	71	55	—	—	76	—
—	—	—	104	36·5	51·5	77	78	71	71	78	48	—	—	—	81°
—	—	—	97·5	32·5	43	—	80	—	77	—	—	—	—	—	—
51	43	126·5	105·5	31·5	45	—	77	—	72	—	60	77	78	84	—
57·5	47	121	99·5	35	46	80	81	78	79	67	61	—	76	82	83°
—	—	119·5	108	32	45·5	—	84	—	72	70	53	—	—	—	—
56	43·5	121·5	102·5	32·5	48	77	78	74	74	68	53	82	—	77	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	81	—	—	—	—	—	—	—	—
—	—	—	—	—	—	74	73	75	74	—	50	85	83	—	—
—	—	—	—	—	—	75	75	78	77	—	51	79	—	—	—
—	—	—	—	—	—	—	75	—	79	—	46	—	—	—	—
57	36	—	—	—	—	—	75	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	75	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	70	—	—	—	—	—	—	—	—
—	—	117	86·5	30	47·5	74	74	71	71	—	50	—	79	—	—
—	—	104·5	85·5	28	43	—	74	—	—	—	—	—	—	—	—
—	—	117	97	39	—	—	—	—	—	—	—	—	—	—	—
—	—	—	96·5	37·5	47·5	—	76	—	72	—	—	—	—	—	—
—	—	—	89	30	37	—	76	—	68	—	50	—	—	—	—
—	—	—	—	—	—	—	86	—	76	—	—	—	—	—	—
—	—	—	103	33·5	42·5	—	81	—	75	81	48	81	80	—	—

MEASUREMENTS

Number of Skull	I	I'	I''	I'''	L	B	L'	H'	H	OH	LB	fmt	fmb	t	S	Q	G'H	GB	J	NH	NB	$\frac{O_1}{R}$	R	L	$\frac{O_2}{R}$	GL	G_1	G_2	W_1	W_2	h_1	J	B/L'	B/L	H'/L'	H/L	G'H/GB	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
3			182·5	145	129·5	97·5								523																	33	43·5										
4	188	183	188	13·5		98·5								518																				69						—	—	
9																														33	45		35							—	
18																														115·5	35	46·5									
19	198	199·5		137·5										537	—		77					51	22·5						55		41	46·5	69				44				
21	197	193	194	197	140	119·5	101·5	112	142·5	122	106·5	39	31	545	400	320		92				51	22	37·5	37·5	33	33·5				71	71	72	72			43	88	89	—	
22	17	189	189	191·5	137	110	95	135	134	117	96·5	30·5		538	393	310	94	98	136	48·5	22	37	37·5	30·5	31	86	53			72	72	71	71	65	45	82	83	—	86°
25		188		141	114·5	99	137	137·5	117	103						312	79	99·5	127	52·5	25·5	40·5	40·5	32·5	31·5	96				—		70	49	80	78	—	84°
38	197	198	194	191	143	123	98	139	130	12	106	33·5	29·5	546	399	322		93	139	50·5	22·5	39·5	39·5	35	34·5				—			73	72	71	70	45	89	87	—	—
48	208	205	208	137	124	103									394		81			55	24·5					123·5			38	42			45						—
40																	70			49	23·5		35·5	—	33		56·5	41			48		93	73	—
59	193·5	187	197·5	142	115							33·5	28·5	538	374		71	91·5		50·5	23	38·5		32·5			111	105	37·5	45	75	78	45	84	—
62	194·5	186·5	195·5																								—	99·5	31	46	66		—
65	192	181·5	186·5	185·5	133	110·5	110	120	125·5	111	96·5	37·5	28·5	511	362	301	70	92		49·5	20·5	36·5	36	31	31·5	97	55·5	—	33	45	73	73	69	69	76	41	85	87	—	81°
72																						42		28				—		66		—
77	198	197·5	193	193·5	142	121·5	102·5	113	144		192·5		31·5	550	400	325	68·5	96·5		48	26·5				68	58	44	125·5	112	30·5	46·5	72	72	72	73	71	55		70		—
80	191	192·5	190	182	140	125·5	97·5	137	136·5	115	185·5	37	30	553	377	321	70	97		56·5	27				100·5		—	104	30·5	51·5	77	78	71	71	78	48		81
81			187	152	123	95·5		146						540	300														97·5	32·5	43	80		77	—
82		184	182·5	190·5	142	123·5	98		133		94	31	33·5	527	384		66·5			41	24·5	37·5	37	29	29	89	51	43	126·5	105·5	31·5	45	77	72	60	77	78	84
83	187	185	183	86	14	193	97	140	146	120	104·5	30	33	535	370	333	67	91·5		42	25·5	30			29·5	68	57·5	47	121	99·5	35	46	80	81	78	70	67	61	76	82	83
84		181		175	152	128	18		130		103	33·5	32·5	535	358		64·5	91		50·5	27					66			119·5	108	32	45·5		84		72	70	53
86	19	188	184·5	188·5	17	124	99	141	140	129	106·5	30	27	541	399	340	64·5	94·5		46	24·5	38			31		91·5	50	43·5	121·5	102·5	32·5	48	77	78	74	74	68	53	82	77
98		176·5	175·5	171·5	145	121·5	98							518				98									—				82		

LONG BARROW. ♀.

13		172	171·5	171·5	139·5		101		—	—	—			502	—	—	—		92·5	132·5	49	24·5	36·5	37·5	31	31	—	—	—	—		81	—	—	—	—	50	85	83	—	—
24	187	188	187	187·5	138	119	92	141·5	140	118	100·5	34	31	530	389	311	—			45·5	23	36·5	—	29	—	—	—	—	—	—	74	73	75	74	—	50	85	83	—	—
26	181	182	180·5	180·5	136	113	89·5	142	140·5	121	98	33·5	30·5	508	387	314	—			46	21	36·5	—			—	—	—	—	—	75	75	78	77	—	51	79	—	—	—
27		176·5	175	182	132				139·5	—	96·5			—	382	—	64·5	—		46	21					85·5	—	—	—	—	75	75	—	79	—	46	—
28							90		141·5	—	98	36·5	29·5	—	—	—	65·5	92		50	22·5	35·5	36·5	31	31	98	57	36	—	—	70	—	—	—
30	—	181·5	173·5	182·5	136	113·5	85·5		133	—				517	377	—	—			50						—	—	75	—	—	—
31		188·5	187	185·5	131	115·5	87·5				—			—	—	—	—	98	—							—	—	70	—	—	—
39	188	187	186	188	139·5	115	93	133	131·5	115	99	34	29·5	524	384	311	—	98	—	44	22	—	38	—	30	—	—	117	86·5	30	47·5	74	74	71	71	—	50	—	79	—
43		186	184	—	138	—	—				—			—	—	—	—	—	—							—	—	104·5	85·5	28	43	74	—	—
46		193	190·5	196	—		100·5				—			—	—	—	—	—	—							—	—	117	97		39	—	—
75	—	190	188·5	194·5	145·5	122	97·5		135·5	—	89·5			549	405	—	—			44·5	22·5	—		—		93·5	—	—	—	96·5	37·5	47·5	76	76	—	72	—	50	—
78		178	174·5	180·5	135	113	98		121	—	93			510	362	—	65·5			44·5	22·5	—		—		93·5	—	—	—	89	30	37	76	76	—	68	—	50	—
79	—	160	170·5	140	120·5			128·5	—		35	30·5	—	—	—	—									—	—				86	—	76	—
85	—	182·5	180	182·5	147	121	100·5		138	—	100	34	30·5	—	370	—	71·5	88·5	—	50	24	39	39·5	31·5	31·5	93·5	—	—	—	103	33·5	42·5	—	81	—	75	81	48	81	80	—	—

TABLE III.

L	G_1	G_2	W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
—	—	—	—	106·5	—	42	—	82	—	73	—	—	—	—	—	—
2·5	—	38	—	—	—	—	—	66	—	68	82	46	—	—	—	—
6	52	37·5	123	93	37	42·5	—	—	—	75	65	51	72	70	72	—
—	—	39·5	—	—	29·5	44	—	67	—	74	—	—	—	—	—	—
—	—	—	122	107	36·5	49·5	—	74	—	—	—	—	—	—	—	—
—	—	—	105·5	93·5	—	—	—	81	—	74	—	—	96	95	—	—
—	62·5	38·5	—	109	—	44	—	70	—	—	—	55	—	73	62	—
—	—	—	—	—	—	—	—	69	—	—	—	—	—	—	—	—
5	54	42·5	119	101·5	35·5	43	—	71	—	74	75	49	86	82	79	—
—	58·5	40·5	—	—	37	47·5	—	72	—	—	72	50	—	—	69	—
—	—	—	—	—	—	—	—	68	—	—	—	—	—	—	—	—
—	57	46·5	129	107	34	46·5	—	87	—	72	75	45	83	78	82	—
—	—	—	—	—	—	—	—	81	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	80	—	—	63	—	—	—	—	—
—	—	—	—	—	—	—	—	67	—	—	—	—	—	—	—	—
—	—	—	—	93	36	—	76	76	—	—	74	51	79	—	—	—
—	—	—	135	101·5	32	46·5	—	—	—	—	—	—	—	—	—	—
1·5	55·5	44	—	107	32·5	—	79	79	77	78	73	45	—	80	79	82°
—	—	—	—	—	—	—	—	75	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	79	—	71	—	—	81	80	—	—
—	57·5	42	130	119	36·5	49	—	83	—	—	73	47	88	81	73	—
—	—	—	—	107·5	32·5	—	—	81	—	74	—	—	—	—	—	—
—	—	—	—	—	—	—	—	77	—	68	—	—	—	70	—	—
—	—	—	115·5	—	35	51·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	84	—	79	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	83	—	—
—	—	—	129·5	110·5	34	44	—	83	—	—	—	—	78	82	—	85°
—	—	46	—	103·5	32·5	45·5	81	83	—	—	68	52	—	—	—	—
—	—	—	126·5	105·5	35·5	42	—	—	—	—	—	—	77	76	—	84°
—	—	—	117·5	—	34	45	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	30	43·5	—	—	—	—	—	—	—	—	—	—
—	55·5	41	117	102·5	31·5	—	77	77	74	74	73	—	—	75	74	85°
—	—	—	—	—	—	—	—	—	—	—	—	—	—	79	—	—
—	—	—	129	98·5	32·5	43·5	72	72	—	—	—	—	80	79	—	—
·5	54·5	37	120·5	89	—	41	73	74	70	71	78	42	—	—	68	82°
—	—	—	117·5	98·5	—	49·5	—	—	—	—	—	—	87	—	—	—
·5	51·5	44·5	—	—	31	—	—	—	—	—	—	53	85	82	86	—
—	—	—	—	122	35·5	45	83	82	—	—	—	—	78	79	—	82°
—	—	—	—	110	—	41·5	—	83	—	—	—	—	—	81	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	37·5	—	—	—	—	—	79	—	—	66	54	83	86	—	—
—	53	36	—	—	—	—	82	82	—	77	—	48	90	—	68	89°
—	—	—	—	102·5	—	48·5	—	—	—	—	—	—	—	81	—	—

MEASUREMENTS

Number of Skull	T'	L	L''	F	B	B''	B'	H'	H	OH	LB	jml	fmb	U	S	Q	G'H	GB	J	NH	NB	O₁ L	O₁ R	O₂ L	O₂ R	GL	G₁	G₂	W₁	W₂	h₁	f	B/L'	B/L	H'/L'	H/L	G'H/GB	NB/NH	O₂/O₁ L	O₂/O₁ R	G₂/G₁	P
90		185	177	178·5	151·5	128	98	—	135·5	—	98·5	—			535	387											102·5	—	38		106·5		42		82		73						
93		201	197	194	132	114·5	96·5	—	137	—	112	—			535			77	94	130	56	25·5	—											66	—	68	82	46	—			
96		179·5	175·5	176·5	—	123	99·5	—	135·5	—	100·5	32·5	30·5			369		66	101		50	25·5	37·5	38	27	20·5	96	52	37·5	123	93	37	42·5				75	65	51	72	70	72
102		195·5	188	193	—	—	—	—	—	—	—			547														39·5				29·5	44		—							
111	—	200	195·5	197·5	134	115·5	96·5	—	149	—	104·5	—		545			70		—											122	107	36·5	49·5		67	—	74			—	
114		184		181·5	136	122·5	100·5	—	—	—	—			520			—																	74	—							
117		182	180·5	184·5	145	123·5	93	—	134·5	—	107·5	33	29	524	367		—			35·5	36·5	34	34·5				62·5	38·5	105·5	93·5		44		81	—	74			96	95	
118		202·5	199	199	141	125	105·5	—	—	—	—			558	385		69		130	48	20·5	—	39·5	—	29	—		—	109				70	—			55	—	73	62	
119		192·5	188·5	193·5	132·5	—	95	—	—	—	—			540			—			—			—											69	—				—	—	—	
122		190	186	193	148·5	121·5	104·5	—	—	—	—			540			—																	—								
123		199·5	194·5	193·5	137	—	98·5	—	145·5	—	117·5	—		534	382		75·5	100·5	—	51·5	25	38·5	39·5	33	32·5	105	54	42·5	119	101·5	35·5	43		71	—	74		86	82	79
125		197·5	194·5	194·5	139	115·5	101	—	—	—	—			544	382		73	101·5	—	48	24	—	—	—	—		58·5	40·5		37		47·5		72		72		50	—	69	
126		194	192	193	132·5	—	96·5	—	—	—	—			526			—																	68							—	
127		183	182	180	139	130	100	—	132	—	99	37	34	543	376·5		75	103	—	57·5	26	38	38·5	31·5	30	—	57	46·5	129	107	34	46·5		87	—	72	75	45	83	78	82
132		191	—	195·5	154	—	100·5	—	—	—	—			550	390		—																	81	—							
133		191·5	188	180	152·5	121·5	100·5	—	120	—	99	—		550	390		—																	80		63						
134		199	196·5	196	134·5	108·5	97	—	—	—	—			535			—																	67								
135	184	182·5	180·5	180	139	118·5	100·5	—	129	—	—			529	372	347	70	94·5		40	23·5	39	38·5		30·5	—			93	36			76	76	—	—	74	51	79		
137		—	—	—	—	—	—	—	—	—	—																		135	101·5	32	46·5									
138	182	181	178	183	143·5	127	98	141	141	119	100·5	37·5	32·5			330	67	92	140·5		22·5	—	40·5	—	32·5	91·5	55·5	44		107	32·5	—		70	79	77	78	73	45	—	80	79	82
141		191	—	187	166·5	147·5	117·5	99·5	—	—	—			557			—																	75	—				—	—	—	
149		187·5	187	186·5	148	125·5	97·5	—	132·5	—	105	33·5	30	535	378		—			40	—	40	32·5	32										79	—	71		—	—	81	80	
153		191·5	187·5	190	138·5	—	104	—	—	—	—	38	29	557			73	100		50	23·5	39	41·5	34·5	33·5	—	57·5	42	130	119	36·5	49		83	—	73	47	88	81	73	
156		180	180	183·5	146	127	103	—	133·5	—	115·5	—		534			—			—		—	34·5	—	—		—	—	—	107·5	32·5	—		81	—	74		—	—	—	—	
185		191	—	178	148	137	—	—	129	—	—	38·5	31·5	540	355		—						40·5	—	32·5	—								77		68				70	—	
169		—	—	—	—	—	—	—	—	—	—																													—	—	
173		174	171	177	147	121·5	97	—	138	—	100	35	30·5	515	370		—						40·5	—	33·5	—				115·5	35	51·5							—	83	—	
178		—	—	—	—	—	—	—	—	—	—	41	29·5				—						41	—	33·5	—								84		79				—	—	
185	187	184	181	180	152	128	100·5	—	—	122	—			536		331	65·5	96	—	48	25	39	38·5	31	31·5	—	—	46	129·5	110·5	34	44		81	83		68	52	78	82		85°
186		—	—	—	—	—	—	—	—	122	—	40·5	32				—					40	38·5	31	30·5	—	—	—	126·5	103·5	35·5	45·5		83	—				77	76		84°
187		182	180	181·5	—	—	—	—	—	—	—	40·5	32	542			—					40	40	31	30·5	—	—	—	117·5	105·5	34	42		—					—	—		
188		—	—	—	—	—	—	—	—	—	—																			—	30	43·5	45		—							
190	191	191·5	186·5	190	147	128·5	99·5	141	141	120	106·5			545	382	328	71·5	98·5	—	42·5		—	32	94	55·5	41	117	102·5	31·5				77	77	74	74	73	—	75	74	85°
191		—	179·5	182	—	—	—	—	—	—	—	38	33				—			40·5		32	—			40·5	—	32											—	79		
193	189	189	183·5	185·5	137	—	100·5	—	128	—	39	32·5	532		335	—			—	40·5	40·5	32·5	32	—	—	129	98·5	32·5	43·5			72	72		—	80	79		82°
195	189	187	185	187	138·5	116	91·5	132	132	120	95·5	37·5	31·5	527	380·5	321	70·5	90·5	—	51	21·5	—	—	92·5	54·5	37	120·5	89		41		73	74	70	71	78	42	—	68	82°
196		183	177·5	183	—	—	—	—	—	—	100·5	39	33	—			—			41		39	35·5	—		117·5	98·5	—	49·5			87	—			—	—	87			
197		—		—	—	—	—	—	—	100·5	—	34·5	26·5				69			51	27	39	39	33	32	94·5	51·5	44·5		31				85	82	86		53	—		
207	183	185·5	180	184	151·5	126	99	—	126	—	34·5	26·5			335	—				43	—	43	33·5	34	—	—	122	35·5	45	83	82		82	—	—	—	—	78	79	82°
214		185·5	180·5	180·5	153·5	123·5	101·5	—	—	126	—			542			—			—		40	40		32·5	—	—	110		41·5			83	—			—	—	81		
215	—	182	—	185	—	—	92	—	—	—	—						—						—		—								—					—	—		
217		191·5	189·5	185	150	—	97	—	—	—	—			550			66	100·5	—	46	25	39	39·5	32·5	34	—	—	37·5	—				79	—		66	54	83	86	—	—
218	181	181	177	180	149	117	100·5	—	139	125	101	—		522	372	340	69	—		49·5	24	36·5	37	33	30	92	53	36	—	102·5	—	48·5		82	82	—	77	48	90	—	68	89°
222		—	—	—	—	—	—	—	—	—	—						—						37	—		—								81	—				—	81	—	

TABLE IV.

G_1	G_2	W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
—	—	—	—	—	—	—	77		—	—	—	—	—	—	—
53·5	37·5	107·5	92·5	34·5	45	—	72	—	74	—	54	—	86	70	—
—	—	—	—	—	—	—	81	—	74	—	—	—	—	—	—
48	37	—	86	33·5	39	69	69	74	73	79	48	88	87	77	84°
—	—	—	—	—	—	—	74	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
51·5	39	—	—	—	—	85	88	—	—	79	44	—	—	75	85°
—	40·5	—	—	31·5	43	—	73	—	—	—	49	—	76	—	—
—	—	117·5	98	26	44	84	84	—	—	—	—	—	81	—	86°
—	—	123·5	—	27·5	41·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	80	—	77	—	—	—	—	—	—
—	—	—	—	—	—	—	74	—	—	—	—	—	—	—	—
—	—	—	97·5	26·5	43·5	—	81	—	72	—	—	—	—	—	—
—	—	—	—	—	—	—	80	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	76	—	—	—	—	—	—
54	40·5	—	—	—	—	72	72	—	76	—	42	86	—	75	84°
—	—	—	103·5	33	45	—	—	—	—	—	—	—	—	—	—
—	—	112	93·5	32	45·5	75	75	75	74	70	—	87	—	—	75°
—	—	—	100·5	34·5	46	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	76	—	71	—	—	—	—	—	—
—	—	—	—	—	—	—	69	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	78	—	—	—	—	—	—	—	—
49	36	—	92	30	45	—	78	—	79	71	51	88	85	73	—
—	—	—	—	—	—	74	76	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	83	—	—	—	—	—	—	—	—
—	—	123	95	27	—	74	74	—	—	69	51	—	77	—	—
—	—	123	99·5	32	43	75	75	—	—	—	—	82	—	—	—
49	41·5	115	86	36·5	43·5	—	81	—	80	73	52	82	—	85	90°
—	—	123·5	—	29·5	41·5	78	78	—	—	—	—	78	79	—	—
—	—	—	—	43	—	—	84	—	—	—	—	—	86	—	—
—	—	—	—	31·5	—	77	78	—	—	—	—	—	—	—	—
—	—	113	89	29·5	46	—	—	—	—	—	—	79	—	—	—
49·5	41·5	—	—	—	—	—	—	—	79	—	49	—	—	84	—
—	—	—	—	—	—	—	70	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
51	38·5	115·5	100	30	44·5	77	78	—	—	—	—	88	—	75	—
—	—	—	—	—	—	—	79	—	68	—	—	—	—	—	—

MEASUREMENTS

Number of Skull	L'	L	L''	F	B	B'''	B'	H'	H	OH	LB	fml	fmb	U	S	Q	G'H	GB	J	NH	NB	O₁ L	O₁ R	O₂ L	O₂ R	GL	G₁	G₂	W₁	W₂	h₁	f	B|L'	B|L	H'|L'	H|L	G'H|GB	NB|NH	O₂|O₁ L	O₂|O₁ R	G₂|G₁	P
91	—	155	183·5	180	143	—	100	—	—	—	—	—	—	528	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	77	—	—	—	—	—	—	—	—	—
94	—	184	182	181·5	132·5	110	97	137	—	102·5	33	29	513	372	—	68·5	—	—	44·5	24	—	39	—	33·5	98·5	53·5	37·5	107·5	92·5	34·5	45	—	72	—	74	—	54	—	86	70	
98	—	—	—	—	—	111	97	127	—	95	30·5	30	404	345	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	81	—	74	—	—	—	—	—	
99	180	183·5	183	184	134	—	—	134	133	112	96·5	—	—	465	368	288	70	89	—	52·5	25	37·5	38·5	33	33·5	83	48	37	—	86	33·5	39	69	69	74	73	79	48	88	87	77	84
100	—	185	—	192·5	130	—	98	—	—	—	—	—	525	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	74	—	—	—	—	—	—	—	
103	—	169	167	170·5	138	—	98	—	—	—	—	—	480	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	
104	160	163	162·5	170	143·5	110·5	93	—	—	110	—	—	498	350	316	68	86·5	—	49·5	22	—	—	—	—	51·5	39	—	—	—	—	—	85	88	—	—	79	44	—	—	75	85
105	—	187	184	184	130·5	113·5	97	—	—	—	—	—	529	—	—	65·5	—	—	48	23·5	—	40·5	—	31	—	—	40·5	—	—	31·5	43	—	73	—	—	—	49	—	76	—	
110	166	166·5	166	168	140	—	93·5	—	117	—	—	—	—	—	317	—	—	—	—	—	—	40·5	—	33	—	—	—	117·5	98	26	44	84	84	—	—	—	—	—	81	—	86
112	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	123·5	—	27·5	41·5	—	—	—	—	—	—	—	—	—	
113	—	170	171	174·5	136·5	—	97	131·5	91	—	—	—	498	370	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	80	—	77	—	—	—	—	—	
115	—	187·5	184	186·5	135·5	100	—	—	—	—	—	—	365	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	74	—	—	—	—	—	—	—	
121	—	175·5	175	180	141	115	100	126·5	—	92	40	29·5	517	369	—	—	—	—	—	—	—	—	—	—	—	—	97·5	26·5	43·5	—	81	—	72	—	—	—	—	—	
130	—	174·5	172·5	173	140	110·5	97·5	—	—	—	—	—	505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	80	—	—	—	—	—	—	—	
136	—	199	195·5	195·5	139·5	112	101	128	—	93	—	—	498	348	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	70	—	—	—	—	—	
142	187	186	185	185·5	134·5	109·5	99	142	117	108·5	34·5	—	521	375	314	74	—	52	22	36·5	—	31·5	—	54	40·5	—	—	—	—	—	72	72	—	76	—	42	86	—	75	84
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	103·5	33	45	—	—	—	—	—	—	—	—	—	—	
144	181	183	181·5	182	135	112	94	138	136·5	121	114·5	30	28	508	375	314	62	89	—	38·5	39	33·5	—	104	—	112	93·5	32	45·5	75	75	75	74	70	—	87	—	75
148	—	—	—	174·5	133·5	124	87	124	—	—	—	498	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
150	—	178·5	176	181·5	136	109·5	96	126·5	—	95·5	—	505	373	—	—	—	—	—	—	—	—	—	—	—	100·5	34·5	46	—	76	—	71	—	—	—	—	—	
151	—	190·5	189	181·5	132	101·5	95	—	—	—	—	524	380	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	69	—	—	—	—	—	—	—	
159	—	175·5	175	173·5	137	—	—	—	—	—	—	502	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	78	—	—	—	—	—	—	—	
160	—	175	171	175·5	137	113·5	92·5	139	—	97	34	26·5	499	364	—	63	88·5	—	40·5	23·5	36·5	36·5	32	31	86	49	36	—	92	30	45	—	78	—	79	71	51	88	85	73	
162	192	188·5	185	189	143	—	97·5	—	119	—	—	542	—	332	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	74	76	—	—	—	—	—	—	
166	—	179	175·5	181	148	—	95·5	—	—	—	—	523	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	83	—	—	—	—	—	—	—	
171	178	178·5	176	177·5	132	110	99	111	—	—	—	508	—	294	62	90	—	48	24·5	40	41	—	31·5	92	—	—	123	95	27	—	74	74	—	69	51	—	77	—	
177	184	184	180	183	138·5	110·5	95	—	118	—	—	518	—	316	—	—	—	—	—	—	33	—	—	—	—	123	99·5	32	—	75	75	—	—	—	—	—	—	
180	—	175	174·5	172·5	142·5	114	100	140·5	—	100	37·5	28	504	367	—	65·5	90	—	43	22·5	39·5	40·5	32·5	—	98·5	49	41·5	115	86	36·5	43·5	—	81	—	80	73	52	82	—	85	90
192	178	178	176·5	178·5	138·5	116	99	—	123	—	—	512	—	324	—	—	—	—	41	42	32	33	—	—	—	123·5	—	29·5	41·5	78	78	—	78	79	—	—	—	
199	167	166	169	141·5	—	92	—	—	—	—	—	490	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	43	—	84	—	—	—	—	—	—	—	
200	170	175	174·5	170·5	136	120	—	—	121	—	—	500	—	312	—	—	—	—	—	36	—	31	—	—	—	—	31·5	—	77	78	—	—	—	86	—	—	—	
201	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
202	—	173·5	172	175	—	—	—	136·5	—	102·5	39·5	30·5	—	—	—	—	—	—	—	41·5	—	33	—	—	—	113	89	29·5	46	—	—	—	—	—	79	—	—	
204	—	183·5	—	186	128	—	99·5	—	—	—	—	513	—	—	63·5	—	—	50	24·5	—	—	—	—	91	49·5	41·5	—	—	—	—	—	79	—	49	—	84	
206	—	182	—	184·5	149	—	93	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	70	—	—	—	—	—	—	—	
211	—	168	—	173	137·5	—	—	—	—	—	—	483	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	
212	181	179·5	177·5	177·5	139·5	113	100	—	120	—	—	517	356	322	—	99	—	—	38	—	33·5	—	—	51	38·5	115·5	100	30	44·5	77	78	—	—	—	88	—	75	—	
221	—	182	181·5	186	143·5	118·5	93	123	—	95	31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	79	—	68	—	—	—	—	—	

DESCRIPTIVE CATALOGUE.

In the accompanying remarks :

        Calvaria = Dome = Schädeldach.
        Cranium = Braincase.
        Skull = Braincase and face bones.
        ad. = Adult.
        B. B. = Greenwell's *British Barrows.*
        G. C. = Greenwell Collection in Oxford Museum.

The page of the *British Barrows*, together with the line on the page in which the particular skull is referred to are, when possible, cited. The Roman numerals give the number of the special excavation in the same work.

LONG BARROWS (MALE).

No. 3.—Calvaria, *ad.* Mandible without ascending portion of left ramus. Alveolar and palatal portions of maxillae, also left occipital condyle in box. Posterior sagittal depression. B. B. p. 484, ccxxi. Ebberston, N. R. Yorkshire. G. C.

No. 4.—Calvaria, old *ad.* Torus occipitalis. B. B. p. 484, ccxxi. Ebberston, N. R. Yorkshire. G. C. Cf. also MS. Catalogue, Oxford.

No. 9.—Imperfect Calvaria, *ad.* Mandible without ascending portion of right ramus ; the left coronary process broken. Maxillae detached (*query* from same individual ?). Supra-inial depression. B. B. p. 509, ccxxvii. Over Silton, N. R. Yorkshire. G. C. Cf. MS. Catalogue, Oxford.

No. 18.—Skull, *ad.* The right face-bones and the basi-cranial axis wanting, the mandible imperfect. Distorted by pressure. B. B. pp. 497—501, p. 501, l. 1, ccxxiv. Rudstone, E. R. Yorkshire. G. C.

No. 19.—Skull, old *ad.* Imperfect : basis cranii missing : mandible with most of the right ramus missing. B. B. p. 501, l. 6, and pp. 612 *et seq.* (figured and measured). Rudstone, E. R. Yorkshire. G. C.

No. 21.—Skull, old *ad.* Right parietal and squamosal imperfect. Thurnam, *Mem. Anthrop. Soc.* I. p. 132, p. 478. Dinnington, near Rotherham, W. R. Yorkshire.

No. 22.—Skull, old *ad.* Occipital imperfect. Metopic suture. Thurnam, *loc. cit.* Dinnington, near Rotherham, W. R. Yorkshire.

No. 25.—Skull, *adolescent.* Frontal defective (from abscess ?) ; left palate broken. Paired precondylar eminences, short torus palatinus. Thurnam, *loc. cit.* Dinnington, near Rotherham, W. R. Yorkshire.

No. 38.—Skull, *ad.* Ossicles in lambdoid ; vestiges of transverse occipital suture ; left infra-orbital suture in face. *Crania britannica*, No. lxviii. Vol. I. pp. 244, 245, Vol. II. pp. 59 (3). Rodmarton, Gloucestershire. Thurnam Collection, No. 163.

No. 40.—Skull, young *ad.* Very defective ; left temporal and basis cranii absent. Metopic suture. *Crania britannica*, No. lxx. Vol. I. pp. 244, 245, figured, Vol. II. 59 (4). Rodmarton, Gloucestershire. Thurnam Collection, No. 166.

No. 48.—Skull with mandible, *ad.* Sphenoid, palatine region, much of right frontal and parietal absent. Mandible without angle of right ramus. Rolleston, *Journ. Anthrop. Inst.* v. p. 151 (1875). Nether Swell, Gloucestershire.

No. 59.—Skull with mandible, *ad.* Basis cranii wanting : other extensive defects. Slight torus occipitalis ; minute ossicles in lambdoid. B. B. p. 521, ccxxx. Rolleston, *loc. cit.* p. 169. Swell, Gloucestershire.

No. 62.—Skull with mandible, young *ad*. Very fragmentary. Mandible wanting both condyles, and not belonging to skull. B. B. pp. 524—527, l. 37, ccxxxii. Upper Swell, Gloucestershire.

No. 65.—Skull with mandible, *adolescent*. Mandible with greater part of right ramus wanting. Metopic suture ; right infra-orbital suture on face, left *m* 3 just appearing. B. B. p. 529, l. 5. Upper Swell, Gloucestershire.

No. 72.—Imperfect calvaria, *ad*. Part of right side of face and temporal bone attached. B. B. p. 531, l. 11. Upper Swell, Gloucestershire.

No. 77.—Skull with mandible, *ad*. Much broken. G. Y. Ackerman, *Archaeologia*, xxxvii. p. 432. Thurnam, *Archaeologia*, xlii. Crawley, near Witney, Oxfordshire.

No. 80.—Skull with mandible, young *ad*. Mandible broken at symphysis. Ossicles in lambdoid : slight bathrocephaly. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 81.—Calvaria, with separate maxillae and mandible, *ad*. Mandible with right condyle broken. Metopic suture. Trace of torus occipitalis. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 82.—Skull with mandible, *ad*. Defects. Right canine displaced in front of lateral incisor ; mastoids very small. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 83.—Skull with mandible, *ad*. (young ?). Right zygoma broken. Ossicles of pterion ; the right 30 × 10 mm., the left 20 × 7 mm. Two ossicles in left coronal suture, the larger 20 × 20 mm. Ossicles of lambdoid, two on right, three on left side, the largest 20 × 15 mm. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 86.—Skull with mandible, *ad*. Skull much broken. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 87.—Skull, *ad*. Basis cranii and most of face wanting. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

LONG BARROW (IMMATURE, OR SEX DOUBTFUL).

No. 7.—Calvaria (♀ ?). Very imperfect. B. B. p. 484, ccxxi. Ebberston, N. R. Yorkshire. G. C.

No. 23.—Skull (sex ?). Thurnam, *loc. cit.* No. 21. Dinnington near Rotherham, W. R. Yorkshire.

No. 29.—Skull (♀ ?), *ad*. Left temporal and left side of face wanting. Very long mastoid. Thurnam, *loc. cit.* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 51.—Calvaria with mandible. Child 6 years. Metopic suture. Rolleston, *loc. cit. sub* No. 48, p. 156, l. 7, and p. 158, l. 21. Swell, Gloucestershire.

No. 54.—Skull with mandible. Face and basis cranii very imperfect. Rolleston, *loc. cit. sub* No. 48, p. 161. Swell, Gloucestershire.

No. 66.—Calvaria with left temporal (♂ ?), *ad*. Slight sagittal ridge ; torus occipitalis ; ossicles of lambdoid. B. B. p. 530. Upper Swell, Gloucestershire.

LONG BARROW (FEMALE).

No. 13.—Calvaria, *ad*. Imperfect : additional fragments. B. B. pp. 491, 492, l. 8, ccxxiii. Weston, E. R. Yorkshire.

No. 24.—Skull, old *ad*. Ossicle of lambdoid 37 × 25 mm. : faint post-coronal depression : small pterygo-spinous ridge. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 26.—Skull, old *ad.* Right malar wanting. Teeth lost. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 27.—Skull, *ad.* Both zygomata, the right temporal, and part of the occipital wanting. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 28.—Skull, *ad.* Left temporal wanting, occipital and left parietal injured. Posterior sagittal and lambdoid ossicles. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 30.—Cranium, old *ad.* Occipital condyles wanting. Sagittal groove. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 31.—Calvaria, *ad.* Thurnam, *loc. cit. sub* 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 39.—Skull with mandible, *ad.* Right infra-orbital suture on face : perforation in anterior wall of left external auditory meatus. *Crania britannica*, Vol. I. pp. 244, 245, cvii. Rodmarton, Gloucestershire.

No. 43.—Calvaria with temporals, detached maxillae, and mandible, young *ad.* A tongue, 20 × 10 mm., projecting from supra-occipital into right parietal, in the middle of the right lambdoid (a fused Wormian ?). Rolleston MS., "Barrows not as yet described." Birdlip, Gloucestershire.

No. 46.—Skull with mandible, *ad.* Very imperfect. Rolleston, *Journ. Anthrop. Inst.* v. p. 149 (1875). Swell, Gloucestershire.

No. 63.—Skull with mandible, old *ad.* Very fragmentary. B. B. p. 528, l. 13. Upper Swell, Gloucestershire.

No. 75.—Skull with mandible, *ad.* Both skull and mandible injured. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 78.—Skull with mandible, *ad.* Much broken. Mandible without right condyle. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 79.—Cranium, young *ad.* Left temporal, frontal and right parietal broken. Ossicle of left pterion, 20 × 12 mm. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 85.—Skull with mandible, young *ad.* Broken : defect on left side involving parietal occipital and temporal; condyles gone. *m* 3 coming into place. Bilateral perforation in anterior wall of external auditory meatus. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

ROUND BARROWS (MALE).

No. 90.—Cranium with mandible, old *ad.* Basis cranii much broken, left condyle of mandible wanting. Maxillae and fragments in box. Post-parietal flattening with slight sagittal groove : torus occipitalis. B. B. p. 382, clxvii. Ashfell, Kirkby Stephen, Westmoreland. G. C.

No. 93.—Skull, *ad.* Sphenoid, ethmoid, left temporal and left malar wanting. Large healed wound in right frontal ; slight sagittal ridge ; torus occipitalis ; imperfect porus crotaphitico-buccinatorius. B. B. p. 136, l. 41, ii. figured, p. 602. Langton Wold, E. R. Yorkshire. G. C.

No. 96.—Skull with mandible, young *ad.* Left temporal and parietal broken. Signs of injury to left frontal. B. B. p. 141, v. figured on p. 578. Heslerton Wold, E. R. Yorkshire. G. C.

No. 102.—Calvaria, old *ad.* B. B. p. 147, ix. Sherburn Wold, E. R. Yorkshire. G. C.

No. 111.—Skull with mandible, young *ad.* Ethmoid, sphenoid and right malar wanting : fragments in a box. B. B. p. 163, l. 25. Ganton, E. R. Yorkshire. G. C.

No. 114.—Calvaria with temporals, and mandible, *ad.* B. B. p. 166, l. 34, xxii. Ganton, E. R. Yorkshire. G. C.

No. 117.—Cranium with mandible, young *ad.* Malars *in situ* ; sphenoid wanting. Ossicles of lambdoid. B. B. p. 174, l. 32, xxvii. Ganton, E. R. Yorkshire. G. C.

No. 118.—Skull with mandible, *ad.* Basis cranii wanting, the rest much broken. Metopic suture. B. B. p. 176, l. 7, xxviii. Ganton, E. R. Yorkshire. G. C.

No. 119.—Calvaria, with portion of left temporal and malars, mandible, *ad*. B. B. p. 176, l. 26, xxviii. Ganton, E. R. Yorkshire. G. C.

No. 122.—Calvaria with temporals and imperfect mandible, old *ad*. Ossicles of lambdoid partly fused ; faint torus occipitalis, narrow auditory meatus. B. B. p. 183, l. 11 from bottom, xxxiv. Willerby, E. R. Yorkshire. G. C.

No. 123.—Skull with mandible, young *ad*. Left temporal, pterygoid, and most of occipital wanting ; right squamosal wanting. Slight torus occipitalis. B. B. p. 183, l. 5–16, xxxiv. Willerby, E. R. Yorkshire. G. C.

No. 125.—Skull with mandible, *ad*. Basis cranii and right temporal wanting, mandible without left condyle. B. B. p. 191, last line, xli. figured p. 616. Helperthorpe, E. R. Yorkshire. G. C.

No. 126.—Calvaria with portions of temporals, *ad*. B. B. p. 193, xlii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 127.—Skull with mandible, *ad*. Somewhat distorted ; sphenoid and ethmoid wanting. Metopic suture ; infra-orbital suture on face. B. B. p. 193, l. 14, xlii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 132.—Calvaria with left malar and mandible, *ad*. Lower jaw without right coronoid process. Fragments of other bones in box. Two ossicles in lambdoid, 85 × 15 mm. and 20 × 16 mm. B. B. p. 195, l. 18, xliii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 133.—Cranium, *ad*. Left occipital condyle wanting. The rest much broken. Box with loose fragments. Torus occipitalis. B. B. p. 195, l. 33, xliii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 134.—Calvaria with temporals and mandible, *ad*. Ascending portion of right ramus of mandible broken. Slight torus occipitalis. B. B. p. 198, l. 11, xliv. figured p. 619.

No. 135.—Skull with mandible, young *ad*. Basis cranii wanting. B. B. p. 200, l. 19, xlvi. figured ? p. 570. Weaverthorpe, E. R. Yorkshire. G. C.

No. 137.—Calvaria, loose maxillae, malars, etc., and mandible, *ad*. B. B. p. 201, l. 6, xlvi. Weaverthorpe, E. R. Yorkshire. G. C.

No. 138.—Skull with mandible, *ad*. Ethmoid gone. Parietals much broken : mandible without left condyle. B. B. p. 206, l. 11, xlix. Helperthorpe, E. R. Yorkshire. G. C.

No. 141.—Skull, old *ad*. Basis cranii and face much broken. Ossicles in lambdoid. Torus occipitalis. B. B. p. 207, l. 7, xlix. Helperthorpe, E. R. Yorkshire. G. C.

No. 149.—Skull, young *ad*. The right half of the sphenoid, the ethmoid, and the right maxilla wanting. B. B. p. 217, l. 10, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 153.—Skull with mandible, *ad*. Much broken. Ossicles in lambdoid. B. B. p. 226, l. 8, lix. figured p. 586. Cowlam, E. R. Yorkshire. G. C.

No. 155.—Calvaria with mandible and fragments, *ad*. Slight torus occipitalis ; *post mortem* compression. B. B. p. 230, l. 17, lxi. Rudstone, E. R. Yorkshire. G. C.

No. 156.—Skull with mandible, *ad*. Ethmoid, squamosals and occipitals broken ; left malar almost entirely wanting. Metopic suture ; a few ossicles in the lambdoid ; flattening of obelion. B. B. p. 231, l. 1, lxi. Rudstone, E. R. Yorkshire. G. C.

No. 165.—Skull with mandible, *ad*. Left malar and ethmoid wanting. B. B. p. 248, l. 3 from bottom, lxiii. figured p. 590. Rudstone, E. R. Yorkshire. G. C.

No. 169.—Skull with mandible, *ad*. Basis cranii, ethmoid, and right squamosal wanting. Perforation in anterior wall of right external auditory meatus. B. B. p. 265, l. 24, lxviii. Rudstone, E. R. Yorkshire. G. C.

No. 173.—Cranium, with nasals and nasal portion of right maxilla, *ad*. Slight torus occipitalis. B. B. pp. 271, 272, l. 1. Flixton Wold, E. R. Yorkshire. G. C.

No. 178.—Skull with mandible, young *ad*. Basis cranii mostly wanting, right petrosal and squamosal wanting. Ossicle in lambdoid 35 × 17 mm. B. B. p. 274, l. 12, lxx. Flixton Wold, E. R. Yorkshire. G. C.

No. 185.—Skull with mandible, *ad*. (young ?). Sphenoid wanting. Metopic suture. B. B. p. 300, xc. Goodmanham, E. R. Yorkshire. G. C.

No. 186.—Skull with mandible, *ad.* Plagiocephalic ; right frontal flattened, right parietal protuberant ; bilateral post-coronal depression (distortion partly *post mortem*). Goodmanham, E. R. Yorkshire (?). G. C.

No. 187.—Skull with mandible, young *ad.* Sphenoid, ethmoid and much of both malars broken ; mandibular condyles broken. Marked plagiocephaly (*post mortem* ?). B. B. p. 301, l. 9, xcii. Goodmanham, E. R. Yorkshire. G. C.

No. 188.—Calvaria with portions of malars and temporals attached ; maxillae loose, *ad.* Mandible without angle on right side, without condyle on left. B. B. p. 303, l. 3, xciv. Goodmanham, E. R. Yorkshire. G. C.

No. 190.—Skull with mandible, *ad.* Ethmoidal and presphenoidal regions deficient. Slight sagittal ridge ; prominent inion. B. B. p. 308, l. 21, xcix. Goodmanham, E. R. Yorkshire. G. C.

No. 191.—Skull with mandible, *ad.* Left side of head and face largely deficient ; mandible without ascending portion of left ramus ; condyle and coronary process wanting on right side. Inion pronounced ; ossicle of right pterion 30×5 mm., between alisphenoid, parietal and frontal. B. B. p. 312, l. 3, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 193.—Skull with mandible, *ad.* Sphenoid and ethmo-nasal regions deficient. Ossicles in right lambdoid. B. B. p. 313, l. 19, ciii. Goodmanham, E. R. Yorkshire. G. C.

No. 195.—Skull with mandible, *ad.* Presphenoidal region deficient. Ossicle in lambdoid 40 × 25 mm. B. B. p. 315, l. 9 from bottom, cv. Goodmanham, E. R. Yorkshire. G. C.

No. 196.—Skull with mandible, young *ad.* Very defective on right side. B. B. p. 318, l. 11 from bottom, cx. Goodmanham, E. R. Yorkshire. G. C.

No. 197.—Skull with mandible, young *ad.* Right side of cranium largely deficient ; right ascending ramus of mandible wanting. B. B. p. 318, l. 6 from bottom, cx. Goodmanham, E. R. Yorkshire. G. C.

No. 207.—Skull with mandible, old *ad.* B. B. p. 323, l. 9, cxiii. Goodmanham, E. R. Yorkshire. G. C.

No. 214.—Skull with mandible, *ad.* Base of skull very defective. Mandible without the right condyle. Plagiocephalic. B. B. p. 329, cxx. Goodmanham, E. R. Yorkshire. G. C.

No. 215.—Calvaria with part of left temporal and fragments of mandible, *ad.* (young ?). B. B. p. 330, l. 11, cxxi. Goodmanham, E. R. Yorkshire. G. C.

No. 217.—Skull, *ad.* The greater part of the base and all the left temporal wanting. B. B. p. 330, l. 27, cxxi. Goodmanham, E. R. Yorkshire. G. C.

No. 218.—Skull with mandible, *ad.* (young ?). Left occipital condyle and sphenethmoid region injured. Mandible with angle broken away on right side. B. B. p. 332, l. 4, cxxiii. Londesborough, E. R. Yorkshire. G. C.

No. 222.—Skull with mandible, old *ad.* Basis cranii and left temporal region absent. Left condyle of mandible broken. Slight torus occipitalis. B. B. p. 447, l. 14 from bottom, ccxvi. Nether Swell, Gloucestershire.

ROUND BARROW (IMMATURE, OR SEX DOUBTFUL).

No. 124.—Calvaria with broken petrosals (♂ ?). B. B. p. 185, xxxviii. p. 186, l. 6. Willerby Wold, E. R. Yorkshire. G. C.

No. 140.—Imperfect calvaria (♀ ?) with mandible, post-coronal depression. B. B. p. 206, l. 27, lix. Helperthorpe, E. R. Yorkshire. G. C.

No. 157.—Calvaria, very imperfect (♀ ?) with mandible and box containing maxillae, etc. B. B. p. 232, l. 4, lxi. Rudstone, E. R. Yorkshire. G. C.

No. 181.—Skull with mandible (♀ ?). Right malar, sphenethmoids and nasal region gone. Maxillae separate. Somewhat distorted. B. B. p. 276, l. 23, lxxi. Flixton Wold, E. R. Yorkshire. G. C.

No. 213.—Calvaria, defective. Ossicle of bregma, 26 × 14 mm. (See photograph below.) Group of five ossicles in lambda, the largest about 17 × 20 mm. B. B. p. 328, l. 5, cxviii. Goodmanham, E. R. Yorkshire. G. C.
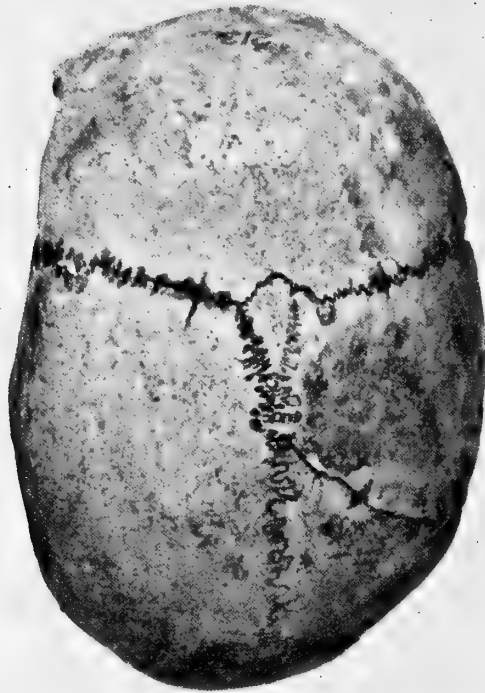
No. 106.—Skull with mandible, about 6 years. Sphenethmoid region wanting, *m* 1 just coming into place. B. B. p. 161, l. 29, xxi. Potter Brompton Wold, E. R. Yorkshire. G. C.

No. 131.—Skull with mandible, about 6 or 7 years. Base, with sphenoid, ethmoid, nasal and left temporal regions wanting. B. B. p. 195 (query line ?). Weaverthorpe, E. R. Yorkshire. G. C.

No. 139.—Calvaria (young). B. B. p. 206, l. 26, xlix. Helperthorpe, E. R. Yorkshire. G. C.

No. 179.—Skull with mandible (about 12 years). Base imperfect; sphenoid, ethmoid and nasals gone. Ossicle in right lambdoid 15 × 15 mm. B. B. p. 275, l. 7. Flixton Wold, E. R. Yorkshire. G. C.

No. 209.—Skull with mandible (about 8 years). Base and right temporal region wanting. B. B. p. 324, l. 6, cxiv. Goodmanham, E. R. Yorkshire. G. C.

No. 216.—Skull with mandible (about 12 years). Much broken, base wanting. Markedly prognathous. B. B. p. 330, l. 19, cxxi. Goodmanham, E. R. Yorkshire. G. C.

ROUND BARROW (FEMALE).

No. 91.—Calvaria, *ad.* Flattening of obelion. Small ossicle in left lambdoid. Wellburn, N. R. Yorkshire. Cf. Oxford MS. Catalogue.

No. 94.—Skull with mandible, old *ad.* B. B. p. 137, l. 26, ii. Langton Wold, E. R. Yorkshire. G. C.

No. 98.—Cranium, young *ad.* Base largely wanting; otherwise defective. Metopic suture persistent; ossicle in right lambdoid. B. B. p. 146 (line ?), vii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 99.—Skull with mandible, *ad.* Occiput protuberant. B. B. p. 146 (line ?), vii. (figured, p. 608). Sherburn Wold, E. R. Yorkshire. G. C.

No. 100.—Calvaria with right temporal, *ad.* B. B. p. 146 (line ?), vii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 103.—Skull with mandible, *adolescent.* Base, ethmoidal and right temporal regions wanting. Ascending portion of right ramus of mandible wanting. Metopic suture persistent. B. B. p. 150, l. 36, xii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 104.—Skull with mandible, young *ad.* Base defective. Ossicle of lambda 25 × 30 mm.; another ossicle in left lambdoid 20 × 15 mm.; small ossicle of left pterion. B. B. p. 152, l. 23, xiii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 105.—Skull with mandible, young *ad.* Basis cranii imperfect, left malar wanting. Part of occipital, with one condyle, loose in box. Triple ossicle of lambdoid. B. B. p. 154, l. 1, xiii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 110.—Skull without basi-occipital, young *ad.* B. B. p. 163, l. 17, xxi. Ganton, E. R. Yorkshire. G. C.

No. 112.—Skull with mandible, young *ad.* Right side of face and right squamosal and part of frontal wanting. B. B. p. 163, l. 34, xxi. Ganton, E. R. Yorkshire. G. C.

No. 113.—Calvaria, *ad.* Part of base and left temporal attached. Metopic suture persistent. B. B. pp. 166, 167, xxii. Ganton, E. R. Yorkshire. G. C.

No. 115.—Cranium, *ad.* Much broken. Box containing loose parts of face and broken mandible. B. B. p. 166, l. 38, xxii. Ganton, E. R. Yorkshire. G. C.

No. 121.—Cranium with mandible, *ad.* Mandible much broken. Imperfect facial bones in box. Metopic suture persistent. B. B. p. 183, l. 7, xxxiii. Willerby Wold, E. R. Yorkshire. G. C.

No. 130.—Cranium with malar bones and mandible, old *ad.* B. B. p. 195, l. 3, xliii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 136.—Imperfect cranium with broken mandible, young *ad.* Bathrocephalic; ossicles in lambdoid. B. B. p. 200, l. 39, xlvi. Weaverthorpe, E. R. Yorkshire. G. C.

No. 142.—Skull with mandible, *ad.* Base and ethmoidal regions imperfect; mandible without condyles; box with fragments. Metopic suture persistent; numerous ossicles in left lambdoid. B. B. p. 210, l. 4, li. Cowlam, E. R. Yorkshire. G. C.

No. 143.—Skull with mandible, *ad.* Base, sphenoid, ethmoid, left malar, squamosal and parts of frontal and parietal gone. Marked alveolar prognathism. B. B. p. 211, l. 8, lii. Cowlam, E. R. Yorkshire. G. C.

No. 144.—Skull with mandible, *ad.* Sphenoid and ethmoid regions wanting. B. B. p. 211, l. 21, liii. Cowlam, E. R. Yorkshire. G. C.

No. 148.—Cranium with mandible, young *ad.* Most of base gone. Right condyle and coronary process of mandible broken. Box containing maxillae. Ossicles in lambdoid; synostosis of atlas and occipital. B. B. p. 215, l. 37, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 150.—Cranium with mandible, *ad.* Mandible without ascending portion of right ramus. Occipital slightly protuberant. B. B. p. 218, l. 27, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 151.—Cranium with mandible and box of fragments, *ad.* Mandible without right condyle. B. B. p. 219, l. 3, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 159.—Cranium with mandible, *ad.* (old ?). Very defective; mandible without most of left ramus. Box of fragments. Perforation in anterior end of right external auditory meatus. B. B. p. 236, l. 22, lxii. Rudstone, E. R. Yorkshire. G. C.

No. 160.—Skull with mandible, young *ad.* Ossicle of lambdoid 30 × 20 mm. Left imperfect porus crotaphitico-buccinatorius. B. B. p. 237, l. 19, lxii. Rudstone, E. R. Yorkshire. G. C.

No. 162.—Skull with mandible, *ad.* Base, sphenoid and right maxilla wanting. Box con-

taining fragments. Slight post-parietal groove ; ossicles in lambdoid ; slight bathrocephaly. B. B. p. 248, l. 12, lxiii. Rudstone, E. R. Yorkshire. G. C.

No. 166.—Calvaria, petrosals attached, with mandible, *ad.* B. B. p. 250, l. 19, lxiii. Rudstone, E. R. Yorkshire. G. C.

No. 171.—Skull with mandible, *ad.* Base entirely gone, together with sphenoid and ethmoid. Faint torus occipitalis. B. B. p. 556, l. 7, ccxxxiv. Rudstone, E. R. Yorkshire. G. C.

No. 177.—Skull with mandible, *ad.* Whole basis and right malar wanting. B. B. p. 273, l. 14, lxx. Flixton Wold, E. R. Yorkshire. G. C.

No. 180.—Skull with mandible, young *ad.* Ossicle of left pterion 15 × 8 mm. between alisphenoid and frontal. Ossicle of left asterion 10 × 12 mm. Tongue 15 × 8 mm. from occipital into left parietal. B. B. p. 275, l. 33, lxxi. Flixton Wold, E. R. Yorkshire. G. C.

No. 192.—Skull with mandible, *ad.* Basi-occipital and sphenoidal regions wanting. Large ossicles in left lambdoid nearly fused. B. B. p. 313, l. 3, ciii. Goodmanham, E. R. Yorkshire. G. C.

No. 199.—Skull with mandible, young *ad.* Basi-cranial axis wanting ; mandible without left condyle. Ossicle of lambdoid 37 × 33 mm. B. B. p. 319, l. 17, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 200.—Skull with mandible, *ad.* (young ?). Basi-cranial axis and left malar wanting. Left angle of mandible wanting. Traces of torus occipitalis. B. B. p. 319, l. 36, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 201.—Calvaria with temporals and malars attached, and with mandible, *ad.* B. B. p. 230, l. 12, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 202.—Skull with mandible, *ad.* Right malar, temporal and much of frontal and parietal wanting. Mandible without condyle or angle on right side. Slight posterior sagittal groove. B. B. p. 321, l. 14, cxii. Goodmanham, E. R. Yorkshire. G. C.

No. 204.—Imperfect cranium with mandible, *ad.* Metopic suture persistent. Ossicle 23 × 15 about 20 mm. behind bregma. Faint post-coronal depression. B. B. p. 322, l. 5, cxiii. Goodmanham, E. R. Yorkshire. G. C.

No. 206.—Calvaria with left temporal, *ad.* Slight flattening of obelion. B. B. p. 323, l. 4, cxiii. Goodmanham, E. R. Yorkshire. G. C.

No. 211.—Cranium with loose maxillae and mandible, *ad.* (young ?). Basis cranii wanting. Mandible broken at symphysis and otherwise imperfect. Faint torus occipitalis ; double ossicle of lambdoid about 40 × 20 mm. B. B. p. 326, l. 33, cxvii. Goodmanham, E. R. Yorkshire. G. C.

No. 212.—Skull with mandible, young *ad.* Basis and sphenoid imperfect. Metopic suture persistent. Ossicles in lambdoid. B. B. p. 327, l. 15, cxvii. Goodmanham, E. R. Yorkshire. G. C.

No. 221.—Skull, *ad.* (old ?). Sphenoid, ethmoid and left malar defective. Post-coronal depression. Ossicles of lambda and in lambdoid suture. Slight bathrocephaly. Gatcombe Park, Minchinhampton, Gloucestershire.

THE EGG OF CUCULUS CANORUS.

AN ATTEMPT TO ASCERTAIN FROM THE DIMENSIONS OF THE CUCKOO'S EGG IF THE SPECIES IS TENDING TO BREAK UP INTO SUB-SPECIES, EACH EXHIBITING A PREFERENCE FOR SOME ONE FOSTER-PARENT.

SECOND MEMOIR.

By OSWALD H. LATTER.

(1) *On the Existence of Cuckoo Gentes.*

In my previous paper (*Biometrika*, Vol. I. No. 2, January 1902, pp. 164—176) I examined this and certain other questions relating to the egg of the Cuckoo. On that occasion I had measurements of only 243 eggs. I have now been able to obtain the dimensions of 1572 specimens (including those dealt with previously) and am therefore in a position to speak with more confidence. Of this number, 717 eggs were measured by myself; and I must here take the opportunity of recording my grateful thanks to Prof. A. Newton of Cambridge, Dr E. Hartert of the Tring Museum, Mr F. Norgate of Bury S. Edmunds, Mr J. H. Gurney of Keswick Hall, Norwich, and many other private collectors for their courtesy in granting me help and access to their collections. I am further indebted to a large number of my pupils at Charterhouse for assistance in tabulating the measurements and in checking the accuracy of the calculations. The remaining 855 specimens were measured by Eugene Rey of Leipzig, and the dimensions published in *Zoologische Vorträge herausgegeben von W. Marshall*, Leipzig, 11th Heft, 1892; *Ornithologische Monatschrift d. Deutschen Vereins z. Schutze der Vogelwelt*, XIX. Jahrgang, 1894, No. 5; and *Journal für Ornithologie*, XLIII. Jan. 1895.

In the present instance I have confined myself to the attempt to ascertain if the eggs of Cuckoos deposited in the nests of any one species stand out as a set apart from Cuckoo's eggs deposited elsewhere. The series includes eggs taken from the nests of no less than eighty-four different species of foster-parents, in addition to a number whose history was not ascertainable. From the nests of six- teen species of foster-parents sufficient Cuckoo's eggs were obtained to make it

worth while to test the point in question in respect of the dimensions of length and breadth. The method employed is fully explained in my previous paper, but it will be convenient to re-state it. The mean (M) length or breadth, as the case may be, of all the Cuckoo's eggs is first computed, thence the standard deviation (σ) is obtained by the formula $\sigma^2 = \dfrac{\text{sum} \, (M - x)^2}{n}$, where $x =$ the measurement of any one egg and $n =$ the number of eggs measured: the coefficient of variation, $\dfrac{100 \, \sigma}{M}$, can then be found. To test whether the deviation occurring in Cuckoo's eggs deposited in the nests of any particular species of foster-parent is significant, M_r is taken as the mean of the whole race of Cuckoos, and M_s as the mean of Cuckoo's eggs found in the nests of the foster-parent species: the standard deviation (σ_s) of such eggs is also ascertained. The value of $M_r - M_s$ is then compared with that of $0 \cdot 67449 \sqrt{\dfrac{\sigma_r^2}{n_1} + \dfrac{\sigma_s^2}{n_2}}$ (where $n_1 =$ total number of Cuckoo's eggs, and $n_2 =$ the number of Cuckoo's eggs in the nests of the species in question), which is the probable error of $M_r - M_s$ due to random sampling. If the value of $M_r - M_s$ be not at least 2 to 3 times as great as the value of the other expression, then the difference of M_r and M_s is not definitely significant. It will be seen by reference to the tabulated summary below that in the case of four of the sixteen sets examined, viz. Robin-Cuckoos, Wren-Cuckoos, Whitethroat-Cuckoos, and Hedge-Sparrow-Cuckoos, the differences are significant in respect of both length and breadth. Of the remainder, five, viz. Garden-Warbler-Cuckoos, Barred-Warbler-Cuckoos, Marsh-Warbler-Cuckoos, Reed-Warbler-Cuckoos, and Tree-Pipit-Cuckoos, exhibit differences significant in one dimension, but doubtful in the other; two, viz. Sedge-Warbler-Cuckoos and White-Wagtail-Cuckoos, exhibit significant differences in one dimension only; one, viz. Meadow-Pipit-Cuckoos, exhibit doubtfully significant differences in but one dimension; and four, viz. Redstart-Cuckoos, Red-backed-Shrike-Cuckoos, Pied-Wagtail-Cuckoos, and Yellow-Ammer-Cuckoos, do not present differences significant in either dimension. The balance of evidence is thus decidedly in favour of there being distinct sets of Cuckoos. As a result it now seems fairly certain that Prof. A. Newton's suggestion (*Dictionary of Birds*, p. 123) is correct, and that there are certain "gentes" of Cuckoos whose members being closely related lay eggs of somewhat similar dimensions, and in the main confine their attentions, generation after generation, each to its own particular variety of foster-parent. In other words, the evidence here adduced points to the conclusion that the species *Cuculus canorus* is tending to break up into a number of sub-species, each with its own particular habits in respect of selecting a foster-parent. There are recorded numerous instances of individual Cuckoos adhering to one species of foster-parent whenever possible, but it appears that in the event of failure to find a nest of the species preferred the Cuckoo places her egg in any nest that comes handy. This fact probably accounts for the very large number of species in whose nests Cuckoo's eggs have been found, and emphasises the importance of obtaining large numbers of measurements for investigations of this character.

TABLE I.

Summary of Results.

Group	Number of Eggs	Length in Millimetres				Breadth in Millimetres				Remarks
		Mean (M)	Standard Deviation (σ)	Coefficient of Variation	Signi-ficance Test*	Mean (M)	Standard Deviation (σ)	Coefficient of Variation	Signi-ficance Test*	
Cuckoo (Race)	1572	22·3	·9642	4·32	—	16·5	·5952	3·60	—	—
Robin-Cuckoo	78	22·8	1·1638	5·105	5·5	16·8	·7127	4·242	6	L. and B. significant
Garden-Warbler-Cuckoo ...	91	21·9	·7860	3·587	7·01	16·4	·4900	2·980	2·8	L. significant and B. probably so
Meadow-Pipit-Cuckoo ...	86	22·3	·8851	3·969	—	16·6	·5238	3·149	2·57	B. probably significant
Wren-Cuckoo	54	21·4	·9375	4·380	10	16·2	·6578	4·060	5·0	L. and B. significant
Redstart-Cuckoo ...	64	22·4	·8840	3·857	1·3	16·6	·5225	3·16	1·6	Neither significant
Sedge-Warbler-Cuckoo ...	48	22·1	·8642	3·91	11·1	16·4	·6121	3·730	—	L. only significant
Barred-Warbler-Cuckoo ...	35	21·8	·8617	3·952	5·0	16·3	·6090	3·736	2·85	L. significant and B. probably so
Whitethroat-Cuckoo ...	37	21·8	·8545	3·919	5·2	16·2	·6071	3·747	4·4	L. and B. significant
Marsh-Warbler-Cuckoo ...	37	22·1	·8330	3·769	2·1	16·3	·4647	2·850	4·0	L. perhaps : B. significant
Red-backed-Shrike-Cuckoo ...	307	22·3	·7606	3·58	—	16·5	·4909	2·975	—	Neither significant
White-Wagtail-Cuckoo ...	115	22·4	·7976	3·398	1·6	16·7	·6766	4·051	4·6	B. probably significant
Pied-Wagtail-Cuckoo ...	85	22·3	·9192	4·122	—	16·5	·5242	3·176	—	Neither significant
Reed-Warbler-Cuckoo ...	66	22·6	·9514	4·51	3·2	16·6	·5768	3·470	2·04	L. significant and B. perhaps so
Tree-Pipit-Cuckoo ...	48	22·6	·9514	4·209	3·0	16·6	·4394	2·647	2·3	L. significant and B. perhaps so
Hedge-Sparrow-Cuckoo ...	58	22·6	·8759	3·875	3·75	16·8	·4970	2·958	6·6	L. and B. significant
Yellow-Ammer-Cuckoo ...	31	22·2	·9846	4·43	0·8	16·5	·4981	3·0	—	Neither significant

4 species differ significantly in both dimensions.
5   ,,   ,,   ,,   one and perhaps in both.
2   ,,   ,,   ,,   one only.
1   ,,   ,,   ,,   perhaps one only.
4   ,,   ,,   ,,   neither dimension.

* Ratio of difference of racial and group means to the probable error of this difference.

We note that judged by the coefficient of variation the Cuckoo's egg is more variable than the eggs for the sub-groups for

Length in 12 out of 16 cases,

Breadth in 10 out of 16 cases.

It is less variable in both breadth and length in the cases of Robin-Cuckoo and Wren-Cuckoo, less variable in length only in Yellow-Ammer-Cuckoo and Reed-Warbler-Cuckoo, and less variable in breadth only in Sedge-Warbler-Cuckoo, Barred-Warbler-Cuckoo, Whitethroat-Cuckoo, and White-Wagtail-Cuckoo. We therefore see that as a rule the egg of the *gens* is less variable than that of the race.

(2) *On the Relation of the Gens Egg to the Foster-Mother's Egg.*

Taking these 16 cases and correlating the dimensions of the Cuckoo's egg with those of the foster-mother, we find:

Correlation of length of Cuckoo's and foster-mother's egg* = ·10,

Correlation of breadth of Cuckoo's and foster-mother's egg = ·24.

These are not very large values, and the number of cases dealt with is only 16, but it does appear that there is some relationship between the size

TABLE II.

Bulk-Modulus of Cuckoo's and Foster-Mother's Eggs.

Species	Size of Foster-Mother's Egg	Size of Frequenting Cuckoo's Egg
Robin	4791	6435
Garden-Warbler ...	5197	5890
Meadow-Pipit ...	4142	6145
Wren	2855	5616
Redstart ...	3724	6173
Sedge-Warbler ...	3234	5944
Barred-Warbler ...	5478	5792
Whitethroat ...	3831	5721
Marsh-Warbler ...	3724	5872
Red-backed-Shrike	6359	6071
White-Wagtail ...	4877	6247
Pied-Wagtail· ...	4690	6071
Reed-Warbler ...	2871	6228
Tree-Pipit ...	4560	6228
Hedge-Sparrow ...	4322	6379
Yellow-Ammer ...	5504	6044
Mean	4385 mm.^{3}	6053·5 mm.^{3}
Standard Deviation	959·13 mm.^{3}	225·72 mm.^{3}

* Mean Length of Cuckoo's Egg, 22·225 ; of Foster-Parent's Egg, 19·806 ; the mean breadths are respectively 16·500 and 14·763 ; the corresponding standard deviations are: Length ·3544 and 1·6414 ; Breadth ·1837 and 1·0971.

of the foster-parents' eggs and those which the Cuckoo lays in their nest. It is worth while considering this relationship from another standpoint. The bulk of an egg must be approximately proportional to its length × (breadth)², and we may take this roughly as a bulk-modulus. Using it we obtain the foregoing Table II. for the bulk-moduli in cubic millimetres. This leads to a correlation of ·1746, or ·17, say, and confirms the previous results. There thus appears to be really a small relation between the bulk of the Cuckoo's egg and that of the foster-parent. The results are based on only 16 classes, which is, of course, small for determining a correlation, but it would seem that we must, since all three measurements give a positive correlation, assume some slight relationship between size of Cuckoo's egg and that of the foster-parent. This relation might be clearer and less irregular, if we had measurements of fairly long series of the eggs of the foster-parents. It is perhaps worth noting that the breadth measurement is the one in which the resemblance is closer. We may therefore ask: Is it possible that the differentiation in size of egg, which we find as we pass from the foster-parent of one to the foster-parent of a second species, is merely due to a slight constriction of the oviduct, instinctive or otherwise, following on the choice of a foster-parent? Were this the case, it would be unnecessary to suppose separate Cuckoo *gentes*. Now the relation between length and breadth of egg has been worked out for Thrush, Blackbird, House-Sparrow and Linnet and the value of the correlation lies between ·35 and ·40. We should expect therefore that if the differentiation of the Cuckoo's egg were produced solely by such a constriction of the oviduct, that the length of the egg would be increased, or the above correlation between length and breadth be at any rate reduced. We find, however, that if the length and breadth of the 16 Cuckoo groups be correlated, the resulting value is ·91. There can hardly be a doubt therefore that the variation in the Cuckoo's egg as we pass from one species of foster-parent to a second is a real variation in bulk, the length and breadth changing in a closely correlated manner. Thus although the egg is only slightly correlated with the size of the foster-parent's egg it varies as a whole in a manner not consonant with a mere constriction of the oviduct, and the fact that the breadths of foster-mother's and Cuckoo's eggs are more closely correlated than their lengths, does not enable us to dispense with a theory of *gentes*.

Table IV. shows the dimensions of the eggs of the foster-parents for comparison with those of the Cuckoo frequenting their nests. The approximate sizes of the birds are also inserted.

(3) *On the Distribution of Frequency in the Cuckoo's Egg.*

A consideration of the first two columns of totals under the heading " Race " in Table III., shows that Rey has measured the great bulk of his eggs to only ·5 mm. accuracy; he has probably not used a vernier and the result is that his frequencies are crowded up on the whole and half millimetres; a similar crowding although less conspicuous is obvious in the additional 717 measurements. This unconscious

TABLE III. *Measurements of Cuckoo's Eggs.*
Length of Egg.

Length in mm.	RACE			GENTES															
	Rey	Latter	Total	Robin	Garden-Warbler	Meadow-Pipit	Wren	Redstart	Sedge-Warbler	Barred-Warbler	White-throat	Marsh-Warbler	Red-backed-Shrike	White-Wagtail	Pied-Wagtail	Reed-Warbler	Tree-Pipit	Hedge-Sparrow	Yellow-Ammer
19·1		1	1								1								
19·6		1	1			1													
19·7	1	1	2																
19·8		1	1				1												
19·9		2	2																
20·0	13	3	16	1	2		6						1	1	1	1			1
20·1	2	4	6			1			2				1		1				
20·2	4	4	8					1		1		1				1			
20·3	1	1	2				1				1								
20·4	1	1	2				1												
20·5	15	2	17		3		1			4	1		4	1	1				
20·6	6	2	8	1		1	1						2	1					
20·7	8	2	10		1	1	1			1	1		3		1				
20·8	7	6	13	1	2		3					1	1	1					
20·9	4	15	19			2	3		1				1		2	1			
21·0	56	18	74	5	7	1	7	4	3	3	5	1	13	4	2	3	2	2	1
21·1	6	13	19	2	2	1					1		1	2			2		3
21·2	14	17	31		4		1	1	2			1	4	3	2	3	2		1
21·3	12	13	25	1	1	1	4				1		4	2	1	1			1
21·4	9	9	18		3	1		2			1	2	1		2				
21·5	46	8	54	1	8	2	3		4	2	1		9	4	4	4	2	2	1
21·6	11	8	19		2	2	1	1	3	2	1	1	10	1	2	2		3	1
21·7	20	16	36	1	1	4		2	2		2	1	10	1	5	1	2		
21·8	17	24	41	1	2	2		1	2	2	1	4		3	6	2	1		
21·9	17	39	56		1	8		1	2		2	4	9			1			2
22·0	131	58	189	5	11	10	6	16	6	7	3	4	48	18	5	5	6	9	2

22·1	22·2	22·3	22·4	22·5	22·6	22·7	22·8	22·9	23·0	23·1	23·2	23·3	23·4	23·5	23·6	23·7	23·8	23·9	24·0	24·1	24·2	24·3	24·4	24·5	24·6	24·7	24·8	24·9	25·0	25·1	25·4	25·5	26·5	Totals
—	1	1	1	2	1	2	—	1	2	—	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	31
5	—	2	—	7	1	3	4	—	5	3	1	1	—	1	—	1	2	1	2	—	—	—	—	—	—	—	1	—	2	—	—	—	—	58
1	1	—	2	3	—	2	—	2	1	1	2	4	1	1	1	—	3	—	4	—	2	—	—	—	—	—	—	—	—	—	—	—	—	48
2	3	2	—	4	—	1	3	1	6	2	1	—	—	6	2	1	—	—	4	—	—	—	—	2	—	—	—	—	—	—	—	—	—	66
1	3	4	1	4	2	2	9	2	4	4	3	1	1	1	2	—	1	1	3	—	—	—	—	—	—	—	—	1	—	—	—	—	—	85
2	5	4	5	9	7	8	1	4	14	3	1	3	—	1	—	2	—	—	4	—	—	—	—	1	—	—	—	—	—	—	—	—	—	115
10	13	18	8	18	7	16	12	6	31	5	6	7	1	7	3	5	1	1	6	1	—	2	—	1	—	—	—	—	—	—	—	—	—	307
1	1	3	—	1	—	2	1	2	2	2	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	37
1	1	3	2	1	—	2	—	2	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	37
—	2	1	—	6	—	—	—	—	2	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	35
2	3	1	2	2	—	—	2	1	2	—	1	1	1	1	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	48
1	2	2	—	4	—	2	1	—	6	1	3	1	—	4	—	2	—	—	3	—	—	—	—	1	—	—	—	—	—	—	—	—	—	64
3	—	1	2	3	—	—	2	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
2	5	6	2	5	5	1	4	2	2	—	—	4	1	1	—	1	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	86
2	3	7	2	12	2	2	—	4	3	1	2	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	91
2	1	1	4	3	3	2	2	3	11	4	—	1	—	2	2	1	1	3	7	—	—	1	—	—	1	—	—	2	—	—	—	1	1	78
47	59	67	37	92	38	54	59	43	113	38	33	38	6	30	11	15	15	17	44	2	8	4	3	8	1	5	3	1	7	1	—	1	1	1572
29	29	33	21	29	19	19	33	29	39	24	23	28	5	8	7	5	14	14	17	1	7	1	3	1	1	2	3	1	1	—	1	—	—	717
18	30	34	16	63	19	35	26	14	74	14	10	10	1	22	4	10	1	3	27	1	1	3	—	7	—	3	—	—	6	1	—	1	—	855

TABLE III.—(*continued*).

Breadth of Egg.

Breadth in mm.	RACE			GENTES															
	Rey	Latter	Total	Robin	Garden-Warbler	Meadow-Pipit	Wren	Redstart	Sedge-Warbler	Barred-Warbler	Whitethroat	Marsh-Warbler	Red-backed-Shrike	White-Wagtail	Pied-Wagtail	Reed-Warbler	Tree-Pipit	Hedge-Sparrow	Yellow-Ammer
14·0	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—
14·5	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·7	1	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
14·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·9	—	2	2	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—
15·0	9	2	11	—	—	—	2	2	—	2	—	—	—	2	1	—	—	—	—
15·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15·2	3	3	6	1	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—
15·3	5	6	11	—	—	—	2	—	—	—	1	—	1	—	3	—	—	—	—
15·4	2	7	9	—	1	—	—	—	2	—	—	—	—	—	1	—	1	—	—
15·5	22	6	28	1	4	—	5	—	1	2	1	1	2	—	1	1	—	—	1
15·6	6	11	17	1	—	1	—	—	2	1	1	1	—	3	—	—	—	—	—
15·7	27	10	37	—	3	3	4	2	3	2	2	1	7	1	1	3	1	1	—
15·8	16	25	41	1	1	2	1	1	2	1	3	3	5	—	2	—	1	1	2
15·9	11	34	45	2	1	2	3	—	2	2	—	2	6	2	4	1	—	—	3
16·0	136	68	204	10	24	2	10	10	4	4	8	3	45	13	8	10	5	3	3
16·1	15	36	51	1	7	4	1	2	3	1	—	2	5	1	3	5	—	2	2
16·2	38	64	102	3	6	8	4	4	—	2	2	4	19	4	10	3	3	3	2
16·3	29	33	62	3	—	4	—	1	2	1	2	3	18	2	5	1	4	2	2
16·4	22	31	53	3	3	6	2	1	2	—	3	1	12	1	2	1	1	2	1
16·5	124	34	158	7	8	4	3	14	7	6	4	2	44	16	6	7	6	4	2
16·6	23	22	45	2	4	2	1	1	1	—	2	2	10	3	3	2	2	2	—
16·7	45	32	77	2	5	3	1	2	1	2	—	3	20	5	5	2	4	8	2
16·8	54	38	92	3	6	10	3	2	—	1	—	2	35	1	3	—	3	3	1
16·9	18	51	69	4	—	6	—	1	1	1	3	2	6	4	8	3	1	3	—
17·0	128	70	198	7	12	13	4	17	7	4	3	2	34	24	11	11	8	7	7
17·1	17	38	55	4	2	4	2	1	3	1	—	2	11	2	3	3	2	5	—
17·2	17	25	42	2	1	3	1	1	1	1	—	1	8	1	3	3	—	2	2
17·3	15	21	36	4	1	4	1	—	1	—	—	—	4	1	3	3	2	2	1
17·4	6	7	13	3	—	—	1	—	1	—	—	—	1	2	—	—	—	—	—
17·5	29	9	38	5	2	1	1	1	—	1	1	—	8	6	1	5	1	4	—
17·6	6	3	9	—	—	1	—	—	—	—	—	—	—	3	1	—	—	1	—
17·7	12	7	19	2	—	—	1	1	2	—	—	—	1	6	1	1	—	2	—
17·8	5	7	12	2	—	1	—	—	—	—	—	—	—	2	—	—	1	1	—
17·9	4	4	8	—	—	—	—	—	—	—	—	—	—	4	—	1	2	—	—
18·0	9	4	13	3	—	1	—	—	—	—	—	—	—	2	1	—	—	—	—
18·1	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18·2	1	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18·8	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19·2	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	855	717	1572	78	91	86	54	64	48	35	37	37	307	115	85	66	48	58	31

Of the remaining 332 Cuckoo's eggs the measurements of which are given in the totals : 46 were not classed, 11 were in Reed-Bunting's nests, 17 in Wood-Warbler's, 17 in Linnet's, 14 in Chaffinch's, 4 in Bullfinch's, 14 in various Lark's, 14 in Yellow-Wagtail's, 15 in Blueheaded-Yellow-Wagtail's, 10 in Spotted-Flycatcher's, 15 in Willow-Wren's, 5 in Nightingale's, 15 in Greenfinch's, 20 in Blackcap's, 7 in Whinchat's, 14 in Lesser-Whitethroat's, 6 in Wheatear's nests, and the remainder in numbers of 5 and under in the nests of various other species.

TABLE IV.

	Cuckoo's Eggs in mm.		Foster-Parent's Egg in mm.		Approximate Length of Bird in inches
	Length	Breadth	Length	Breadth	
Robin	22·8	16·8	20·2	15·4	5¾—6
Garden-Warbler ...	21·9	16·4	*20·3	16·0	5⅓
Meadow-Pipit ...	22·3	16·6	19·7	14·5	5¾—6
Wren	21·4	16·2	*17·7	12·7	3¾
Redstart	22·4	16·6	*19·0	14·0	5⅓
Sedge-Warbler ...	22·1	16·4	*16·5	14·0	4¾
Barred-Warbler ...	21·8	16·3	*22·8	15·5	6¼
Whitethroat ...	21·8	16·2	*19·0	14·2	5½
Marsh-Warbler ...	22·1	16·3	*19·0	14·0	5⅓
Red-backed-Shrike..	22·3	16·5	*22·8	16·7	7½
White-Wagtail ...	22·4	16·7	*20·3	15·5	7¼
Pied-Wagtail ...	22·3	16·5	*20·3	15·2	7½
Reed-Warbler ...	22·6	16·6	*17·8	12·7	5½
Tree-Pipit ...	22·6	16·6	20·0	15·1	6 —6½
Hedge-Sparrow ...	22·6	16·8	20·0	14·7	5½—6
Yellow-Ammer ...	22·2	16·5	21·5	16·0	6½—7
Cuckoo	22·3	16·5	—	—	14

TABLE V.
Frequency Distributions of Cuckoo's Eggs.

A.   With ·5 mm. unit of grouping.

Length	Number		Breadth	Number	
	Rey + O. H. L.	O. H. L. only		Rey + O. H. L.	O. H. L. only
18·75—19·25	1	1	13·75—14·25	1	1
19·25—19·75	3	2	14·25—14·75	2	1
19·75—20·25	33	14	14·75—15·25	19	7
20·25—20·75	39	8	15·25—15·75	102	40
20·75—21·25	156	69	15·75—16·25	443	227
21·25—21·75	152	54	16·25—16·75	395	152
21·75—22·25	392	179	16·75—17·25	456	222
22·25—22·75	288	121	17·25—17·75	115	47
22·75—23·25	286	148	17·75—18·25	37	18
23·25—23·75	100	53	18·25—18·75	0	0
23·75—24·25	86	53	18·75—19·25	2	2
24·25—24·75	21	8			
24·75—25·25	12	5			
25·25—25·75	2	1	Totals	1572	717
25·75—26·25	0	0			
26·25—26·75	1	1			
Totals	1572	717			

* Computed from measurements given in inches by W. J. Gordon in *Our Country's Birds.*

B. With 1 mm. unit of grouping.

Length	Number			Breadth	Number		
	Rey + O. H. L.	Rey only	O. H. L. only		Rey + O. H. L.	Rey only	O. H. L. only
18·75—19·75	4	1	3	13·75—14·75	3	1	2
19·75—20·75	72	50	22	14·75—15·75	121	74	47
20·75—21·75	308	185	123	15·75—16·75	838	459	379
21·75—22·75	680	380	300	16·75—17·75	571	302	269
22·75—23·75	386	185	201	17·75—18·75	37	19	18
23·75—24·75	107	46	61	18·75—19·75	2	0	2
24·75—25·75	14	8	6				
25·75—26·75	1	0	1	Totals	1572	855	717
Totals	1572	855	717				

prejudice in favour of the half and whole millimetres may have tended to obscure some of the points involved and renders it difficult to deal satisfactorily with the distribution of frequency. The multimodal character of the previous distributions (see *Biometrika*, Vol. I. pp. 175 and 176) was chiefly due to the manner in which this emphasis of certain values upset the grouping. Admitting the defect the only legitimate grouping is that which takes an entire millimetre as its unit, but this is very coarse, especially in the case of the breadths, for the purpose of drawing any conclusions. In Table V. B the frequencies are given with 1 mm. unit of grouping and we see that the ·5 mm. modes have disappeared, but the grouping is very coarse. Working, however, from these we find:

TABLE VI.

Authority	Mean in mm.		Standard Deviation in mm.	
	Length	Breadth	Length	Breadth
Rey	22·266	16·559	·9450	·5955
O. H. L. ...	22·486	16·613	·9546	·6021
Total ...	22·366	16·583	·9557	·5992

An examination of this table seems to indicate that the English Cuckoo's egg is for both dimensions slightly larger and more variable than the German. The difference, however, is not such that we can lay any stress upon it, and in round numbers we may certainly take:

Length of egg 22·4, Breadth 16·6, standard deviation of Length = ·96 and of Breadth = ·60.

The following results were obtained by calculating the normal curves for four sets of frequencies*:

* The determination of P, the goodness of fit, from χ^2 has been made by aid of Palin Elderton's Tables, *Biometrika*, Vol. I. p. 159.

Length				Breadth			
Rey		Latter		Rey		Latter	
Obs.	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.	Calc.
1	3·32	3	1·49	—	—	—	—
50	43·14	22	23·27	1	1·01	2	0·71
185	203·66	123	133·24	74	73·43	47	53·44
380	344·74	300	278·73	459	460·36	379	369·01
185	210·39	201	213·77	302	300·75	269	272·68
46	46·06	61	59·16	19	19·35	18	21·02
8	3·57	6	7·11	0	·10	2	0·14
0	·10	1	0·23	—	—	—	—
$\chi^2 = 16·36$ $P < ·025$		$\chi^2 = 8·09$ $P = ·325$		$\chi^2 = ·62$ $P > ·99$		$\chi^2 = 28·59$ $P < ·00004$	

The odds are thus more than 40 to 1 against Rey's lengths of eggs fitting a normal distribution. But only once in 100 random samples of 855 eggs should we expect a better result than we have got for the breadths, if the distribution be really normal. Turning to the 717 measurements we find the length distribution reasonable on the basis of a normal curve; once in every three random samples of 717 eggs we should get a worse result, but the breadths are quite impossible. This impossibility arises, however, entirely from the two giant breadths of 18·8 and 19·2. They are undoubtedly abnormalities and if they be excluded, the fit is a good one, i.e. about 55 samples in 100 would give a worse result. With regard to Rey's length distribution the sources of improbability are seen on analysis to be (a) the crowding up of seven eggs on 25·0 mm., (b) of 131 eggs on 22·0, and (c) of 63 on 22·5 mm. These contribute more than half the value of χ^2! In other words there is little reason to doubt that the variability in length of Cuckoo's eggs would follow a normal curve, were it not for Rey's tendency to heap up observations on the ·5 and ·0 millimetre groups*. It does not therefore seem worth while pursuing the distribution of variability further on the present observations. We see that it is quite possible that the normal curve would really suffice to describe the frequency. On the other hand, if the *gentes* theory be considered as established, we should naturally expect the heterogeneity to show itself in some deviations from any smooth curve of frequency.

In conclusion I must gratefully acknowledge the generous assistance of Prof. Karl Pearson in the preparation of this paper and in the statistical examination of the measurements obtained: indeed without his aid I should have been quite unable to perceive their significance, nor would this paper have been written.

* The evil of this heaping up may be easily seen if we take the first group 19·05—20·05 instead of 18·75—19·75 still grouping by mm. The mean will now be found to be 22·10 and the standard deviation 1·0014 instead of 22·27 and ·9450 respectively!

MISCELLANEA.

Notes on Statistical Processes.

By W. PALIN ELDERTON, *Actuary.*

I.

An Alternative Method of calculating the Rough Moments from the Actual Statistics.

Let s be the position of any term in the series and $f(s)$ the frequency corresponding, then the following table (page 375) explains itself.

Consider the line opposite the first term and we notice that the sum of the series is given by the first summation S_1 and that the second which we will call S_2 gives the first moment of the whole distribution about a vertical through a point situated at unit distance before the point corresponding to $f(1)$. Still considering only the first line we see that S_3 gives each function multiplied by $\dfrac{n(n+1)}{2}$ or $\dfrac{n^2+n}{2}$, i.e. $(m_2'+m_1')\div 2$ where m' is written for the rough moment. Similarly S_4 gives $\dfrac{n^3+3n^2+2n}{6}$ and S_5 gives $\dfrac{n^4+6n^3+11n^2+6n}{24}$, and hence

$$S_2 = m_1',$$

$$S_3 = \frac{m_2'+m_1'}{2},$$

$$S_4 = \frac{m_3'+3m_2'+2m_1'}{6},$$

$$S_5 = \frac{m_4'+6m_3'+11m_2'+6m_1'}{24}.$$

A numerical example is as follows.

TABLE II.

Function	1st sum	2nd sum	3rd sum	4th sum
239	524	978	1648	2587
157	285	454	670	939
93	128	169	216	269
29	35	41	47	53
6	6	6	6	6
524	978	1648	2587	3854

TABLE I.

No. of terms s	Function $f(s)$	Sum of function from s to n	Sum of previous column from s to n (second sum)	Sum of previous column from s to n (third sum)	Sum of previous column from s to n (fourth sum)	Sum of previous column from s to n (fifth sum)
1	$f(1)$	$f(1)+f(2)+\ldots+f(n)$	$f(1)+2f(2)+\ldots+nf(n)$	$f(1)+3f(2)+\ldots$ $+\dfrac{n(n+1)}{2}f(n)$	$f(1)+4f(2)+\ldots$ $+\dfrac{n(n+1)(n+3)}{3!}f(n)$	$f(1)+5f(2)+\ldots$ $+\dfrac{n(n+1)(n+2)(n+3)}{4!}f(n)$
2	$f(2)$	$f(2)+\ldots+f(n)$	$f(2)+2f(3)+\ldots+(n-1)f(n)$	$f(2)+3f(3)+\ldots$ $+\dfrac{n(n-1)}{2}f(n-1)$	$f(2)+4f(3)+\ldots$ $+\dfrac{(n-1)n(n+1)}{3!}f(n-1)$	$f(2)+5f(3)+\ldots$ $+\dfrac{(n-1)n(n+1)(n+2)}{4!}f(n-1)$
$n-2$	$f(n-2)$	$f(n)+f(n-1)+f(n-2)$	$f(n-2)+2f(n-1)+3f(n)$	$f(n-2)+3f(n-1)+5f(n)$	$f(n-2)+4f(n-1)+10f(n)$	$f(n-2)+5f(n-1)+15f(n)$
$n-1$	$f(n-1)$	$f(n)+f(n-1)$	$f(n-1)+2f(n)$	$f(n-1)+3f(n)$	$f(n-1)+4f(n)$	$f(n-1)+5f(n)$
n	$f(n)$	$f(n)$	$f(n)$	$f(n)$	$f(n)$	$f(n)$

It is unnecessary to form the fifth sum in detail as the total of the fourth gives the only term required. The totals check the work throughout. The rough moments (not reduced to unit frequency) from the equations given above are 978, 2318, 6612 and 21458 which agree with the result we should obtain by direct calculation.

If the rough moments about the mean are required the following is a convenient form :

$$\nu_2 = 2S_3/S_1 - d(1+d),$$
$$\nu_3 = 6S_4/S_1 - 3\nu_2(d+1) - d(1+d)(2+d),$$
$$\nu_4 = 24S_5/S_1 - 2\nu_3\{1 + 2(d+1)\} - \nu_2\{6(1+d)(2+d) - 1\} - d(1+d)(2+d)(3+d),$$

where d is the distance between the mean and the point used for calculating the untransferred rough moments. Obviously $d = S_2/S_1$.

I find that if there are few terms to be dealt with the direct method is quicker but for longer series the above has a distinct advantage. The summation method is really due to Mr G. F. Hardy, but though he has remarked that it is practically the same as the method of moments, I have not traced any previous note of the actual connection between the two methods as statistical processes, though Professor Pearson tells me he believes the summation method of reaching moments is given by Coradi in a paper on the Abdank-Abanowitz integrator, and J. Massau, "Mémoire sur l'intégration graphique et ses applications," Paris, 1887, gives the connection between successive integrations and the moments. I understand that the integraph does not give very satisfactory results for the higher moments, but though the method is like the one with which we are dealing, the latter is clearly not open to the same objection.

The method can be extended to enable us to deal with correlation tables. In order to find the coefficient of correlation we require the means and standard deviations of the x's and y's and the xy-moment. The mean and s. D.'s can be found by treating the totals of the x-columns or y-rows in the same way as we did the entries in Table II. but a quicker method can be devised, which however will be easier to explain with the help of an example.

TABLE III. *Correlation Table.*

		x					Totals
		1	*2*	*3*	*4*	*5*	
y	*1*	2	6	10	20	3	41
	2	1	5	30	9	2	47
	3	—	9	28	30	7	74
	4	1	—	11	16	10	38
Totals		4	20	79	75	22	200

We now obtain from this table another (Table IV.) in the same form giving the y-sum of it by summing each column continuously, and then obtain Table V. by summing Table IV. across continuously.

TABLE IV. *y-sum of Table III.*

		x					Totals
		1	*2*	*3*	*4*	*5*	
y	*1*	4	20	79	75	22	200
	2	2	14	69	55	19	159
	3	1	9	39	46	17	112
	4	1	—	11	16	10	38
Totals		8	43	198	192	68	509

TABLE V.

x-sum of Table IV., i.e. a table giving all cases for x, y group and over in Table III.

		\(x\)					Totals
		1	2	3	4	5	
\(y\)	1	200	196	176	97	22	691
	2	159	157	143	74	19	552
	3	112	111	102	63	17	405
	4	38	37	37	26	10	148
Totals		509	501	458	260	68	1796

The totals in the right-hand column of Table IV. give the first sum of the total in the right-hand column of Table III. and are the same as the column $x=1$ in Table V. The total of Table III. or the sum of col. 1 in Table IV. gives the mean of the y's $(509 \div 200)$; similarly the sum of the row $y=1$ gives the mean of the x's $(691 \div 200)$.

The total of Table V. gives the xy-moment (1796) and the x-standard deviation is found by forming from the first row the series 691, 491, 295, 119, 22 and summing it, i.e. 1618. The second moment about the mean can then be found; the numerical working being as follows:

$$x\text{-mean} = 691 \div 200 = 3\cdot455,$$

$$\nu_2 = 2S_3/S_1 - d(1+d) \text{ by formula given above}$$

$$= \frac{1618 \times 2}{200} - 3\cdot455 \times 4\cdot455$$

$$= \cdot788.$$

Similarly with the y-moments

$$y\text{-mean} = 509 \div 200 = 2\cdot545,$$

$$\nu_2 = \frac{(509 + 309 + 150 + 38) \times 2}{200} - 2\cdot545 \times 3\cdot545$$

$$= 1\cdot038.$$

The xy-moment $= \dfrac{1796}{200} - 2\cdot545 \times 3\cdot455$

$$= \cdot1870,$$

$$r = \cdot1870 \div \sqrt{\cdot788 \times 1\cdot038}$$

$$= \cdot207.$$

In this example the moments have not been adjusted as the totals of the rows did not seem to warrant the assumption of high contact.

Table V. can be formed directly from Table III. but it is a little troublesome to do so at first. The method can best be understood by seeing how any item is made up; thus

$$63 = 30 + 17 + 26 - 10, \quad 97 = 20 + 74 + 22 - 19, \quad 102 = 28 + 63 + 37 - 26 \text{ and so on.}$$

It is not difficult to see how Table V. succeeds in giving the xy-moment; we have merely to remember the numbers by which the various terms in a correlation table have to be multiplied to produce the required result. The final item in Table III. for instance has to be multiplied by 20. It occurs in every item in the $x=5$ column of Table IV. and therefore in every item of Table V.; there are 20 items. Similarly the entry 28 in Table III. has to be multiplied by 9;

it occurs in $x=3$ and $y=1, 2$ and 3 in Table IV. and consequently in all the first 9 groups of Table V. and it is therefore correctly treated.

I have found when dealing with correlation tables that the summation method saves time when the table is fairly large. The method might be extended to simplify some of the calculations which are required when non-linear regression is investigated (see Professor Pearson's Drapers' Research Memoir, Biometric Series, II. *On Skew Correlation*).

II.

The Application of Certain Quadrature Formulae.

The usual quadrature formulae express an area in terms of the ordinates at the beginning and end of the base and a varying number of intermediate ordinates, but in some statistical work it is convenient to have values for the areas in terms of ordinates both within and without the base on which the area stands. Symbolically we have to express $\int_{-\frac{1}{2}}^{\frac{1}{2}} y \, dx$ in terms of $y_0, y_1, y_{-1}, y_2, y_{-2}$, etc.

Let
$$y_x = a + bx + cx^2 + dx^3 + ex^4,$$

then
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = a + \frac{c}{12} + \frac{e}{80},$$

and
$$y_0 = a,$$
$$y_{-1} + y_{+1} = 2(a + c + e),$$
$$y_{-2} + y_{+2} = 2(a + 4c + 16e).$$

Now assume the required integral can be put in the form
$$hy_0 + k(y_1 + y_{-1}) + l(y_2 + y_{-2}),$$

substitute the values given just above and equate coefficients of a, c and e respectively to 1, $\frac{1}{12}$ and $\frac{1}{80}$ and we obtain

I.
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y \, dx = \frac{1}{5760} \{5178 y_0 + 308 (y_1 + y_{-1}) - 17 (y_2 + y_{-2})\}.$$

II. If
$$y = a + bx + cx^2,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y \, dx = \frac{1}{24} \{y_{-1} + 22 y_0 + y_1\}.$$

III. If
$$y_x = a + bx + cx^2 + dx^3 + ex^4,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{5760} \{6463 y_0 - 2092 y_1 + 2298 y_2 - 1132 y_3 + 223 y_4\}.$$

IV. If
$$y_x = a + bx + cx^2 + dx^3,$$
$$\int_{-\frac{1}{2}}^{\frac{3}{2}} y_x dx = \frac{1}{24} \{27 y_0 + 17 y_1 + 5 y_2 - y_3\}.$$

V. If
$$y_x = a + bx + cx^2,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{24} \{25 y_0 - 2 y_1 + y_2\}.$$

VI. If
$$y_x = a + bx + cx^2,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{24} \{13 (y_{-\frac{1}{2}} + y_{\frac{1}{2}}) - (y_{-\frac{3}{2}} + y_{\frac{3}{2}})\}.$$

VII.   If
$$y_x = a + bx + cx^2 + dx^3 + ex^4,$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{1440} \{802 (y_{-\frac{1}{2}} + y_{+\frac{1}{2}}) - 93 (y_{-\frac{3}{2}} + y_{\frac{3}{2}}) + 11 (y_{-\frac{5}{2}} + y_{\frac{5}{2}})\}.$$

VIII.   If
$$y_x = a + bx + cx^2 + dx^3 + ex^4,$$

$$\int_{-\frac{1}{2}}^{\frac{3}{2}} y_x dx = \frac{1}{90} \{29 y_{-\frac{1}{2}} + 124 y_{\frac{1}{2}} + 24 y_{\frac{3}{2}} + 4 y_{\frac{5}{2}} - y_{\frac{7}{2}}\}.$$

IX.   If
$$y_x = a + bx + cx^2,$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{12} \{5 y_{-\frac{1}{2}} + 8 y_{\frac{1}{2}} - y_{\frac{3}{2}}\}.$$

The most obvious purpose for which these formulae can be used in biometric work is for calculating the areas from the values of ordinates when the equation to the curve has been determined. It is clearly convenient to calculate as few ordinates as possible and I have found Formulae I. and II. especially convenient when expressed in the form of differences; thus II. can be written

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y\, dx = y_0 + \frac{1}{24} \{\Delta y_0 - \Delta y_{-1}\},$$

and the adjustment from the mid-ordinate is easily calculated.

Formula I. can be written

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y\, dx = y_0 + \frac{291}{5760} \{\Delta y_0 - \Delta y_{-1}\} - \frac{17}{5760} \{\Delta y_1 - \Delta y_{-2}\}$$

and the fractions are ·0505 and ·00295.

Formulae VI. and VII. are useful when the ordinates at the beginning and end of each group have been calculated.

A numerical example of the use of these formulae in a case where their accuracy can be tested will be of interest. We will assume that the ordinates of the normal curve have been calculated for the equidistant values of x and the areas corresponding to these as mid-ordinates are required.

TABLE VI.

x	Value of ordinate	Δ	Area by Formula I.	Area by Formula II.	True value
·80	·2896916	·0785785			
·40	·3682701	·0306722	·366223	·366274	·366218
·00	·3989423	− ·0306722	·396307	·396386	·396298
·40	·3682701	− ·0785785	·366223	·366274	·366218
·80	·2896916	− ·0955055	·288992	·288986	·288994
1·20	·1941861	− ·0832653	·194740	·194696	·194746
1·60	·1109208	− ·0569298			
2·00	·0539910				

The second and the last columns of this table were found from W. F. Sheppard's Tables, *Biometrika*, Vol. I. pp. 182 *et seq.* For frequency curve work we do not generally require a large number of figures, and as the total frequency does not often exceed 5000, Formula II. would be sufficiently accurate while Formula I. would give a satisfactory answer in almost all possible cases.

III.

Adjustment of Moments.

The quadrature formulae given above can be used very conveniently for adjusting the statistical moments.

We may first take the calculation of moments when the ordinates at equidistant points are known. Let $y_0, y_1, y_2 \ldots y_{n-1}$ be given and we require $\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx$. Now

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx + \int_{\frac{1}{2}}^{\frac{3}{2}} y_x dx + \ldots + \int_{n-\frac{3}{2}}^{n-\frac{1}{2}} y_x dx.$$

If we now apply Formula I. we see that we can use it for all the integrals on the right-hand side of this equation except the first two and the last two and the value of these can be taken from Formula IV. Summing the values we have

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx = \frac{1}{5760} \{ 6463 y_0 + 4371 y_1 + 6669 y_2 + 5537 y_3$$
$$+ 5760 (y_4 + y_5 + \ldots + y_{n-5}) + 5537 y_{n-4}$$
$$+ 6669 y_{n-3} + 4371 y_{n-2} + 6463 y_{n-1} \} \ \ldots\ldots\ldots\ldots \text{ X.,}$$

which means that we can multiply the first and last ordinates by $\frac{6463}{5760}$ (or $1\cdot122205$), the second and last but one by $\frac{4371}{5760}$ (or $\cdot758854$), the third and last but two by $\frac{6669}{5760}$ (or $1\cdot157813$), the fourth and last but three by $\frac{5537}{5760}$ (or $\cdot961285$), leave all the other ordinates unaltered and work out the moments in the usual way from the modified series of ordinates which will now give the proper values for equating to the moments from the formula for the curve.

If there be high contact at each end of the curve—and ordinates are known—there is no reason why Formula I. should not be used even for the end groups, and this tells us that the rough statistical moments require no modification in such a case.

The above rules have been applied in actual cases, thus Formula VI. was found when tested, by using 12 ordinates to approximate to $\int_0^1 \frac{dx}{1+x}$, to give an error of $-\cdot000000205$ which, though it is greater than the best formulae given by Professor Pearson in *Biometrika*, Vol. I. pp. 278, 279, is sufficiently accurate for almost all conceivable purposes. The rule in cases of high contact was tested by adding 12 ordinates of the normal curve calculated to 5 decimal places, which gave $1\cdot24998$ instead of $1\cdot25000$. A type III. curve which had high contact gave 24473 with 9 ordinates instead of 24475, so that in each case the result was practically exact. A further test can however be applied, for if we assume that areas and not ordinates are known we can apply the formulae and reach Sheppard's adjustments. For if areas are known we are given as the rough t^{th} moment the expression

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx\, X^t + \int_{\frac{1}{2}}^{\frac{3}{2}} y_x dx\, (X+1)^t + \int_{\frac{3}{2}}^{\frac{5}{2}} y_x dx\, (X+2)^t + \ldots\ldots + \int_{n-\frac{3}{2}}^{n-\frac{1}{2}} y_x dx\, (X+n-1)^t,$$

and we require $\int_{-\frac{1}{2}}^{n-\frac{1}{2}} (X+x)^t y_x dx$. The series of integrals can be written by the help of Formula I. as

$$\frac{1}{5760} \{ \ldots + [5178\, h^t + 308 \{ (h-1)^t + (h+1)^t \} - 17 \{ (h-2)^t + (h+2)^t \}] y_h + \ldots \},$$

where X is neglected in order to simplify, and working out this general coefficient we have

$$\frac{1}{5760} \{ 5760\, h^t + 240 t\, (t-1)\, h^{t-2} + 3t\, (t-1)\, (t-2)\, (t-3)\, h^{t-4} \text{ etc.} \}.$$

If $t=2$ this becomes h^2+12 and if $t=4$, $h^4+\frac{1}{2}h^2+\frac{1}{80}$. We have however already noticed that if there is high contact the value of $\int_{-\frac{1}{2}}^{n-\frac{1}{2}}(X-x)^t y_x\,dx$ is given by the sum of the ordinates, i.e. the second moment is given by a series of which the general term is $h^2 y_h$ and the fourth by one whose general term is $h^4 y_h$; hence if μ be written for the true and ν for the unadjusted moment we have

$$\mu_2+\frac{1}{12}=\nu_2,$$

and

$$\mu_4+\frac{1}{2}\mu_2+\frac{1}{80}=\nu_4,$$

or

$$\nu_2-\frac{1}{12}=\mu_2,$$

and

$$\nu_4-\frac{1}{2}\nu_2+\frac{7}{240}=\mu_4.$$

The odd moments are seen to require no modification.

It is interesting to note that Formula II. will give the adjustment for the second moment but that for the fourth it gives $\nu_4-\frac{1}{2}\nu_2+\frac{1}{24}=\mu_4$ which only differs from the true result by $\frac{3}{240}$. To demonstrate Sheppard's correction for the nth moment a parabola of at least the nth order must be used.

There is another method of a very simple character by which adjustments when areas are given can be reached; it consists of finding ordinates from the given areas and then working on the values obtained.

It is easy to obtain formulae suitable for our purpose in the following way:

Let $y_x=a+bx+cx^2+dx^3+ex^4$, then

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x\,dx=A_0 \quad =a+\frac{2c}{24}+\frac{2e}{160},$$

$$\int_{\frac{1}{2}}^{\frac{3}{2}} y_x\,dx=A_1 \quad =a+b\ +\frac{26}{24}c+\frac{80}{64}d+\frac{242}{160}e,$$

$$\int_{-\frac{3}{2}}^{-\frac{1}{2}} y_x\,dx=A_{-1}=a-b\ +\frac{26}{24}c-\frac{80}{64}d+\frac{242}{160}e,$$

$$\int_{\frac{3}{2}}^{\frac{5}{2}} y_x\,dx=A_2 \quad =a+2b+\frac{98}{24}c+\frac{544}{64}d+\frac{2882}{160}e,$$

$$\int_{-\frac{5}{2}}^{-\frac{3}{2}} y_x\,dx=A_{-2}=a-2b+\frac{98}{24}c-\frac{544}{64}d+\frac{2882}{160}e.$$

These equations can easily be solved and we obtain the following result:

$$a=\frac{1}{1920}\{2134A_0-116(A_{-1}+A_{+1})+9(A_{-2}+A_{+2})\}\ \dots\dots\dots\dots\text{XI.},$$

$$b=\frac{1}{1920}\{\qquad -1360(A_{-1}-A_{+1})+200(A_{-2}-A_{+2})\},$$

$$c=\frac{1}{1920}\{-2640A_0+1440(A_{-1}+A_{+1})-120(A_{-2}+A_{+2})\},$$

$$d=\frac{1}{1920}\{\qquad 320(A_{-1}-A_{+1})-160(A_{-2}-A_{+2})\},$$

$$e=\frac{1}{1920}\{480A_0-320(A_{-1}+A_{+1})+80(A_{-2}+A_{+2})\}.$$

But $y_0 = a$, and if there be high contact we can form all the ordinates $y_0, y_1, y_2 \ldots \ldots y_{-1}, y_{-2}, \ldots$ by using the expression given for a, hence, if X be the distance of y_0 from the vertical about which moments are calculated we have as the general term for the nth moment

$$[2134 X^n - 116 \{(X-1)^n + (X+1)^n\} + 9 \{(X-2)^n + (X+2)^n\}],$$

which after reduction becomes

$$\left[1920 \, X^n - \frac{n(n-1)}{2!} \cdot 80 \cdot X^{n-2} + \frac{n(n-1)(n-2)(n-3)}{4!} \cdot 28 \cdot X^{n-4} \right],$$

and this will be found to give Sheppard's adjustments.

The values of b, c, d and e are given above as they may be useful in some cases other than those with which we are now dealing, e.g. for some work when parabolas are being used. They enable us to find, for instance,

$$y_{-2} = \frac{1}{1920} \{1689 A_{-2} + 684 A_{-1} - 746 A_0 + 364 A_1 - 71 A_2\},$$

which, by the way, if applied to all terms of the series also give Sheppard's adjustments when high contact is assumed.

The method in a slightly modified form might be applied to enable us to deal with the cases in which the curve rises sharply from zero, but the difficulty is that in all such cases the actual starting point makes a very great difference to the result we obtain. In fact it is possible if we assume the curve to start at the beginning of one of the groups to obtain a negative ordinate at the middle point of the group and the result is in consequence extremely unsatisfactory, though it gives some idea of where the curve really starts.

Using the same notation as above, viz. $\int_{r-\frac{1}{2}}^{r+\frac{1}{2}} y \, dx = A_r$, and also the same method, the following formulae can be obtained :

If $\qquad y_x = a + bx + cx^2 + dx^3 + ex^4,$

$y_0 = \frac{1}{1920} \{2134 A_0 - 116(A_{-1} + A_1) + 9(A_{-2} + A_2)\}$ as above,

$y_1 = \frac{1}{1920} \{-71 A_2 + 2044 A_1 - 26 A_0 - 36 A_{-1} + 9 A_2\}$ .... XII.,

$y_2 = \frac{1}{1920} \{1689 A_2 + 684 - 746 A_0 + 364 A_{-1} - 71 A_{-2}\}$ ....XIII.

If $\qquad\qquad y = a + bx + cx^2,$

$y_0 = \frac{1}{24} \{-A_{-1} + 26 A_0 - A_1\}$................................XIV.,

$y_1 = \frac{1}{24} \{23 A_1 + 2 A_0 - A_{-1}\}$ ....XV.

In order to apply these formulae to the problem of correcting moments we first find the ordinates corresponding to the given areas by means of Formulae XI. to XV. and then use Formula X. or some similar simpler expression such as

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x \, dx = \frac{1}{24} \{26 y_0 + 21 y_1 + 25 y_2 + 24 y_3 + 24 y_4 + \ldots\} \ldots\ldots\ldots\ldots\text{XVI.,}$$

which is formed from equations II. and V. in order to find the terms to be used for giving the corrected moments. The exact area may not be reproduced, for if we find the first term y_0 by XV. and the other terms by XIV. and then use XVI. we have the following result :

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx = \frac{1}{24} \{26 y_0 + 21 y_1 + 25 y_2 + 24 y_3 + ...\}$$

$$= \frac{1}{24} \times \frac{1}{24} \{26 \times 23 A_0 + 26 \times 2A_1 - 26 A_2$$

$$- 21 A_0 + 21 \times 26 A_1 - 21 A_2$$

$$- 25 A_1 + 25 \times 26 A_2 - 25 A_3$$

$$- \text{etc.}\}$$

$$= \frac{1}{576} \{577 A_0 + 573 A_1 + 579 A_2 + 575 A_3 + 576 A_4 + ...\},$$

or the area is overstated by

$$\frac{1}{576} \{A_0 - 3A_1 + 3A_2 - A_3\}.$$

When a parabolic curve is being fitted these formulae can be applied as they stand, for in such cases we can generally choose our range to some extent, that is the formula itself does not fix an actual starting place for our curve ; but in much frequency curve work the difficulty of adjusting the moments is, as has been already remarked, considerably increased owing to the rapid rise of the curve from zero. In such cases, especially when the terms are few in number or there seems a likelihood from the original statistics that the curve does not start at the beginning of the base of the first group, I think the formulae should be applied neglecting the first group, which should be examined separately. .

Taking the following unpromising series (col. 1) from *Phil. Trans.* Vol. 197 A, pp. 454—456, the following table was made.

TABLE VII.

Frequency (1)	Ordinate by XIV. & XV.	Modified ordinate by XVI.
47	(neglected)	(neglected)
762	742·7	804·6
160·5	141·2	123·6
20	14·8	15·4
5	4·5	4·5
1·5	1·3	1·3
996		

If the curve starts at the middle of the base of the 47 group its mid-distance from the middle point of the base of the 762 group is really $\frac{3}{4}$ and this should be taken into account in calculating the moments. Now it is easy to see that the first group probably relates to a small base because the mid-ordinate calculated from $\frac{1}{24}(23 A_0 + 2A_1 - A_2)$ is -8. If in order to

simplify matters we assume that half a base is the only alternative to a whole one the moments can be calculated as follows :

<div align="center">TABLE VIII.</div>

Modified ordinate f	Distance d from 804·6 ordinate	fd	fd^2	fd^3	fd^4
47·0	− ·75	− 35·2	+ 26·4	− 19·8	+ 14·8
804·6	—	—	—	—	—
123·6	1	123·6	123·6	123·6	123·6
15·4	2	30·8	61·6	123·2	246·4
4·5	3	13·5	40·5	121·5	364·5
1·3	4	5·2	20·8	83·2	332·8
996·4		137·9	272·9	431·7	1082·1

The series of numbers from which Professor Pearson found the graduated figures given in col. (1) of Table VII. was 1, 49, 798, 123, 18, 5, 2 ; but owing almost entirely to the differences of 2·6 between 18 and 15·4 and ·7 between 2 and 1·3 the totals that are obtained are 996, 131, 325, 473, 1393, which differ widely from those given above. The mean is very close in comparison to that obtained by using the figures as they stand in col. (1) Table VII. but the other moments are little better. I give the result of the attempt to find adjustments in this awkward case because it shows that the method can give a series of numbers very like that required to give a true result, but the small differences make the result unsatisfactory.

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TABLES FOR FACILITATING THE COMPUTATION OF PROBABLE ERRORS.

By WINIFRED GIBSON, B.Sc., Jessel Student, University College, London.

THE following tables have been prepared with the view of rendering the computation of the probable errors of the mean, the standard deviation and the coefficient of variation as brief as possible. It is well known that if σ be the standard deviation, V the coefficient of variation, and n the number of observations, then:

(i)   The Probable Error of the Mean $= \cdot6744898 \, \sigma/\sqrt{n}$.

(ii)   The Probable Error of the Standard Deviation $= \cdot6744898 \, \sigma/\sqrt{(2n)}$.

(iii)   The Probable Error of the Coefficient of Variation

$$= \cdot6744898 \, V \left\{ 1 + 2 \left(\frac{V}{100} \right)^2 \right\}^{\frac{1}{2}} \Big/ \sqrt{(2n)}.$$

It is clear, therefore, that the tabling of the expressions $\cdot6744898/\sqrt{n}$, $\cdot6744898/\sqrt{(2n)}$, and $V \left\{ 1 + 2 \left(\frac{V}{100} \right)^2 \right\}^{\frac{1}{2}}$ would make the computations of these probable errors much easier.

The question then arose whether the actual values or the logarithms of these expressions should be given. To those who work with a calculator, the latter would be of little service, while the former would still be helpful to those who have not this advantage. As nearly all biometricians now use a Tate, Brunsviga, or other mechanical computator, it was settled that the actual values should be given.

With the numbers usually available in biometric work, exclusive of human vital statistics, n rarely exceeds 1000. Hence

$$\chi_1 = \cdot6744898/\sqrt{n} \quad \text{and} \quad \chi_2 = \cdot6744898/\sqrt{(2n)}$$

have been tabled for $n = 1$ to 1000.   Clearly for any even value of n between 1000 and 2000 the value of χ_1 can be found from the value of χ_2 for $\frac{1}{2}n$.   Thus χ_1 for $n = 1594$ is χ_2 for $n = 797$, and is found to be ·01689.   As the first difference for $\chi_2 > 500$ does not exceed ·00002, and the second difference is zero to five figures, it will be clear that χ_1 can for odd numbers between 1000 and 2000 be found by the simple process of halving the sum of the adjacent values of χ_2 for $\frac{1}{2}(n-1)$ and $\frac{1}{2}(n+1)$.   Further, for any even value of n between 1000 and 2000, χ_2 may be found by halving the value of χ_1 for $\frac{1}{2}n$, and for any odd value of n between 1000 and 2000 χ_2 may be found by quartering the sum of the values of χ_1 for $\frac{1}{2}(n-1)$ and $\frac{1}{2}(n+1)$.   Thus suppose $n = 1731$; we have $\frac{1}{2}(n-1) = 865$ and $\frac{1}{2}(n+1) = 866$. Hence $\chi_1 = \frac{1}{2}(·01622 + ·01621) = ·016215$, and $\chi_2 = \frac{1}{4}(·02293 + ·02292) = ·0114625$. The actual values are $\chi_1 = ·016212$ and $\chi_2 = ·011463$.   Such differences will hardly ever have any statistical importance.   To determine the probable error of the coefficient of variation we use Table III.   Here we find

$$\psi = V\left\{1 + 2\left(\frac{V}{100}\right)^2\right\}^{\frac{1}{2}}$$

tabled for values of V from 0 to 50.   The probable error of V must therefore be found by taking out the value of χ_2 corresponding to the given value of n, and multiplying it by the ψ found for the given value of V by interpolation from this table.

Thus suppose $V = 23·65$ for 583 observations.   We have by the usual advancing difference formula (*Biometrika*, Vol. II. p. 175)

$$\psi = 24·18612 + ·65 \times 1·15861 - \frac{·65 \times ·35}{2} \times ·01316 + \frac{·65 \times ·35 \times 1·35}{2 \times 3} \times ·00045$$

which evaluated by the Brunsviga $= 24·93774$*.   Further, from Table II., $\chi_2 = ·01975$. Thus the probable error $= \psi \chi_2 = ·49252$.   The value actually found by direct calculation is ·49259, the difference, which is of no importance for practical statistics, depending upon cutting off χ_2 at the fifth figure.

While Tables I. and II. are the work of the author, Table III. is due to Dr Raymond Pearl and J. Blakeman.   All the values were found to seven figures, but it seemed sufficient for practical work to register them to five.

---

* Actual value 24·937,739, so that the ψ table is amply sufficient.

TABLES I. AND II. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
1	·67449	·47694	51	·09445	·06678	101	·06711	·04746
2	·47694	·33724	52	·09353	·06614	102	·06678	·04722
3	·38942	·27536	53	·09265	·06551	103	·06646	·04699
4	·33724	·23847	54	·09179	·06490	104	·06614	·04677
5	·30164	·21329	55	·09095	·06431	105	·06582	·04654
6	·27536	·19471	56	·09013	·06373	106	·06551	·04632
7	·25493	·18026	57	·08934	·06317	107	·06521	·04611
8	·23847	·16862	58	·08856	·06262	108	·06490	·04589
9	·22483	·15898	59	·08781	·06209	109	·06460	·04568
10	·21329	·15082	60	·08708	·06157	110	·06431	·04547
11	·20337	·14380	61	·08636	·06107	111	·06402	·04527
12	·19471	·13768	62	·08566	·06057	112	·06373	·04507
13	·18707	·13228	63	·08498	·06009	113	·06345	·04487
14	·18026	·12747	64	·08431	·05962	114	·06317	·04467
15	·17415	·12314	65	·08366	·05916	115	·06290	·04447
16	·16862	·11923	66	·08302	·05871	116	·06262	·04428
17	·16359	·11567	67	·08240	·05827	117	·06236	·04409
18	·15898	·11241	68	·08179	·05784	118	·06209	·04391
19	·15474	·10942	69	·08120	·05742	119	·06183	·04372
20	·15082	·10665	70	·08062	·05700	120	·06157	·04354
21	·14719	·10408	71	·08005	·05660	121	·06132	·04336
22	·14380	·10168	72	·07949	·05621	122	·06107	·04318
23	·14064	·09945	73	·07894	·05582	123	·06082	·04300
24	·13768	·09735	74	·07841	·05544	124	·06057	·04283
25	·13490	·09539	75	·07788	·05507	125	·06033	·04266
26	·13228	·09353	76	·07737	·05471	126	·06009	·04249
27	·12981	·09179	77	·07687	·05435	127	·05985	·04232
28	·12747	·09013	78	·07637	·05400	128	·05962	·04216
29	·12525	·08856	79	·07589	·05366	129	·05939	·04199
30	·12314	·08708	80	·07541	·05332	130	·05916	·04183
31	·12114	·08566	81	·07494	·05299	131	·05893	·04167
32	·11923	·08431	82	·07448	·05267	132	·05871	·04151
33	·11741	·08302	83	·07403	·05235	133	·05849	·04136
34	·11567	·08179	84	·07359	·05204	134	·05827	·04120
35	·11401	·08062	85	·07316	·05173	135	·05805	·04105
36	·11241	·07949	86	·07273	·05143	136	·05784	·04090
37	·11088	·07841	87	·07231	·05113	137	·05763	·04075
38	·10942	·07737	88	·07190	·05084	138	·05742	·04060
39	·10800	·07637	89	·07150	·05056	139	·05721	·04045
40	·10665	·07541	90	·07110	·05027	140	·05700	·04031
41	·10534	·07448	91	·07071	·05000	141	·05680	·04017
42	·10408	·07359	92	·07032	·04972	142	·05660	·04002
43	·10286	·07273	93	·06994	·04946	143	·05640	·03988
44	·10168	·07190	94	·06957	·04919	144	·05621	·03974
45	·10055	·07110	95	·06920	·04893	145	·05601	·03961
46	·09945	·07032	96	·06884	·04868	146	·05582	·03947
47	·09838	·06957	97	·06848	·04843	147	·05563	·03934
48	·09735	·06884	98	·06813	·04818	148	·05544	·03920
49	·09636	·06813	99	·06779	·04793	149	·05526	·03907
50	·09539	·06745	100	·06745	·04769	150	·05507	·03894

TABLES I. AND II.—(*continued*).

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
151	·05489	·03881	201	·04757	·03364	251	·04257	·03010
152	·05471	·03868	202	·04746	·03356	252	·04249	·03004
153	·05453	·03856	203	·04734	·03347	253	·04240	·02998
154	·05435	·03843	204	·04722	·03339	254	·04232	·02993
155	·05418	·03831	205	·04711	·03331	255	·04224	·02987
156	·05400	·03819	206	·04699	·03323	256	·04216	·02981
157	·05383	·03806	207	·04688	·03315	257	·04207	·02975
158	·05366	·03794	208	·04677	·03307	258	·04199	·02969
159	·05349	·03782	209	·04666	·03299	259	·04191	·02964
160	·05332	·03771	210	·04654	·03291	260	·04183	·02958
161	·05316	·03759	211	·04643	·03283	261	·04175	·02952
162	·05299	·03747	212	·04632	·03276	262	·04167	·02947
163	·05283	·03736	213	·04622	·03268	263	·04159	·02941
164	·05267	·03724	214	·04611	·03260	264	·04151	·02935
165	·05251	·03713	215	·04600	·03253	265	·04143	·02930
166	·05235	·03702	216	·04589	·03245	266	·04136	·02924
167	·05219	·03691	217	·04579	·03238	267	·04128	·02919
168	·05204	·03680	218	·04568	·03230	268	·04120	·02913
169	·05188	·03669	219	·04558	·03223	269	·04112	·02908
170	·05173	·03658	220	·04547	·03216	270	·04105	·02903
171	·05158	·03647	221	·04537	·03208	271	·04097	·02897
172	·05143	·03637	222	·04527	·03201	272	·04090	·02892
173	·05128	·03626	223	·04517	·03194	273	·04082	·02887
174	·05113	·03616	224	·04507	·03187	274	·04075	·02881
175	·05099	·03605	225	·04497	·03180	275	·04067	·02876
176	·05084	·03595	226	·04487	·03173	276	·04060	·02871
177	·05070	·03585	227	·04477	·03166	277	·04053	·02866
178	·05056	·03575	228	·04467	·03159	278	·04045	·02860
179	·05041	·03565	229	·04457	·03152	279	·04038	·02855
180	·05027	·03555	230	·04447	·03145	280	·04031	·02850
181	·05013	·03545	231	·04438	·03138	281	·04024	·02845
182	·05000	·03535	232	·04428	·03131	282	·04017	·02840
183	·04986	·03526	233	·04419	·03125	283	·04009	·02835
184	·04972	·03516	234	·04409	·03118	284	·04002	·02830
185	·04959	·03507	235	·04400	·03111	285	·03995	·02825
186	·04946	·03497	236	·04391	·03105	286	·03988	·02820
187	·04932	·03488	237	·04381	·03098	287	·03981	·02815
188	·04919	·03478	238	·04372	·03092	288	·03974	·02810
189	·04906	·03469	239	·04363	·03085	289	·03968	·02806
190	·04893	·03460	240	·04354	·03079	290	·03961	·02801
191	·04880	·03451	241	·04345	·03172	291	·03954	·02796
192	·04868	·03442	242	·04336	·03066	292	·03947	·02791
193	·04855	·03433	243	·04327	·03060	293	·03940	·02786
194	·04843	·03424	244	·04318	·03053	294	·03934	·02782
195	·04830	·03415	245	·04309	·03047	295	·03927	·02777
196	·04818	·03407	246	·04300	·03041	296	·03920	·02772
197	·04806	·03398	247	·04292	·03035	297	·03913	·02767
198	·04793	·03389	248	·04283	·03029	298	·03907	·02763
199	·04781	·03381	249	·04274	·03022	299	·03901	·02758
200	·04769	·03372	250	·04266	·03016	300	·03894	·02754

TABLES I. AND II.—(*continued*).

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
301	·03888	·02749	351	·03600	·02546	401	·03368	·02382
302	·03881	·02744	352	·03595	·02542	402	·03364	·02379
303	·03875	·02740	353	·03590	·02538	403	·03360	·02376
304	·03868	·02735	354	·03585	·02535	404	·03356	·02373
305	·03862	·02731	355	·03580	·02531	405	·03352	·02370
306	·03856	·02726	356	·03575	·02528	406	·03347	·02367
307	·03850	·02722	357	·03570	·02524	407	·03343	·02364
308	·03843	·02718	358	·03565	·02521	408	·03339	·02361
309	·03837	·02713	359	·03560	·02517	409	·03335	·02358
310	·03831	·02709	360	·03555	·02514	410	·03331	·02355
311	·03825	·02704	361	·03550	·02510	411	·03327	·02353
312	·03819	·02700	362	·03545	·02507	412	·03323	·02350
313	·03812	·02696	363	·03540	·02503	413	·03319	·02347
314	·03806	·02692	364	·03535	·02500	414	·03315	·02344
315	·03800	·02687	365	·03530	·02496	415	·03311	·02341
316	·03794	·02683	366	·03526	·02493	416	·03307	·02338
317	·03788	·02679	367	·03521	·02490	417	·03303	·02336
318	·03782	·02675	368	·03516	·02486	418	·03299	·02333
319	·03776	·02670	369	·03511	·02483	419	·03295	·02330
320	·03771	·02666	370	·03507	·02479	420	·03291	·02327
321	·03765	·02662	371	·03502	·02476	421	·03287	·02324
322	·03759	·02658	372	·03497	·02473	422	·03283	·02322
323	·03753	·02654	373	·03492	·02469	423	·03279	·02319
324	·03747	·02650	374	·03488	·02466	424	·03276	·02316
325	·03741	·02646	375	·03483	·02463	425	·03272	·02313
326	·03736	·02642	376	·03478	·02460	426	·03268	·02311
327	·03730	·02637	377	·03474	·02456	427	·03264	·02308
328	·03724	·02633	378	·03469	·02453	428	·03260	·02305
329	·03719	·02629	379	·03465	·02450	429	·03256	·02303
330	·03713	·02625	380	·03460	·02447	430	·03253	·02300
331	·03707	·02621	381	·03456	·02443	431	·03249	·02297
332	·03702	·02618	382	·03451	·02440	432	·03245	·02295
333	·03696	·02614	383	·03446	·02437	433	·03241	·02292
334	·03691	·02610	384	·03442	·02434	434	·03238	·02289
335	·03685	·02606	385	·03438	·02431	435	·03234	·02287
336	·03680	·02602	386	·03433	·02428	436	·03230	·02284
337	·03674	·02598	387	·03429	·02424	437	·03227	·02281
338	·03669	·02594	388	·03424	·02421	438	·03223	·02279
339	·03663	·02590	389	·03420	·02418	439	·03219	·02276
340	·03658	·02587	390	·03415	·02415	440	·03216	·02274
341	·03653	·02583	391	·03411	·02412	441	·03212	·02271
342	·03647	·02579	392	·03407	·02409	442	·03208	·02269
343	·03642	·02575	393	·03402	·02406	443	·03205	·02266
344	·03637	·02571	394	·03398	·02403	444	·03201	·02263
345	·03631	·02568	395	·03394	·02400	445	·03197	·02261
346	·03626	·02564	396	·03389	·02397	446	·03194	·02258
347	·03621	·02560	397	·03385	·02394	447	·03190	·02256
348	·03616	·02557	398	·03381	·02391	448	·03187	·02253
349	·03610	·02553	399	·03377	·02388	449	·03183	·02251
350	·03605	·02549	400	·03372	·02385	450	·03180	·02248

TABLES I. AND II.—(*continued*).

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
451	·03176	·02246	501	·03013	·02131	551	·02873	·02032
452	·03173	·02243	502	·03010	·02129	552	·02871	·02030
453	·03169	·02241	503	·03007	·02127	553	·02868	·02028
454	·03166	·02238	504	·03004	·02124	554	·02866	·02026
455	·03162	·02236	505	·03001	·02122	555	·02863	·02024
456	·03159	·02233	506	·02998	·02120	556	·02860	·02023
457	·03155	·02231	507	·02996	·02118	557	·02858	·02021
458	·03152	·02229	508	·02993	·02116	558	·02855	·02019
459	·03148	·02226	509	·02990	·02114	559	·02853	·02017
460	·03145	·02224	510	·02987	·02112	560	·02850	·02015
461	·03141	·02221	511	·02984	·02110	561	·02848	·02014
462	·03138	·02219	512	·02981	·02108	562	·02845	·02012
463	·03135	·02217	513	·02978	·02106	563	·02843	·02010
464	·03131	·02214	514	·02975	·02104	564	·02840	·02008
465	·03128	·02212	515	·02972	·02102	565	·02838	·02006
466	·03125	·02209	516	·02969	·02100	566	·02835	·02005
467	·03121	·02207	517	·02966	·02098	567	·02833	·02003
468	·03118	·02205	518	·02964	·02096	568	·02830	·02001
469	·03115	·02202	519	·02961	·02094	569	·02828	·01999
470	·03111	·02200	520	·02958	·02092	570	·02825	·01998
471	·03108	·02198	521	·02955	·02089	571	·02823	·01996
472	·03105	·02195	522	·02952	·02087	572	·02820	·01994
473	·03101	·02193	523	·02949	·02085	573	·02818	·01992
474	·03098	·02191	524	·02947	·02084	574	·02815	·01991
475	·03095	·02188	525	·02944	·02082	575	·02813	·01990
476	·03092	·02186	526	·02941	·02080	576	·02810	·01987
477	·03088	·02184	527	·02938	·02078	577	·02808	·01986
478	·03085	·02181	528	·02935	·02076	578	·02806	·01984
479	·03082	·02179	529	·02933	·02074	579	·02803	·01982
480	·03079	·02177	530	·02930	·02072	580	·02801	·01980
481	·03075	·02175	531	·02927	·02070	581	·02798	·01978
482	·03072	·02172	532	·02924	·02068	582	·02796	·01977
483	·03069	·02170	533	·02922	·02066	583	·02793	·01975
484	·03066	·02168	534	·02919	·02064	584	·02791	·01974
485	·03063	·02166	535	·02916	·02062	585	·02789	·01972
486	·03060	·02163	536	·02913	·02060	586	·02786	·01970
487	·03056	·02161	537	·02911	·02058	587	·02784	·01969
488	·03053	·02159	538	·02908	·02056	588	·02782	·01967
489	·03050	·02157	539	·02905	·02054	589	·02779	·01965
490	·03047	·02155	540	·02903	·02052	590	·02777	·01964
491	·03044	·02152	541	·02900	·02051	591	·02774	·01962
492	·03041	·02150	542	·02897	·02049	592	·02772	·01960
493	·03038	·02148	543	·02895	·02047	593	·02770	·01959
494	·03035	·02146	544	·02892	·02045	594	·02767	·01957
495	·03032	·02144	545	·02889	·02043	595	·02765	·01955
496	·03029	·02142	546	·02887	·02041	596	·02763	·01954
497	·03026	·02139	547	·02884	·02039	597	·02761	·01952
498	·03022	·02137	548	·02881	·02037	598	·02758	·01950
499	·03019	·02135	549	·02879	·02036	599	·02756	·01949
500	·03016	·02133	550	·02876	·02034	600	·02754	·01947

TABLES I. AND II.—*(continued).*

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
601	·02751	·01945	651	·02644	·01869	701	·02548	·01801
602	·02749	·01944	652	·02642	·01868	702	·02546	·01800
603	·02747	·01942	653	·02639	·01866	703	·02544	·01799
604	·02744	·01941	654	·02637	·01865	704	·02542	·01798
605	·02742	·01939	655	·02635	·01864	705	·02540	·01796
606	·02740	·01937	656	·02633	·01862	706	·02538	·01795
607	·02738	·01936	657	·02631	·01861	707	·02537	·01794
608	·02735	·01934	658	·02629	·01859	708	·02535	·01792
609	·02733	·01933	659	·02627	·01858	709	·02533	·01791
610	·02731	·01931	660	·02625	·01856	710	·02531	·01790
611	·02729	·01929	661	·02623	·01855	711	·02530	·01789
612	·02726	·01928	662	·02621	·01854	712	·02528	·01787
613	·02724	·01926	663	·02620	·01852	713	·02526	·01786
614	·02722	·01925	664	·02618	·01851	714	·02524	·01785
615	·02720	·01923	665	·02616	·01849	715	·02522	·01784
616	·02718	·01922	666	·02614	·01848	716	·02521	·01782
617	·02715	·01920	667	·02612	·01847	717	·02519	·01781
618	·02713	·01919	668	·02610	·01845	718	·02517	·01780
619	·02711	·01917	669	·02608	·01844	719	·02515	·01779
620	·02709	·01915	670	·02606	·01843	720	·02514	·01777
621	·02707	·01914	671	·02604	·01841	721	·02512	·01776
622	·02704	·01912	672	·02602	·01840	722	·02510	·01775
623	·02702	·01911	673	·02600	·01838	723	·02508	·01774
624	·02700	·01909	674	·02598	·01837	724	·02507	·01773
625	·02698	·01908	675	·02596	·01836	725	·02505	·01771
626	·02696	·01906	676	·02594	·01834	726	·02503	·01770
627	·02694	·01905	677	·02592	·01833	727	·02502	·01769
628	·02692	·01903	678	·02590	·01832	728	·02500	·01768
629	·02689	·01902	679	·02588	·01830	729	·02498	·01766
630	·02687	·01900	680	·02587	·01829	730	·02496	·01765
631	·02685	·01899	681	·02585	·01828	731	·02495	·01764
632	·02683	·01897	682	·02583	·01826	732	·02493	·01763
633	·02681	·01896	683	·02581	·01825	733	·02491	·01762
634	·02679	·01894	684	·02579	·01824	734	·02490	·01760
635	·02677	·01893	685	·02577	·01822	735	·02488	·01759
636	·02675	·01891	686	·02575	·01821	736	·02486	·01758
637	·02672	·01890	687	·02573	·01820	737	·02485	·01757
638	·02670	·01888	688	·02571	·01818	738	·02483	·01756
639	·02668	·01887	689	·02570	·01817	739	·02481	·01754
640	·02666	·01885	690	·02568	·01816	740	·02479	·01753
641	·02664	·01884	691	·02566	·01814	741	·02478	·01752
642	·02662	·01822	692	·02564	·01813	742	·02476	·01751
643	·02660	·01881	693	·02562	·01812	743	·02474	·01750
644	·02658	·01879	694	·02560	·01810	744	·02473	·01749
645	·02656	·01878	695	·02558	·01809	745	·02471	·01747
646	·02654	·01876	696	·02557	·01808	746	·02469	·01746
647	·02652	·01875	697	·02555	·01807	747	·02468	·01745
648	·02650	·01874	698	·02553	·01805	748	·02466	·01744
649	·02648	·01872	699	·02551	·01804	749	·02465	·01743
650	·02646	·01871	700	·02549	·01803	750	·02463	·01742

TABLES I. AND II.—(*continued*).

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
751	·02461	·01740	801	·02383	·01685	851	·02312	·01635
752	·02460	·01739	802	·02382	·01684	852	·02311	·01634
753	·02458	·01738	803	·02380	·01683	853	·02309	·01633
754	·02456	·01737	804	·02379	·01682	854	·02308	·01632
755	·02455	·01736	895	·02377	·01681	855	·02307	·01631
756	·02453	·01735	806	·02376	·01680	856	·02305	·01630
757	·02451	·01733	807	·02374	·01679	857	·02304	·01629
758	·02450	·01732	808	·02373	·01678	858	·02303	·01628
759	·02448	·01731	809	·02371	·01677	859	·02301	·01627
760	·02447	·01730	810	·02370	·01676	860	·02300	·01626
761	·02445	·01729	811	·02368	·01675	861	·02299	·01625
762	·02443	·01728	812	·02367	·01674	862	·02297	·01624
763	·02442	·01727	813	·02366	·01673	863	·02296	·01624
764	·02440	·01725	814	·02364	·01672	864	·02295	·01623
765	·02439	·01724	815	·02363	·01671	865	·02293	·01622
766	·02437	·01723	816	·02361	·01670	866	·02292	·01621
767	·02435	·01722	817	·02360	·01669	867	·02291	·01620
768	·02434	·01721	818	·02358	·01668	868	·02289	·01619
769	·02432	·01720	819	·02357	·01667	869	·02288	·01618
770	·02431	·01719	820	·02355	·01666	870	·02287	·01617
771	·02429	·01718	821	·02354	·01665	871	·02285	·01616
772	·02428	·01717	822	·02353	·01664	872	·02284	·01615
773	·02426	·01715	823	·02351	·01662	873	·02283	·01614
774	·02424	·01714	824	·02350	·01661	874	·02281	·01613
775	·02423	·01713	825	·02348	·01660	875	·02280	·01612
776	·02421	·01712	826	·02347	·01659	876	·02279	·01611
777	·02420	·01711	827	·02345	·01658	877	·02278	·01610
778	·02418	·01710	828	·02344	·01657	878	·02276	·01610
779	·02417	·01709	829	·02343	·01656	879	·02275	·01609
780	·02415	·01708	830	·02341	·01655	880	·02274	·01608
781	·02414	·01707	831	·02340	·01654	881	·02272	·01607
782	·02412	·01706	832	·02338	·01653	882	·02271	·01606
783	·02410	·01704	833	·02337	·01652	883	·02270	·01605
784	·02409	·01703	834	·02336	·01651	884	·02269	·01604
785	·02407	·01702	835	·02334	·01651	885	·02267	·01603
786	·02406	·01701	836	·02333	·01650	886	·02266	·01602
787	·02404	·01700	837	·02331	·01649	887	·02265	·01601
788	·02403	·01699	838	·02330	·01648	888	·02263	·01600
789	·02401	·01698	839	·02329	·01647	889	·02262	·01600
790	·02400	·01697	840	·02327	·01646	890	·02261	·01599
791	·02398	·01696	841	·02326	·01645	891	·02260	·01598
792	·02397	·01695	852	·02324	·01644	892	·02258	·01597
793	·02395	·01694	843	·02323	·01643	893	·02257	·01596
794	·02394	·01693	844	·02322	·01642	894	·02256	·01595
795	·02392	·01692	845	·02320	·01641	895	·02255	·01594
796	·02391	·01690	846	·02319	·01640	896	·02253	·01593
797	·02389	·01689	847	·02318	·01639	897	·02252	·01592
798	·02388	·01688	848	·02316	·01638	898	·02251	·01592
799	·02386	·01687	849	·02315	·01637	899	·02250	·01591
800	·02385	·01686	850	·02313	·01636	900	·02248	·01590

TABLES I. and II.—(continued).

n	χ_1	χ_2	n	χ_1	χ_2
901	·02247	·01589	951	·02187	·01547
902	·02246	·01588	952	·02186	·01546
903	·02245	·01587	953	·02185	·01545
904	·02243	·01586	954	·02184	·01544
905	·02242	·01585	955	·02183	·01543
906	·02241	·01585	956	·02181	·01543
907	·02240	·01584	957	·02180	·01542
908	·02238	·01583	958	·02179	·01541
909	·02237	·01582	959	·02178	·01540
910	·02236	·01581	960	·02177	·01539
911	·02235	·01580	961	·02176	·01539
912	·02233	·01579	962	·02175	·01538
913	·02232	·01578	963	·02174	·01537
914	·02231	·01578	964	·02172	·01536
915	·02230	·01577	965	·02171	·01535
916	·02229	·01576	966	·02170	·01535
917	·02227	·01575	967	·02169	·01534
918	·02226	·01574	968	·02168	·01533
919	·02225	·01573	969	·02167	·01532
920	·02224	·01572	970	·02166	·01531
921	·02223	·01572	971	·02165	·01531
922	·02221	·01571	972	·02163	·01530
923	·02220	·01570	973	·02162	·01529
924	·02219	·01569	974	·02161	·01528
925	·02218	·01568	975	·02160	·01527
926	·02217	·01567	976	·02159	·01527
927	·02215	·01566	977	·02158	·01526
928	·02214	·01566	978	·02157	·01525
929	·02213	·01565	979	·02156	·01524
930	·02212	·01564	980	·02155	·01524
931	·02211	·01563	981	·02153	·01523
932	·02209	·01562	982	·02152	·01522
933	·02208	·01561	983	·02151	·01521
934	·02207	·01561	984	·02150	·01520
935	·02206	·01560	985	·02149	·01520
936	·02205	·01559	986	·02148	·01519
937	·02203	·01558	987	·02147	·01518
938	·02202	·01557	988	·02146	·01517
939	·02201	·01556	989	·02145	·01517
940	·02200	·01556	990	·02144	·01516
941	·02199	·01555	991	·02143	·01515
942	·02198	·01554	992	·02142	·01514
943	·02196	·01553	993	·02140	·01514
944	·02195	·01552	994	·02139	·01513
945	·02194	·01551	995	·02138	·01512
946	·02193	·01551	996	·02137	·01511
947	·02192	·01550	997	·02136	·01510
948	·02191	·01549	998	·02135	·01510
949	·02189	·01548	999	·02134	·01509
950	·02188	·01547	1000	·02133	·01508

TABLE III.

Probable Errors of Coefficients of Variation.

V	ψ	Δ +	Δ² +	Δ³ +
0	0·00000	1·00010	0	60
1	1·00010	1·00070	60	60
2	2·00080	1·00190	120	60
3	3·00270	1·00370	180	60
4	4·00639	1·00609	239	59
5	5·01248	1·00908	299	59
6	6·02156	1·01266	358	59
7	7·03422	1·01682	417	58
8	8·05104	1·02157	475	58
9	9·07261	1·02690	533	57
10	10·09950	1·03280	590	57
11	11·13230	1·03927	647	56
12	12·17157	1·04630	703	56
13	13·21787	1·05389	759	55
14	14·27176	1·06202	814	54
15	15·33379	1·07070	868	53
16	16·40449	1·07991	921	53
17	17·48440	1·08965	974	52
18	18·57405	1·09990	1025	51
19	19·67395	1·11066	1076	50
20	20·78461	1·12192	1126	49
21	21·90653	1·13368	1175	48
22	23·04021	1·14591	1223	47
23	24·18612	1·15861	1270	46
24	25·34473	1·17177	1316	45
25	26·51650	1·18539	1362	44
26	27·70190	1·19945	1406	43
27	28·90135	1·21395	1449	42
28	30·11530	1·22886	1491	41
29	31·34416	1·24418	1533	40
30	32·58834	1·25991	1573	39
31	33·84825	1·27603	1612	38
32	35·12428	1·29253	1650	37
33	36·41681	1·30940	1687	36
34	37·72621	1·32664	1723	35
35	39·05285	1·34422	1758	34
36	40·39707	1·36215	1793	33
37	41·75922	1·38041	1826	32
38	43·13962	1·39899	1858	31
39	44·53861	1·41789	1890	30
40	45·95650	1·43709	1920	30
41	47·39359	1·45658	1950	29
42	48·85017	1·47636	1978	28
43	50·32654	1·49642	2006	27
44	51·82296	1·51675	2033	26
45	53·33971	1·53734	2059	25
46	54·87706	1·55818	2084	24
47	56·43524	1·57927	2109	24
48	58·01451	1·60059	2132	23
49	59·61510	1·62214	2155	22
50	61·23724		2177	

COOPERATIVE INVESTIGATIONS ON PLANTS.

III. ON INHERITANCE IN THE SHIRLEY POPPY *.

SECOND MEMOIR.

CONTENTS.

(1) *Material and Manner of Fertilisation.*

A first memoir on the subject of inheritance in the Shirley Poppy was published in November, 1902†. In summing up our results in that paper we stated that:

"Most plant organs being multiple in appearance, we have to apply special methods to deduce the intensity of heredity from multiple observations on the individual. It will probably be better in future experiments to confine attention to the first or principal flower, instead of using the indirect method of homotyposis; but this will involve the observation (previous to harvesting) of individual plants in large series—500 to 1000—and much increase the labour of superintendence and observation‡."

We continued:

"Notwithstanding the difficulties referred to above we find that for a variety of plant characters in the Shirley Poppy the values of hereditary influence found are on the whole in fair agreement with the like values in man."

VA BIGGS FOR THE MEMOIR ON INHERITANCE.
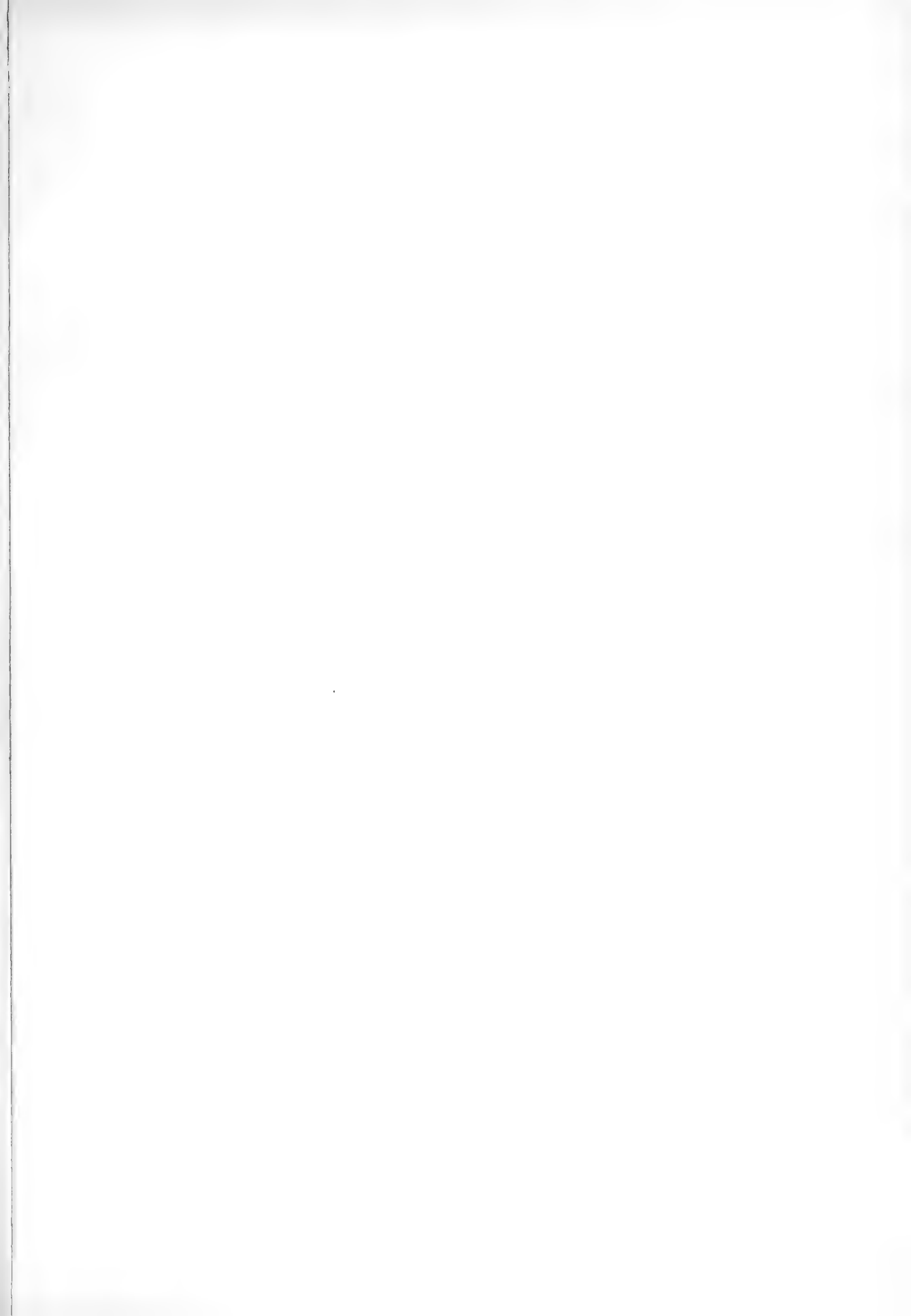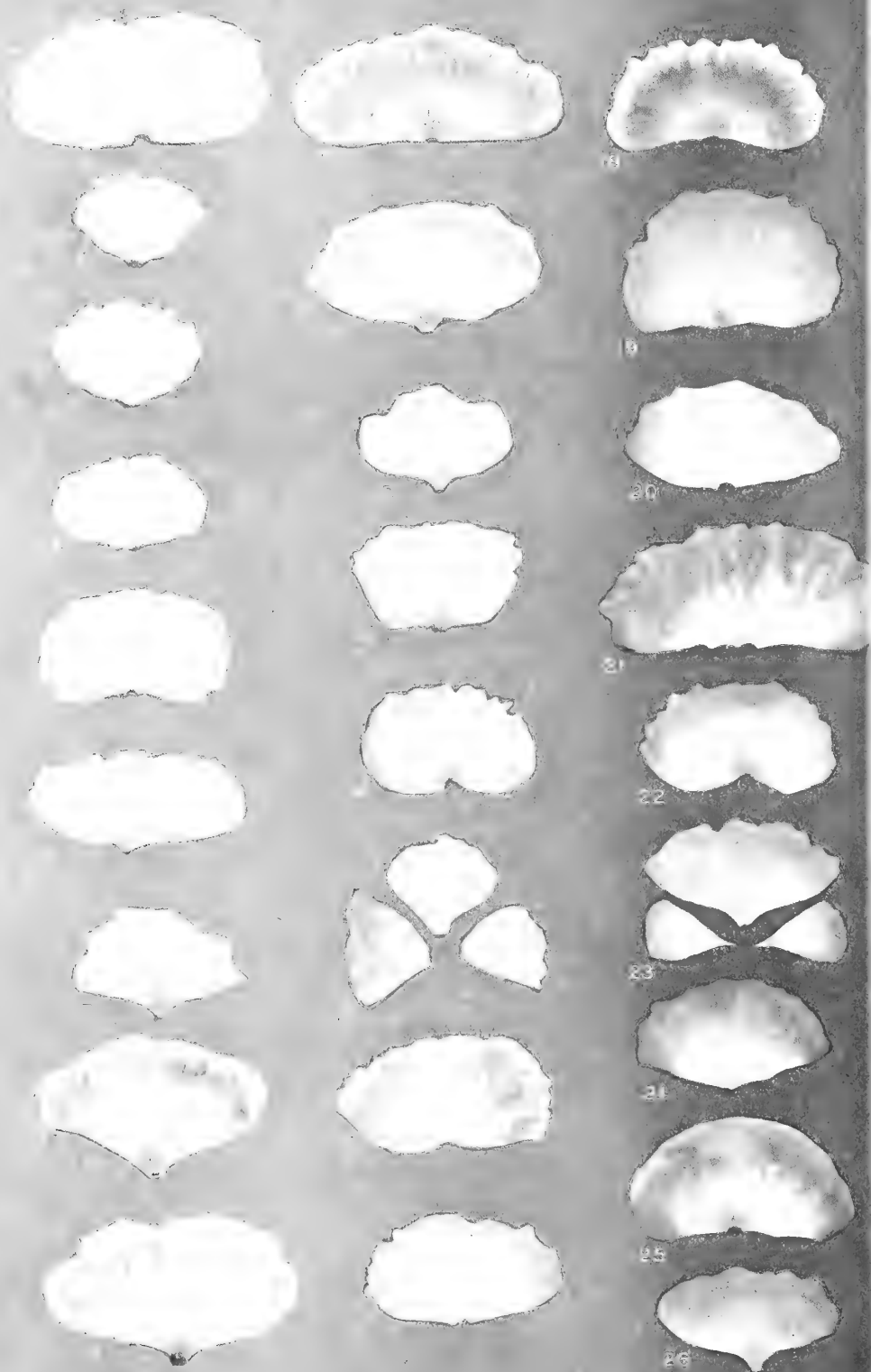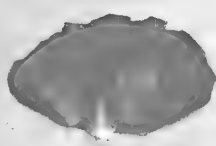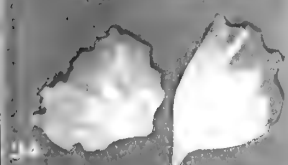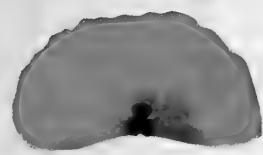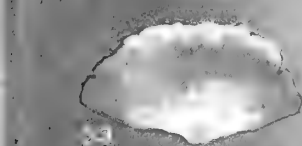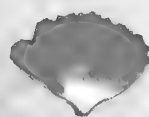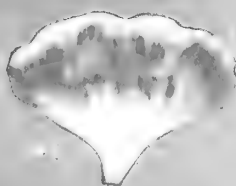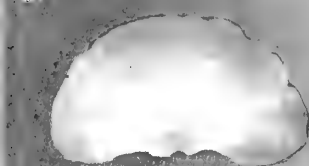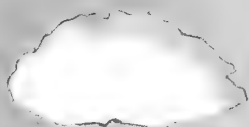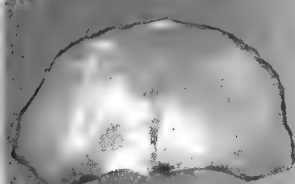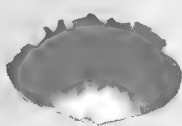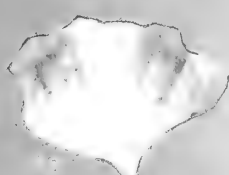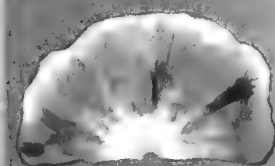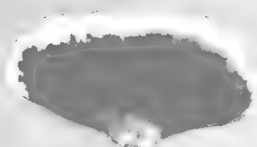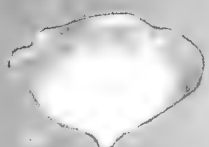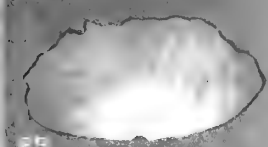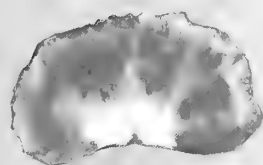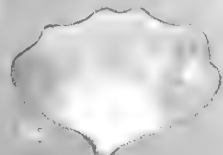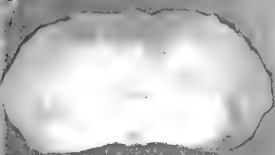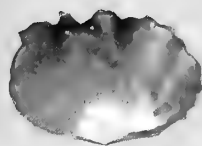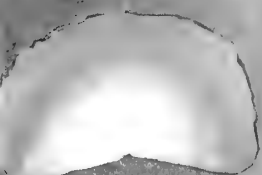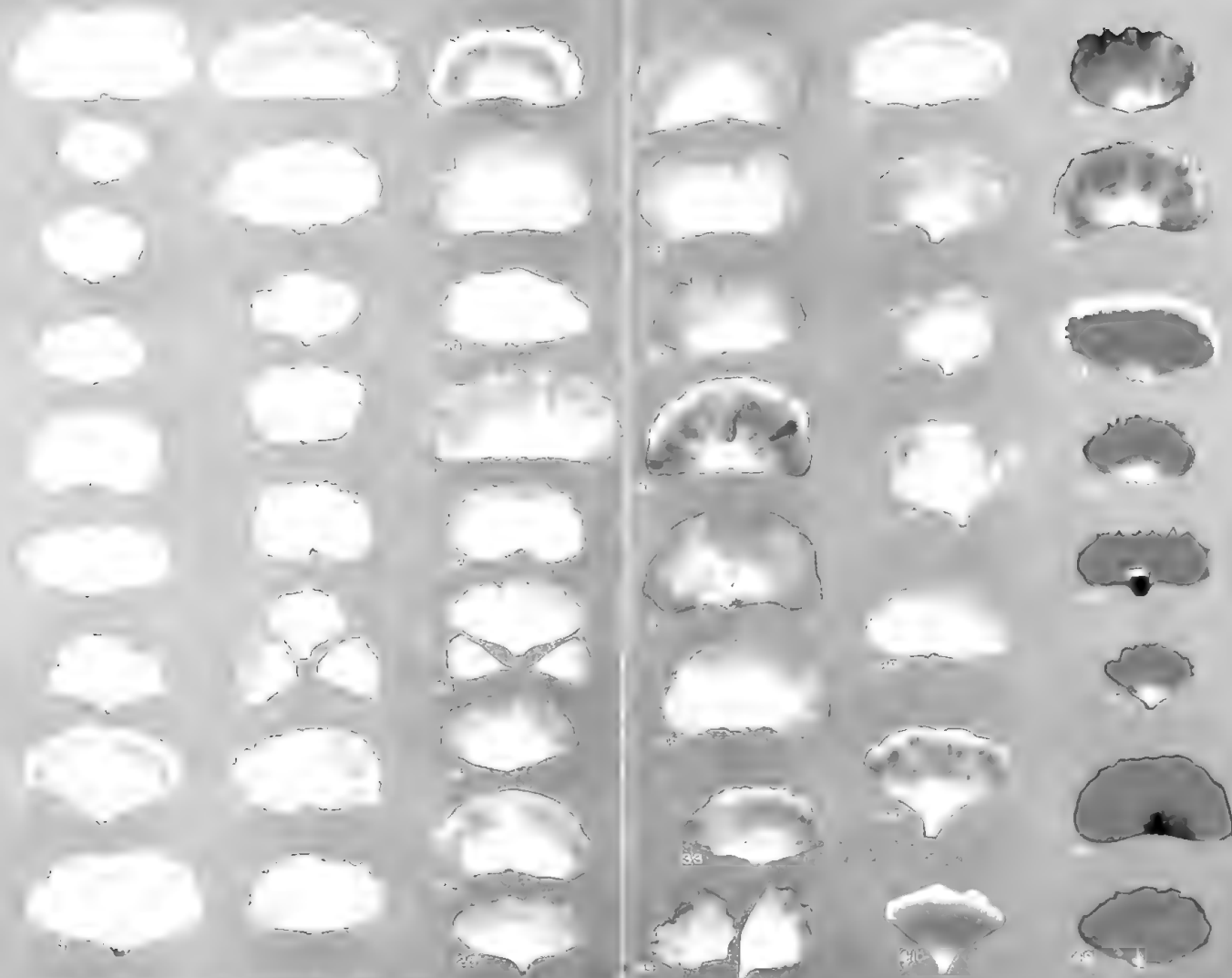
The object of the first series of experiments, like those which have followed it, was to ascertain the intensity of heredity in a free plant *population*, and to compare such heredity with that of similar populations in other types of life. If we have to deal with evolution in actual living forms in which isolation of individuals is not the rule, it is the law of inheritance in *populations* with which we are concerned. We desire to know the statistics of sub-classes, and the proportions in which those sub-classes reproduce themselves. A statistical investigation of this kind cannot be opposed to any ultimate analysis of the result of crossing *individuals*; it can only indeed serve as a criterion of the truth of any hypothesis based upon individual crossings. At the same time it avoids the dangers which arise from neglecting what we may term the "law of small numbers." It is too often forgotten that an imposing display of numbers, obtained by dealing with the numerous offspring of a number of crossings which can be counted on the fingers, must on any hypothesis of parental gametic constancy be given the weight of the number of crossings and not of the number of offspring, when the results are applied to the race as a whole.

In our earlier experiments we were not certain to what extent Shirley Poppies grown in masses were cross or self-fertilised *. This doubt left a double interpretation open when we came to deal with the resemblance of siblings. Our results in the previous investigation were in general accordance with those for man, provided we assumed that the bulk of the plants were *cross-fertilised.*

Another point of considerable importance was the question of whether any really well-defined colour sub-groups could be asserted to exist in the case of the Shirley Poppy. In order to determine this point water-colour sketches of the petals of a long series of the poppies were kindly made for us by Miss Eva Biggs immediately after gathering. It was soon seen that in distribution of colour, amount of colour, and shade of pigmentation there was practical continuity. We found it impossible to draw a rigid line between scarlets, reds, orange-reds, terracottas, rose-pinks, pinks, and pink-whites, although such terminology served fairly well for rough classification. There appeared always to be transitional individuals, although such individuals might be numerically few. The ocular demonstration of this was difficult even in Miss Biggs' paintings, because the colour of the poppy is very difficult to match in water-colours, and being matched is very evanescent. We believe that Miss Biggs' careful work fully achieves what is possible in this direction. But the task becomes harder still when the colour printer has to match the artist's tints. If honest, he frankly tells you that he can approximately catch the artist's shades, but that he can only do so by using non-durable printer's inks. The alternative before the Editors was therefore to produce something like the artist's work which will not be permanent, or something unlike it which will be permanent. The latter seems to fail entirely in its object, for while it might show continuity, it would give colours which we should be told were not even approximately those of the Shirley. We have therefore determined on the second course,

* *Loc. cit.*, p. 62.

namely, to allow the colour-printer to reproduce in possibly evanescent colours, which will show for some time at least the apparent continuity of pigmentation in the Shirley Poppy. It will be seen from our plate that while white-pinks, pinks, rose-pinks, terra-cottas*, and reds are fairly safe broad classifications, there is no absolute discontinuity when we come to deal with individual instances.

In the present experiments the observations were all made on the first or apical flower. They had to be made while the plant was in blossom. The seed of the apical capsule observed was taken and marked with the new pedigree number. A second and sometimes a third bud on the same plant was enclosed in a bag of very fine bolting cloth, or for about half the experiments in oiled paper bags obtained from Cologne. These were invariably used in the second year's experiments. The advantages of the bolting cloth bag are the free admission of air and light, but it becomes heavy in rainy weather and much damage was done in the wet season of 1903. It is possible that very minute insects may occasionally penetrate it, but the risk here is not much greater than with paper bags, however carefully they are tied round the stem. In the spring of 1903 series of 100 packets of pedigree seed were issued to a number of cooperators. Each of these packets contained seed from one *plant* only, and the ultimate origin of the seed was the 1899 Hampden crop of Pearson, although there had been several intervening generations. In the spring of 1904 series of 100 packets of apical capsule seeds of 1903 and of 50 packets of bagged capsule seeds of 1903 were issued, and the same process was followed of collecting the seed of the apical capsules and of a secondary bagged capsule. To test fertility the seed in the free and bagged capsules was merely examined after gathering and not again sown.

The following are the 1903 *crops :*

(A) *Parkstone Crop.* This was grown and observed by H. Blanchard. The crop was sown on May 13th, the first seedlings were observed on the 20th, and the thinning took place between June 13 and 15. Of the 100 packets of single capsule seed provided all produced plants and in most cases more than sufficient plants to get 10 siblings. In one case only one seed and in a second three seeds alone germinated†. But the observer was able to deal with nearly 1000 individual plants. The weather was extremely unfavourable during the harvesting and much harm was done to the bagged capsules. Tabulation and calculations by Alice Lee, D.Sc.

(B) *Arncliffe Crop.* This was grown and observed by the Rev. W. A. Shuffry, and 50 packets of seed were sown on May 30; first seedlings observed June 10, thinned July 13 and 20. Second series of 50 sown June 18, first seed-

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* In the original Hampden Shirley Poppies the terra-cotta was extremely rare, if it occurred at all. It is still scarce, but is creeping up in numbers, and may ultimately need a special class. As meanwhile it is being introduced commercially, we cannot say whether it is a natural evolution or has been introduced by cross-fertilisation into our crops.

† One of these plants also gave a distorted apical capsule with a "stigmatic labyrinth" instead of bands.

lings observed June 25, thinned July 29–30. Out of the earlier sowing, none; out of the later sowing four sets failed to germinate. There were, however, a number of series in which less than ten plants failed to appear or be observed. The lateness of the Dale* season and the badness of the weather were much against this crop. Tabulation and calculation by J. Clerk-Maxwell Garnett.

(C) *Streatham Crop.* This was grown and observed by W. Palin Elderton. Sown on April 30, thinned at end of July. There was no case in which none of the seed of the 95 packets sown failed to germinate, but in some cases very few plants were grown. The weather was very wet during the whole of the flowering season and also very windy; many of the plants were much damaged. The bagged capsules suffered very much, and the seed had to be gathered earlier than was desirable. The tabulation is due to Mary Beeton, and the calculations to Alice Lee and her conjointly.

(D) *Boar's Hill Crop.* This was grown and observed by A. D. Darbishire. Sown April 23–25, first seedlings observed May 7, thinned June 25 to July 1. Out of 42 packets of seed sown at least 40 germinated. But unfortunately the observations were not made on the principal flower, but on the 2nd, 3rd, 4th, and even 5th flowers, so that the material is not comparable with the other observations of this series. Not yet tabled or calculated†.

(E) *Kidderminster Crop.* This was grown and observed by John Notcutt. Seed sown April 17, first seedlings observed April 29, thinned June 25. Of the 100 packets sown all germinated, and there were nearly 1000 plants observed. The tabulation and calculations are by Mary Beeton.

With regard to the seed provided for crops (A) to (E), it must be remarked that in making up the packets fertile capsules were directly chosen and those capsules which provided unfertilised seed deliberately rejected. This method could not be followed in the seed for the 1904 crops, because the harvest showed that the bagged capsules contained apparently no fertile seed. Accordingly the capsules for the seed-sowing in 1904 were taken from the 1903 (A) crop at random, each capsule having been at harvest placed in a bag whether or no it contained fertile seed.

The 1904 *crops were:*

(F) *Crewe Crop.* This was grown and observed by H. E. Ward. Sown May 18, first seedlings observed May 27, thinned June 30. Out of 129 packets of free apical capsule seed 92 germinated. This may be taken to represent the reduction in fertility due to no selection of fertile seed capsules. In 1903 there was practically no failure to germinate. Out of 50 bagged capsule packets 46 failed entirely to germinate, and 4 germinated, giving respectively 5, 2, 1, and 1

* Arncliffe is in Upper Wharfdale.
† It is hoped that enough material may ultimately be accumulated to test the difference between sibling resemblance measured on apical and on other flowers.

actually developed plants; the 92 which germinated of the free capsules gave 400 *actually observed* plants*. The tabulation and calculations are due to Dr A. Lee.

(G) *Bushey Crop.* Grown by H. Payne, but not recorded owing to his appointment to the South African College. The bagged capsules, however, failed to give seed which germinated, while the free capsules gave a very good crop.

(H) *Rushden Crop.* Grown by G. Abbott. Mr Abbott's time was so occupied by other matters during the flowering season that few records were made of the flowers, but the same result occurred in this sowing as at Bushey—the bagged capsules gave no germinating seed, the free capsules a plentiful supply.

We may therefore conclude that while a plentiful supply of fertile seed is obtained when free capsules growing in the midst of a poppy crop are collected, practically no seed is obtained when we deal with bagged capsules, at any rate in seasons like 1903 and 1904. It seems reasonable therefore to infer that the Shirley Poppy is in the bulk cross and not self-fertilised†, and we should accordingly expect in the resemblance of sister plants the intensity to approach what we are familiar with in other types of life as that of *half* and *not whole* siblings. We may remark that the observers noted continual visits of wasps, bees, and other insects to the crops. As far as could be determined from a careful examination of the unused bagged capsules of 1903, they presented no fertile seed whatever.

(2) *Nature of the Observations.*

A full crop consisted of 100 to 150 plots each sown with seed from a single pedigree individual plant. In each plot at a maximum ten and of necessity sometimes fewer individual plants were selected and the characters of their apical flowers observed. The characters were as follows:

(i) *Petals.* (*a*) The number of petals. (*b*) The number of petaloid stamens.

(ii) *Number of stigmatic bands.* These being counted at the periphery and not at the centre.

(iii) *Colour of the "middle third" of the petals.* This was recorded for one petal of the flower only, unless the petals were unlike, when the colours of both petal pairs were given. The colour categories used were:

Red (*r*).	Pink-white (*pw*).	White (*w*).
Red-pink (*rp*).	Veined-white (*vw*).	Red-white (*rw*).
Pink (*p*).		

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* If these beds had not been thinned, and if all the plants in some of the beds had been observed, there would have been far more, i.e. 400 represents the minimum, 9 the maximum fertility of the two groups.

† Another, we consider fairly strong, argument in favour of wide cross-fertilisation can be found in the occurrence of poppies with dark pollen or dark basal patches. Such "rogues" occur where there is no trace of black in the maternal plant, but they generally appear as isolated units; sometimes, but rarely, two or three cases appear in a fraternity. If the whole fraternity were due to one pollen parent, they could hardly fail to be more frequent in the group.

Scarlet was tabled under red, the few terra-cottas and salmon-pink under red-pink, and intermediate colours by intermediate values (e.g. p or pw). If the petal pairs were of different colours, they were recorded as $p \times w$, say, and tabled under intermediate values. Veined-white was used for white faintly pink or for white with distinct pink veins.

(iv) *Breadth of margin.* The classes used were:

Broad (*b*).   Slight (*s*).   None (*n*).

Broad to slight (*bs*).   Slight to none (*sn*).

"None" signifies that the border of the petal was of the same colour as its body or middle-third.

(v) *Colour of the margin, if it existed.* This was only completely noted in the Parkstone crop, and it enabled us to form a classification of the flowers by colour at the border of the petal; this was the same as in the middle-third, if there were no separate margin. Colour scale used as in (iii).

(vi) *Intensity of the wrinkling of the petal.* This was observed some time, 12 to 24 hours, after the opening of the flower. But while frilling is a marked feature of the Shirley, this character was not a satisfactory one to observe, and but little weight can be given to the results. The following classes were adopted:

Very much wrinkled or frilled (f).   Slightly wrinkled (*sw*).

Wrinkled (*w*).   Not wrinkled (*nw*).

(vii) *Base of petal.*

(α) If the petal base differ in colour from the middle-third it may be said to have a basal patch, and the following classification of basal patches was adopted:

None (*n*).   Well defined (*d*).

None to slight (*ns*).   Well defined to large (*dl*).

Slight (*s*).   Large (*l*).

Slight to well defined (*sd*).

(β) If there were a basal patch its colour was noted; white, black and lilac basal patches were recorded.

(viii) The shape of the capsule cover, whether flat or conical, the colour of the pollen, and all abnormalities were further noted in a separate column. Actual shape of capsules was only fully noted in the Crewe crop.

The following are samples of entries from the records:

| Pedigree | Petals | | Bands | Colour | Margin | Wrinkling | Base | | Remarks |
	(*a*)	(*b*)					(*a*)	(β)	
144 30·16 826 23·3 C. F.	4	0	12	*p*	*n*	*sw*	*d*	*w*	Pollen brown

The first number (144) in the first column refers to the original Hampden great-grandmaternal plant, the second (30·16) to the Kidderminster I. grand-maternal capsule, the third (826) to the Parkstone maternal capsule, and the last number is the key number of the Crewe crop. C. F. signifies that it was from a free bud, or 826 capsule was in all probability cross-fertilised.

Again :

Pedigree	Petals (a)	(b)	Bands	Colour	Margin	Wrinkling	Base (a)	(β)	Remarks
149 26·15 D. 47·1	6	3	13	*r*	*b*	*n*	*dl*	*w*	Conical topped capsule

This gives a grand-daughter capsule from Arncliffe of an original Hampden plant. It will be seen that with the crop-books before one it is possible to rapidly look up the pedigree of any plant, although the pollen ancestry must remain unknown.

By observing characters on the apical capsules only we at first considered that we should get rid of the difficulty of having to allow for homotyposis, i.e. the variability of character within the individual plant. This variability, as was pointed out in the first memoir, is very considerable. But as the work went on the question was again raised as to whether the apical flower is as true a measure of individuality as the totality of flowers on the plant. This point has been to some extent discussed in the first memoir on the Shirley*, and is by no means easy to answer. We found in that paper that offspring from seed of all the capsules of a plant had a greater average resemblance to the plant than offspring from seed taken from a single capsule, while the offspring were more like each other when their seed was taken from a single capsule. If the arrangement of buds on a plant were as definite as the digits of a man, we should have no hesitation in comparing the characters of the apical flowers of mother and daughter plants. But this is certainly not the case, and when we deal with characters which are by no means peculiar to the apical flower it becomes a very difficult point to determine how far the variation within the individual plant can or must be allowed for. Characters like "wrinkling," number of petals and number of stigmata have a very considerable variation within the individual. Frequently when the general tendency of the flowers of the plant is to a high number of stigmata, or to an abnormal number of petals, this tendency may fail to be represented on the first flower. Other characters like colour and margin may possibly vary less from flower to flower of the same plant; but are nevertheless sensibly influenced by environment. Lastly, a very considerable selection takes place of the capsules which give any germinating seed. The mother plants which provide germinating seed are far from a random sample of the general Shirley population.

* *Biometrika*, Vol. II. pp. 73, 74, and p. 82.

We have then the following points which must always be borne in mind and which render in our experience the problem of inheritance in plants extremely complex.

(*a*) The wide-reaching effect of a changed environment, whether this be a change of climate, soil, or season.

(*b*) The effect of selection in modifying the intensity of inheritance; the variability of the actual mother plants is for a number of characters far less than the variability in the general population.

(*c*) The difficulty as to the multiplicity of like organs in the individual and the choice as to which of those organs is to represent the individual.

We propose to consider these points in the following sections.

(3) *On the Influence of Environment on the Shirley Characters.*

In the first place let us consider the type and variability in number of stigmatic bands—a character which we can compare with the previous crops. Of course this comparison can only be partial, because in the earlier crops this character was recorded for every flower on the plant, while in 1903–4 only the first or apical flowers were dealt with. Further, for the earlier crops the mean and variability are those found for the homotyposis tables, while in the 1903–4 crops they are taken from the fraternal correlation tables.

TABLE I.

Change of Type and Variability with Environment.

Crop	Mean No. of Stigmata	S. D.	C. of V.
Parkstone, 1903 ...	14·03	1·562	11·13
Enfield I, 1900 ...	13·78	1·518	11·02
Kidderminster II, 1903 ...	13·58	1·880	13·84
Crockham, 1900 ...	13·39	2·055	15·35
Kidderminster I, 1900 ...	13·18	1·702	12·91
Bookham, 1900 ...	13·14	2·330	17·73
Crewe, 1904 ...	13·06	1·687	12·92
Enfield II, 1900 ...	12·72	1·717	13·50
Hampden, 1899 ...	12·61	1·885	14·95
Chelsea, 1899 ...	12·37	1·680	13·58
Arncliffe, 1903 ...	12·12	2·112	17·43
Chelsea II, 1900 ...	12·04	1·694	14·07
Highgate, 1900 ...	12·03	1·890	15·71
Oxford, 1900 ...	11·39	2·173	19·08
Streatham, 1903 ...	11·39	1·507	13·20

Considering that the probable error in these cases is a small quantity in the first or second place of decimals, we reach a very wide range of difference in type and variability occurring in different localities and seasons. We see at once that plants growing under diversities so great as are evidenced above are not directly comparable.

Without laying too much stress on this table we may remark that the mean variation of the first seven entries is 1·82, and of the last eight 1·83. In other

words, it does not appear that the circumstances which lower the mean of the stigmata are the same as those which lower variability. If we classify in order of variability we find the seven most variable stations give a mean of 12·6 bands, and the eight least variable a mean of 12·8 bands, two values, the difference of which is only doubtfully significant. In fact, it will be found that very fertile crops like Kidderminster I., Enfield I., and Parkstone rank in the same class of lesser variability with starveling crops like those of Streatham and Chelsea; while the high variability group embraces low types like those of Highgate, Arncliffe, and Oxford. The high situation, cold and late season of Arncliffe reduced the number of stigmata, but left the plants highly variable, the potting at Oxford produced the same result, but the causes which produced the starveling crop at Streatham were as influential in reducing its variability as in lowering its type. The general inference to be drawn is that both in variability and type the influence of environment, local or seasonal, on plant life is extremely large, and that careful investigations of the correlation of locality, soil, and seasonal meteorological conditions with plant type and variability would give valuable results. Meanwhile we can only adopt methods of studying resemblance, which as far as possible eliminate this influence.

TABLE II.

Percentages of Certain Characters in Different Crops of same Season.

Character	Crop	Percentage
With more than 13 Stigmata	Parkstone	63·0
,, ,, ,, ...	Kidderminster II.	53·3
,, ,, ,, ...	Arncliffe	24·8
,, ,, ,, ...	Streatham	8·8
With more than four Petals or with some petaloid stamens } ...	Parkstone	17·6
,, ,, ,, ,, ...	Kidderminster II.	16·8
,, ,, ,, ,, ...	Arncliffe	16·2
,, ,, ,, ,, ...	Streatham	13·8
Percentages of Flowers with some red ...	Kidderminster II.	35·0
,, ,, ,, ,, ...	Arncliffe	33·8
,, ,, ,, ,, ...	Parkstone	25·5
,, ,, ,, ,, ...	Streatham	25·0
Without margins	Kidderminster II.	72·6
,, ,, ...	Parkstone	71·3
,, ,, ...	Arncliffe	63·9
,, ,, ...	Streatham	61·0
With slight or no basal patches	Streatham	52·4
,, ,, ,, ...	Kidderminster II.	36·4
,, ,, ,, ...	Parkstone	30·7
,, ,, ,, ...	Arncliffe	18·4
With slight or no wrinkling	Parkstone	92·9
,, ,, ,, ...	Kidderminster II.	78·7
,, ,, ,, ...	Streatham	78·6
,, ,, ,, ...	Arncliffe	65·6

Another suggestive manner of examining the influence of environment is to consider the percentage of certain characters observed in the different crops of one and the same season. We first place all the characters together without endeavouring to analyse how far the results may be due to differences of standard in different observers.

Now examining this table we see that although the seed for these four crops was all taken from a single local crop by sampling fertile individuals at random, yet the environment influenced widely the results. This is clear, because in counting the number of stigmata or noting if margins existed there was no room for sensible influence of personal equation. Luxuriant crops like those at Parkstone and Kidderminster had more stigmata, petals, and petaloid stamens than a poor crop like that at Arncliffe, or a starveling one like that at Streatham. Further we see that nearly the same order is preserved for the percentages without margins, or we suspect that poverty in a crop increases the number of individuals in whom the pigment does not reach the edge of the petal. With the other characters the results are not so congruous. Arncliffe and Streatham, which agree in having petals well margined, are different in basal patch frequency, Streatham showing more flowers without basal patch than any other crop and Arncliffe fewer. In wrinkling the results are not very definite, but indicate that the more luxuriant crops have less wrinkling ; while reds appear to be favoured by the bleaker north. The chief point to be noticed is that there is immense diversity from one crop to a second, and that this diversity is hardly greater in characters like colour and base, where personal equation may be influential, than it is in stigmata and number of petals, where it must be of small or no account. All the above percentages have been deduced from the results weighted with the number of sister plants, so as to bring out the influence of change of environment on the results we have to work with in measuring sibling resemblance. These great differences between local crops of the same season forcibly suggest that individual plant-environment within the local crop, depending on subtle and inappreciable differences in soil, aspect, and amount of moisture, will in the case of material thus susceptible produce very sensible differentiations. We believe that a good deal of the irregularity, and possibly the lessened intensity of the inheritance coefficients for plant life, is due to this marked susceptibility of plants to small differences of environment. But until crops can be grown year by year in the same place, and under nearly similar conditions, it will be impossible to measure these conditions, or accurately determine their influence on the intensity of inheritance.

(4) *The Influence of Selection in Modifying the Intensity of Inheritance.*

If ρ be the actually observed correlation, and r the correlation if all members of a race produce offspring, then it is well known that any selection of parents, natural or artificial, tends to reduce ρ below the value of r. If the selection be stringent, then ρ may be so much lower than r that it may appear to have no relation to it at all, and fallacious conclusions be drawn as to the variability of inheritance for different species or for different characters in the same species,

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the real fact of the case being that these species or characters are being subjected to very different intensities of selection.

If the distribution of the character be approximately normal, and if μ be the ratio of the variability of selected parents to the variability of the general population for a given character, then it is known that the true value of inheritance r is closely given by:

$$r = \frac{\rho}{\sqrt{1 - (1 - \mu^2)(1 - \rho^2)}}.$$

Thus with anything like stringency of selection ρ may be quite small, and the value of r completely screened. We may illustrate this on various characters.

(*a*) *Stigmata.* In the case of a crop under very favourable conditions, or where every plant is separately grown as at Oxford, we have a high variability, amounting to 2 or over. But at Parkstone the actual variability was only 1·56, and on a random selection of 129 seed capsules from this crop only 92 sets germinated at Crewe, and of these sets the maternal plants had a stigmatic band variability of 1·4590. Thus $\mu = ·7295$ for the stringency of selection of the maternal plants at Crewe, as compared with Shirley Poppies at large.

(*b*) *Extent of Margin.* If we take the 971 plants at Parkstone and the 408 offspring at Crewe from certain of them we note the following distributions (see p. 417):

	b	bs	s	sn	n	Totals
Parkstone General Population ...	52	73	84	71	691	971
Parkstone Mothers of Crewe Daughters ...	1	58	64	10	275	408

Or, dividing into three groups, we have the percentages:

	Broad Margins	Slight Margins	Insignificant
General Population ...	12·9	8·7	78·4
Mother Plants ...	14·5	15·7	69·8

We see that there has been a considerable alteration in the distribution of margins, between the actual mothers and the general population from which they were selected at random. Let us treat the distribution of margin as approximately normal to get some idea of the stringency of selection. In this case if d be the range of margins classed by this *one* observer as slight we find, if σ be the standard deviation of the general Parkstone population and σ' that of the selected mothers:

$$d = ·3440\sigma \text{ and } d = ·5497\sigma'.$$

Hence $$\mu = \sigma'/\sigma = ·62589.$$

Thus a fairly stringent selection of margins took place between the Parkstone general population and those members of it who actually provided germinating seed, and so daughter-plants at Crewe. But this is not the whole selection. Parkstone has for stigmatic bands a low variability, as compared with what we know of Shirleys in general. It is extremely unlikely that it has not also a reduced variability in extent of margin. How much, however, it is impossible to say, for we cannot compare Mr Blanchard's estimate of what is classed as " slight " in margin with estimates on other crops by other observers. We have no knowledge of the relative personal equations. Thus the above stringency of selection can only be considered as a minimum. μ in reality is probably considerably lower.

(c) *Number of Petals.* If we consider number of petals we find :

	Parkstone General Population	Parkstone Mothers of Crewe Crop	Kidderminster II. General Population
4 Petals	758	333	798
4 Petals and some petaloid stamens	89	21	47
More than 4 Petals with and without petaloid stamens	125	41	122
	972	395	967

Or, reduced to percentages :

4 Petals	78·0	84·3	82·5
4 Petals and some petaloid stamens	9·2	5·3	4·9
More than 4 Petals with and without petaloid stamens	12·8	10·4	12·6

Taking d as the range from normal four petals to the end of the range of normal four petals accompanied by petaloid stamens we have, if σ_P, σ', and σ_K be the respective standard deviations :

$$d = ·3614\sigma_P = ·2613\sigma' = ·2090\sigma_K.$$

We see that in this case the Parkstone maternal plants are an "inverse" selection of the general population, being more variable than the general Parkstone crop, but they are considerably less variable than a typical crop like the Kidderminster. Thus on the general population of Shirleys there is a stringency of selection in the maternal plants of the Crewe crop at least equal to $\mu = ·8$ and possibly more intense. At Streatham even the percentages being 86·1, 2·3, and 11·6 respectively, we have $d = ·1103\sigma_S$, and the stringency of selection of the Crewe maternal plants would give μ as low as ·53. But there are all degrees of petaloid stamens, and all observers may not have noted equally every slight deviation from normality in the stamens of their flowers.

(d)   *Colour of Middle Third of Petal.*   We have the following results :

		Parkstone General Population	Parkstone Mothers of Crewe Crop
Some red in Petal*		240	153
All Pink	...	481	161
Some White	...	252	94
		973	408

Or, in percentages :

Some red in Petal		24·7	37·5
All Pink	...	49·4	39·5
Some White	...	25·9	23·0

Taking the all pink as our reference group we have

$$d = 1\cdot0562\sigma' = 1\cdot3315\sigma,$$

or

$$\mu = \sigma'/\sigma = 1\cdot2606.$$

In other words, there has been such a selection of red mothers that the mothers are *more variable* than the general population at Parkstone. The mean poppy, which in the general population is about half-way † up the pink range, has in the maternal plants crept nearly up to 1/8 from the beginning of the red group‡.

If we turn to the Crewe offspring we find the following values :

Mothers			Daughter Plants	
153	37·5 p.c.		139	34·1 p.c.
161	39·5 p.c.		199	48·8 p.c.
94	23·0 p.c.		70	17·1 p.c.
408			408	

Thus we see regression towards the pinks. In terms of d the pink range we have for σ'' the standard deviation of offspring

$$\sigma'' \times 1\cdot3586 = d,$$

or $\sigma/\sigma'' = 1\cdot02$ nearly. Thus the variability of the general population at Crewe is sensibly the same as that of the general population at Parkstone, but the mothers who provided germinating seed at Crewe were redder than the average Parkstone poppy, and the resulting offspring, though still regressing slightly towards the red, are yet about $\frac{1}{8}$ off the boundary of the red §, instead of half-way across the pink range as in the Parkstone crop.

The exact interpretation of this inverted selection is difficult, and has not yet been theoretically worked out. The formula on p. 404 involves approximate normality of the distribution, and becomes meaningless when the selection curve does not theoretically fall *wholly* inside that of the population from which selection takes place. The distributions of colour percentages at Kidderminster and Arncliffe are :

		Some Red	All Pink	Some White
Kidderminster	...	34·5	40·0	25·5
Arncliffe	...	33·8	40·4	25·8

* The two groups include under red 14 and 2 flowers respectively, which were noted to have a certain amount of white with the red or red-pink.

† ·6851 to 1·3315.　　　　　‡ ·3186 to 1·0562.　　　　　§ ·4106 to 1·3586.

and thus much alike, and very close to the Parkstone mothers of the Crewe crop. It seems therefore impossible to assert that the maternal plants at Crewe, although certainly an "inverse" selection from the Parkstone crop, are really a selection from the general population of Shirleys. We can only consider them as an average sample.

(e) *The Basal Patch.* With the sole exception of wrinkling the basal patch is the most unsatisfactory of the characters observed. Not only do the flowers showing no basal patch vary very much in percentage with the environment, but there is undoubtedly a considerable amount of divergence in the estimations made by different observers of the categories. We have the following results :

	Parkstone General Population	Parkstone Mothers of Crewe Crop	Crewe Crop
$\left.\begin{array}{c} n \\ ns \\ s \end{array}\right\}$ =Slight or none	295	134	53
$\left.\begin{array}{c} sd \\ d \end{array}\right\}$ =Definite ...	342	128	214
$\left.\begin{array}{c} dl \\ l \end{array}\right\}$ =Large ...	332	137	132
	969	399	399

Or, in percentages :

	Parkstone General Population	Parkstone Mothers of Crewe Crop	Crewe Crop
Slight or none	30·4	33·6	13·3
Definite ...	35·3	32·1	53·6
Large ...	34·3	34·3	33·1

Thus we see that while the maternal plants of the Crewe crop were a quite reasonably random sample of the Parkstone population, the distribution of Crewe offspring is totally different. How far is this due to environment, and how far to personal equation? The corresponding percentages at Kidderminster, Arncliffe, and Streatham are:

	Kidderminster General Population	Arncliffe General Population	Streatham General Population
Slight or none	36·5	18·4	52·4
Definite ...	50·4	47·7	30·9
Large ...	13·1	33·9	16·7

If these greater differences are not due to personal equation, which series can be taken as a typical crop of Shirleys? We confess that there does not seem to us any means of separating out the environmental influence from the personal equation in this case. All we can say is that there is no evidence that the maternal plants at Crewe were a selection from the general population of Shirleys, and if they were, we have no means of measuring the stringency of that selection.

(f) *Wrinkling of Petals.* This, which is a marked feature of the beauty of the Shirley, is the most unsuccessful of all the characters recorded. The instruction to observe it between 12 and 24 hours after opening of the bud could not always

be carried out, and there is little doubt that it is not only a feature much modified by season and environment but even by the interval between opening of the bud and the time of record.

The following table gives our results:

		Parkstone General Population	Parkstone Mothers of Crewe Crop	Crewe General Population
Frilled and Wrinkled	...	70	12	137
Slight Wrinkling	...	346	112	234
No Wrinkling	...	553	275	28
		969	399	399

Or, as percentages:

Frilled and Wrinkled	...	7·2	3·0	34·3
Slight Wrinkling	...	35·7	28·1	58·7
No Wrinkling	...	57·1	68·9	7·0

Now it will be clear that while the maternal plants of the Crewe crop are a selected group of the Parkstone crop, yet there is no relation between the percentages in these mothers and in the offspring. The 70 per cent. of not wrinkled maternal plants actually produced four times as many wrinkled and eight times as many slightly wrinkled as not wrinkled plants, and the 3 per cent. wrinkled produced twice as many not wrinkled and slightly wrinkled as wrinkled plants![*] In fact while the Parkstone crop had more non-wrinkled poppies than any other and the maternal selection 12 per cent. more than the general Parkstone population, the Crewe offspring had only 7·0 per cent. of non-wrinkled poppies, less than half that shown by the most wrinkled crop, Arncliffe, which showed about 15 per cent. Whatever changes took place in environment and season between Parkstone and Crewe they were sufficient to totally upset wrinkling as an inherited character! Some possibly, but far from all, of this difference we believe may be due to personal equation. The main feature to be noticed is that whereas the southern crops, Parkstone and Streatham, have more than 45 per cent. of unwrinkled flowers, the northern crops have less than 17 per cent. (Arncliffe 14·9, Kidderminster 16·4), sinking to 7 per cent. at Crewe.

If we confine our attention to the Parkstone crop and take d for the range of slightly wrinkled we have:

$$d = 1·3860 \times \sigma' = 1·2812\,\sigma,$$

Or the stringency of maternal selection

$$= \mu = \sigma'/\sigma = ·9244.$$

If we take the Oxford crop of 1900 as one in which the wrinkling was very carefully observed, we find $d = 1·1946\,\sigma$, or we have $\mu = \sigma'/\sigma = ·8620$; so that it is quite possible that the stringency may be greater than that indicated by the Parkstone crop.

[*] See Appendix I., Table F.

(5) *The Influence of Variability of like Organs in the Plant on Coefficients of Heredity. Homotyposis.*

We have already seen that if all the flowers of a plant were identical, or if the first flower were as markedly differentiated from other flowers as one digit in a man is from a second (i.e. if there were a very marked correlation between order of flowering and character), the problem of inheritance in plants would be much simpler. But the variation of like organs in a plant is very considerable, and in many cases the degree of variation is not highly correlated with the position or order of production. Now if there were no such correlation, it has been shown elsewhere that the extent of the variation of the individual plant—its homotyposis—would be properly measured by the coefficient of correlation of like organs. Hence our selection for record of apical flowers will mean that we get closer to the true values of the inheritance coefficients than by using all or any flowers on the plant, provided 1st there be marked differentiation of the apical flower from other flowers (i.e. if the first flowers of two related plants are much more alike than the first flower of one and any other flower of the second), or again 2nd there be no variation of the character observed in the plant at all. For example, colour and margin in the broad classification of our records do not exhibit much variation in the plant. Number of stigmata, size of local patch and wrinkling, do exhibit much variation, and further this variation is by no means perfectly correlated with position or order of budding. In order therefore to allow for the neglected homotyposis, we ought to have obtained measurements of (i) the correlation between the order of flowering and the characters observed, and (ii) the homotyposis of these characters. The first would have enabled us to measure the differentiation of the first or apical flower from other flowers, and by the aid of the second result we could have made allowance for the neglected homotyposis.

But to record some half-dozen characters on all the flowers of each plant would have multiplied the work of recording tenfold, and the task seemed at present beyond the volunteer labour, which was all that was available. Accordingly we must simply state that our inheritance coefficients are too low because we have at present no adequate measure of the differentiation of the apical flowers, nor of the homotyposis of most of the characters dealt with. But it is satisfactory to note that those characters in which mere inspection has shown least differentiation with position and most variation in the individual, such as number of stigmata, size of basal patch and wrinkling, are precisely those in which the coefficients of inheritance come out lowest. We hope that investigations into character differentiations with position and into homotyposis will be started this season.

Meanwhile the results in sections (3) to (5) will, we hope, suffice to indicate to the reader that in our opinion the study of inheritance in plants is not a simple, straightforward matter. We have not in Shirleys at any rate found the definite and exclusive categories which the Mendelians appear to encounter at every turn. We have found our characters widely modified by environment and season ; we have found selection acting on the seed sown, and both type and variability

susceptible of much alteration. Lastly, although we have classified each character into more sub-groups than we have met with in Mendelian writings, we find many cases in which the category is hard to determine, and we are convinced that personal equation must be given its due weight.

(6) *Parental Correlation.*

As we stated at the beginning of this paper, we intended to base the 1903 experiments on examinations of apical flowers only. We were not therefore able to compare in the bulk of characters the crops of 1903 with the maternal plants. Ample material should, however, have been provided for parental correlation in the three crops of 1904. Unfortunately two of our observers were unable to complete their tasks. We are therefore thrown back on a comparison of the Crewe daughter plants with the Parkstone maternal plants. The fact that the Parkstone and Crewe crops were grown in very different environments is against very good results, but the number of bagged capsules at Parkstone and their comparatively good state led to our selecting that crop as the chief source for the seed supply of 1903. The tables are formed for the apical flowers of the maternal plants at Parkstone and the daughter plants at Crewe, and this was done with six characters. The work of tabling and reduction is due to Dr A. Lee. The coefficients of correlation, except in the case of the stigmata where the method of moments was used, were found by the fourfold table method. We should, if the investigations were now to be started *de novo*, probably use the method of mean square contingency, but it was considered desirable to preserve as far as possible uniformity of treatment throughout the whole round of poppy investigations, and the fourfold table method had been adopted in the first memoir. We refer briefly to the characters dealt with.

Stigmata. There was probably a stringency of selection here of the maternal plants of about ·7295. It is an open question to what extent the homotyposis of about ·5 ought also to be allowed for, at any rate the value found for parental inheritance is a minimum value.

Extent of Margin. This was determined by a fourfold table, the divisions being between 'no' and 'some' margins. The factor of selection of the maternal plants is here ·6259. The homotypic influence is, as in the case of colour, probably very small, the flowers on a plant being usually all whole coloured or all margined.

Number of Petals. This was determined by a fourfold table, division into normal four petals and non-normal flowers. There was inverse selection of maternal plants compared to the general Parkstone population, i.e. they were more variable than that population; but if we compare the maternal group with the Streatham crop as a Shirley population of maximum petal variability there would have been a stringency of selection of ·5278. It is not clear, however, how great a selection we ought to allow for, because the observers may not have been equally definite as to what they reckoned as an incipient petaloid

stamen. Further, we have at present no measure of the homotyposis of plants in relation to supernumerary petals and petaloid stamens.

Colour of Middle Third of Petal. This was worked by a fourfold table, the division being into poppies showing some and those showing no red. We cannot do more than consider the maternal plants as an average random selection of Shirleys (see p. 406). In the matter of body colour, within the classifications red, pink, pink-white, etc., there appears to be small variation within the plant, or we may look upon this broad colour classification as individual to the plant and not to the bud.

The Basal Patch. This was dealt with by a fourfold table, the division being, none to slight, and slight-definite to large. No evidence is available as to whether there was or was not a selection of maternal plants. The extent of basal patch varies considerably on flowers of the same plant, and is probably a good deal influenced by personal equation and homotyposis.

Wrinkling. The division here was by a fourfold table. Mr Blanchard's estimates of wrinkling were so low as compared with Mr Ward's that the division of the maternal plants was made into not wrinkled and some wrinkling, while the Crewe population was divided into wrinkled and slight or no wrinkling. There may be a selection stringency of ·8 to ·9, and probably normal homotyposis, as wrinkling varies considerably on the same plant. But we have no definite knowledge on these points, and we consider wrinkling, although very characteristic of the Shirley, as very unsuitable for record.

We sum up our results in the following table:

TABLE III.

Parental Correlation, Parkstone Maternal and Crewe Daughter Plants.

Character	Raw Correlation	Selection Stringency	Corrected Correlation
Stigmata...	·1717	·7295	·2324
Extent of Margin ...	·3175	·6259	·4718
Number of Petals ...	·2740	·5278 (?)	·4750 (?)
Colour of Middle Third	·4008	0	·4008
Basal Patch	·1305	·6900 (??)	·1864 (?)
Wrinkling	·1595	·8620 (??)	·1842 (?)
Mean	·24	·69	·33

We have not the least hesitation in admitting the full inconclusiveness of these results. All we consider that they show is, that if we could accurately allow for selection of parentage, for the variability of character in a plant with its multiplicity of organs, and for the extreme response to seasonal and environmental changes, and, further, eliminate the effects of personal equation, there is no reason to suppose that the parental resemblances for different characters would

not group themselves between ·4 and ·5, the values found for characters in other types of life. We do not believe that experiments on plant heredity, which employ no measurement and allowance for these modifying factors, can lead to conclusive results. In our opinion these factors are all potent, and modify widely the limits of any category which may be selected for classification of simple characters in plants.

(7) *On Collateral Heredity.*

The following table gives the results obtained for the 1903 crops with a comparison of such characters as are comparable in the Oxford crop of 1900. In all cases the apical capsules in the sibling plants are compared. In the Appendix are given the actual tables from which the results are deduced. It is desirable to point out the methods adopted in the individual cases:

(*a*) *Resemblance in number of Stigmatic Bands.* Found by usual correlation table.

(*b*) *Number of Petals.* Fourfold table, the grouping being into the normal four petals and the abnormal more than four petals, including in the latter class semi-double, double flowers, and flowers with one or more petaloid stamens.

(*c*) *Colour of Middle Third.* Fourfold table, the grouping being into flowers showing some and flowers showing no red.

(*d*) *Breadth of Margin.* Fourfold table, the grouping being into flowers showing some and flowers showing no margin.

(*e*) *Colour of Flower at Margin of Petal.* Fourfold table, the grouping being into flowers showing some and flowers showing no white there.

(*f*) *Extent of Basal Patch.* Fourfold table, the divisions being into none or slight, and large or definite patches.

(*g*) *Wrinkling.* Fourfold table, the divisions being into marked wrinkling ('frilled' or 'wrinkled') and insignificant wrinkling (slight or no wrinkling)*.

Several points are brought out by this table. In the first place we see that the starveling Streatham crop shows a great reduction in the fraternal correlations. Now if we endeavour to interpret this result we should reasonably suppose that much selection took place at Streatham; the mean of the stigmatic bands and their variability has been much lowered. There is further no reason for supposing a correlated selection in brother plants. Now if we suppose two organs A and B with standard deviations σ_1 and σ_2 and correlation r_{12}, to be selected in a non-correlated manner so that their standard deviations become s_1 and s_2, it is easy to show, on the hypothesis of nearly normal distribution, that the correlation after selection will be

$$\rho_{12} = \frac{s_1 s_2 \, r_{12}}{\sigma_1 \sigma_2 \sqrt{1 - r_{12}^2 \left(1 - \frac{s_1^2}{\sigma_1^2}\right)} \sqrt{1 - r_{12}^2 \left(1 - \frac{s_2^2}{\sigma_2^2}\right)}} \quad \dots\dots\dots\dots \text{(i)},$$

and this independently of changes in the means.

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* Shape of the capsules, for which only data at Crewe were available, gave an uncorrected correlation between sister plants of ·15. See Table VV. Appendix.

TABLE IV.

Fraternal Correlation, Record of Apical Flowers.

| Character | Crop | | | | | Mean |
	Oxford, 1900	Kidderminster II, 1903	Arncliffe, 1903	Parkstone, 1903	Streatham, 1903	
Stigmata	·26	·33	·38	·14	·16	·25
Colour of Middle Third...	·34	·43	·38	·41	·31	·38
Breadth of Margin ...	·19	·29	·21	·15	·14	·19
Number of Petals ...	·23	·25	·16	·28	·17	·22
Extent of Basal Patch ...	·22	·24	·16	·18	·06	·17
Colour at Margin ...	—	—	—	·28	—	·28
Wrinkling	·21	·16	·12	·11	·06	·13
Petal Length	·28	—	—	—	—	·28
Mean	·24	·28	·23	·22	·15	·24

Accordingly, applying this to the case of two brothers, with a standard deviation in their generation of s, and an unselected standard deviation σ, we conclude that if a generation of any population be subjected to a selection of stringency $s/\sigma = \mu$, the correlation of brothers r will be reduced to a correlation ρ where

$$\rho = \frac{r\mu^2}{1 - r^2 + r^2\mu^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(ii)}.$$

Now if $r = \frac{1}{4}$ be the true correlation of half siblings, what must be the intensity of selection μ required to reduce this to $\rho = \cdot 15$?

Substituting, we at once find $\mu = \cdot 7645$.

Now can we suppose that a selection of this stringency has been at work at Streatham? It would involve a reduction of about 25 per cent. in the variability below the normal variability of the Shirley Poppy. Now the only data we can judge from are the first three crops which give a mean fraternal correlation of ·25; and the only character for which we know their absolute variability is their number of stigmatic bands. These give a mean variability of 2·055, which is practically equal to that in the general Shirley population. Accordingly the selected variability of the Streatham poppies ought to be $2\cdot055 \times \cdot7645 = 1\cdot571$. The actually observed value is 1·507. In like manner the Parkstone crop which shows the next greatest reduction in the fraternal correlation has for its reduced stigmatic band correlation ·14, which should mark a reduction in stigmatic band variability from about 2 to 1·57, the actual value observed being 1·56. Without laying too much stress on isolated accordance in a very irregular table of this kind we may reasonably conclude that the great apparent reduction in the Streatham correlations, and to a less extent the reduction in the Parkstone, can

most probably be accounted for on the basis of selection, such selection really reducing the correlations in a manner qualitatively and, as far as can be roughly determined, quantitatively in the manner observed.

In the next place, we have here made no allowance for possible selection of maternal plants and none for homotyposis. Hence the value of our fraternal correlation is again a minimum value. Even including the Streatham results the average correlation for all characters is ·23 and without these it is ·26. The chief irregularities are in the values of the correlations for wrinkling, extent of basal patch and margin, where we should expect minute differences of soil or situation, time of observation or personal equation to be more influential*. Now we might at first argue that ·25 would be a satisfactory value to reach for fraternal heredity, as being about *half* the value noted for fraternal correlation of whole brothers, but there are three points to be considered here. (i.) The neglect of any correction for selection, whether of maternal plants or of surviving offspring, has certainly given too low a value to these correlations. (ii.) There is a further correction to be made for the homotyposis which remains after allowing for differentiation of the first flower. This is probably not substantial in some of the characters, but it is considerable in others. It would again tend to raise the observed values. (iii.) In considering the result we have assumed that daughter plants grown from the same capsules are *half* sisters, or that the pollen parents are different for each pair of cases. We have already indicated (p. 398) that we consider that there is a fairly wide cross-fertilisation, i.e. a considerable variety of pollen parents to plants grown from the same capsule, but it is hardly likely to be complete for each pair of sister plants, and will probably vary considerably from capsule to capsule. Thus half-sibling correlation is rather a minimum limit to what we might anticipate, and not what is actually likely to occur.

Hence the apparent reasonableness of a value about ·25 is seen to be rather fortuitous.

Lastly, granted that the sister plants are in the bulk half-sisters, and that the average value of the correlation of whole sisters is about ·5, it appears open to considerable questioning whether we ought really to expect that the half-sibling resemblance will be just *half* the whole-sibling resemblance. This may appear to some to flow at once from *à priori* considerations, but we do not feel so clear on this point now as a few years ago ; for the determinations of whole- and half-sibling relationships which have been made so far do not bear it out†.

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* Experiments at University College on personal equation show a continuous change during a long series of observations. Judgment would thus depend on the time of flowering, and brother plants not being registered usually on the same occasion, it follows that personal equation would be likely to introduce a small spurious correlation between non-brothers, rather than to emphasise the degree of resemblance between brothers.

† As in the case of all vital problems, however, the complexity is very great. Horse and cattle breeders usually inbreed to some extent, and it very often happens that two half-siblings have more than four common great-grandparents, and so should be more alike than normal half-siblings with only four in common.

The following table gives a summary of the results hitherto reached: we have included the Shirley as if the sister plants were true half-siblings:

Half-Sibling Resemblance.

Thoroughbred Horse, through dam (3 series)	·36
Shorthorn Cattle, through sire and dam (6 series) ...	·32
Basset Hound, through dam (1 series)	·22
Shirley Poppy, Co-ovarial Plants. Colour (5 series) ...	·38
„ „ „ All characters (32 series)	·24

In our opinion the horse and the shorthorn are the most reliable of these series, and in both cases the values are sensibly larger than half the whole sibling values. The Shirley when we deal with all the characters is fairly close to ·25 and the Basset Hound value. But the colour resemblance in the Shirley, which we consider least affected by homotyposis and selection, is in closer agreement to what we find for horses and cattle. We feel therefore that differential local environment, maternal and offspring selection, varying personal equation in estimating plant characters, and the complete absence at present of any quantitative allowance for positional differentiation and homotyposis, prevent us from saying more than that the results so far obtained for fraternal correlation in Shirleys are not inconsistent with those found for animals.

Much enlightenment would be reached could we deal with definite pollen parents in very large numbers. Further what is badly needed for our general guidance at the present time is a measure of the relative intensity of whole and half-sibling resemblance in some quantitatively determinable character. Possibly the record of the cephalic indices of whole and half brothers in public primary schools would be a comparatively easy method of reaching a definite result.

(8) *General Conclusions.*

(*a*) We consider that in colour distribution and tint as well as in other characters there is continuity in the variation of the Shirley Poppy.

(*b*) The Shirley Poppy under ordinary conditions of environment and season is cross-fertilised and probably there is a very considerable variety of pollen parent to sister plants grown from the seed of one capsule.

(*c*) The very great influence of environment upon nearly all the characters dealt with, the variation of character within the same plants, and the unknown allowance to be made for homotyposis, even when we confine our attention to apical flowers, render the problem of plant heredity of very great complexity.

(*d*) As suitable measurable characters in a plant are extremely few, one is compelled to select qualitative characters; and their description, especially when up to the present no scale of patterns has been available, is difficult, and liable to considerable modification by the influence of personal equation.

The experience of the present series of observations by no means accords with that of those observers who find it easy to formulate simple alternative categories

for plant characters, and group without apparent hesitation individuals into one or other category.

(e) The *minimum limit* to parental correlation has the average value ·33. In the best class of characters, however, it lies between ·4 and ·48, and we believe if the disturbing factors could be fully allowed for it would be found to lie between ·45 and ·50, its value as determined for other types of life.

(f) The *minimum limit* to fraternal correlation has the average value ·24. This allows for no selection of maternal plants. The real value is undoubtedly higher, and is quite consistent with half-sibling resemblance as found for horses and cattle, which it closely approaches in the case of colour, where the influence of selection and of variation in the plant appears to be least.

(g) Although very considerable differences exist in the values found for the correlations at different centres and for different characters, we do not think these differences are sufficient to justify any assertion that the intensity of heredity varies with character or with environment. They point rather to the great difficulty of dealing with plant life. What is urgently needed is a biometric farm where secular experiments could be conducted under nearly uniform conditions; there a continuous record of all flowers of each plant could be made by one and the same observer using a standard qualitative scale of groups.

APPENDIX. CORRELATION TABLES *.

I. RESEMBLANCE OF DAUGHTER AND MOTHER PLANTS. DAUGHTER PLANT GROWN FROM SEED TAKEN FROM APICAL CAPSULE OF MOTHER PLANT.

A. *Parkstone and Crewe. Apical Flowers.*

Parkstone Mother Plant. Number of Stigmata.

		10	11	12	13	14	15	16	17	18	19	Totals
Crewe Daughter Plant. Number of Stigmata.	9	—	—	—	1	1	—	—	—	—	—	2
	10	1	4	4	6	3	4	—	—	—	—	22
	11	—	6	6	5	12	7	1	—	—	—	37
	12	—	4	13	20	26	13	2	—	—	—	78
	13	1	10	16	23	29	14	2	3	—	—	98
	14	—	3	15	18	22	13	2	3	2	—	78
	15	—	1	6	13	15	11	3	2	1	—	52
	16	—	4	2	1	10	6	2	3	—	—	28
	17	—	1	—	—	2	1	1	—	—	—	5
	18	—	1	—	1	—	—	—	—	—	—	2
	19	—	—	—	—	2	—	—	—	—	—	2
	Totals	2	34	62	88	122	69	13	11	3	0	404

* There is not absolute uniformity in the classification categories of all these tables. Thus one observer may have used "wrinkled" entirely and never the extreme "frilled," while a second may have placed the few "veined whites" in a category denoting white with very little pink.

B. *Parkstone and Crewe. Apical Flowers.*

Parkstone Mother Plant. Number of Petals and Petaloid Stamens.

Crewe Daughter Plant. Number of Petals and Petaloid Stamens.	4 Petals	4 Petals + Petaloid Stamens	5 Petals or more without and with Petaloid Stamens	Totals
4 Petals	261	12	26	299
4 Petals + Petaloid Stamens ...	54	7	14	75
5 or more Petals without and with Petaloid Stamens	18	2	1	21
Totals	333	21	41	395

C. *Parkstone and Crewe. Apical Flowers.*

Parkstone Mother Plant. Colour of Middle Third of Petal.

Crewe Daughter Plant. Colour of Middle Third of Petal.	r	rp	rw	rpw	p	pw	vw	w	Totals
r	37	23	—	—	29	3	1	1	94
rp	4	11	—	—	18	9	1	1	44
rw	—	—	—	—	—	—	—	—	—
rpw	—	1	—	—	—	—	—	—	1
p	31	32	—	—	94	32	5	5	199
pw	2	6	1	1	17	20	6	6	59
vw	—	—	—	—	—	—	1	—	1
w	1	3	—	—	3	2	1	—	10
Totals	75	76	1	1	161	66	15	13	408

D. *Parkstone and Crewe. Apical Flowers.*

Parkstone Mother Plant. Breadth of Margin.

Crewe Daughter Plant. Breadth of Margin.	b	bs	s	sn	n	Totals
b	1	18	12	3	20	54
bs	—	4	7	—	18	29
s	—	2	3	—	8	13
sn	—	11	6	3	42	62
n	—	23	36	4	187	250
Totals	1	58	64	10	275	408

E.   *Parkstone and Crewe.   Apical Flowers.*

Parkstone Mother Plant.   Extent of Basal Patch.

Crewe Daughter Plant. Extent of Basal Patch.		n	ns	s	sd	d	dl	l	Totals
	n	2	2	1	2	—	—	2	9
	ns	1	1	1	3	—	3	—	9
	s	4	3	7	9	7	4	1	35
	sd	1	—	1	3	1	4	1	11
	d	28	3	20	32	37	60	23	203
	dl	—	—	—	—	—	—	—	—
	l	38	6	15	21	13	24	15	132
	Totals	74	15	45	70	58	95	42	399

F.   *Parkstone and Crewe.   Apical Flowers.*

Parkstone Mother Plant.   Wrinkling of Petal.

Crewe Daughter Plant. Wrinkling of Petal.		*f* or *w*	*sw*	*nw*	Totals
	f or *w*	4	47	86	137
	sw	5	60	169	234
	nw	3	5	20	28
	Totals	12	112	275	399

II.   RESEMBLANCE OF SISTER PLANTS, *i.e.* PLANTS GROWN FROM SEED TAKEN FROM THE SAME MATERNAL CAPSULE.

AA.   *Parkstone.   Sister Plants' Apical Flowers.*

First Sister Plant.   Number of Stigmata.

Second Sister Plant. Number of Stigmata.		10	11	12	13	14	15	16	17	18	19	Totals
	10	2	3	18	12	30	13	4	6	—	—	88
	11	3	26	38	53	51	27	18	11	—	1	228
	12	18	38	100	167	184	132	63	27	4	2	735
	13	12	53	167	310	315	278	115	51	7	4	1312
	14	30	51	184	315	404	332	168	80	15	5	1584
	15	13	27	132	278	332	318	168	73	15	1	1357
	16	4	18	63	115	168	168	84	47	13	2	682
	17	6	11	27	51	80	73	47	20	7	2	324
	18	—	—	4	7	15	15	13	7	—	—	61
	19	—	1	2	4	5	1	2	2	—	—	17
	Totals	88	228	735	1312	1584	1357	682	324	61	17	6388

BB. *Arncliffe. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Stigmata.

		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Totals
Number of Stigmata.	3	—	—	—	—	—	—	—	1	2	3	2	1	—	—	—	—	—	9
	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
	6	—	—	—	2	2	4	4	4	2	—	—	—	—	—	—	—	—	18
	7	—	—	—	2	—	4	3	5	4	3	3	3	—	1	—	—	—	28
	8	—	—	—	4	4	18	20	33	19	18	13	7	1	—	—	—	—	137
	9	—	—	—	4	3	20	16	51	44	35	17	5	7	1	—	—	—	203
	10	1	—	—	4	5	33	51	116	127	119	90	54	13	7	—	—	—	620
	11	2	—	—	2	4	19	44	127	198	136	138	92	37	37	1	1	—	838
	12	3	—	—	—	3	18	35	119	136	230	168	132	57	30	2	4	—	937
	13	2	—	—	—	3	13	17	90	138	168	134	126	45	29	6	9	2	782
	14	1	—	—	—	3	7	5	54	92	132	126	104	38	36	9	1	—	608
	15	—	—	—	—	—	1	7	13	37	57	45	38	26	31	3	8	2	268
	16	—	—	—	—	1	—	1	7	37	30	29	36	31	20	13	6	1	212
	17	—	—	—	—	—	—	—	—	1	2	6	9	3	13	6	1	—	41
	18	—	—	—	—	—	—	—	—	1	4	9	1	8	6	1	6	3	39
	19	—	—	—	—	—	—	—	—	—	—	2	—	2	1	—	3	—	8
	Totals	9	0	0	18	28	137	203	620	838	937	782	608	268	212	41	39	8	4748

(Second Sister Plant. — left row label)

CC. *Streatham. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Stigmata.

		6	7	8	9	10	11	12	13	14	15	16	17	Totals
Second Sister Plant. Number of Stigmata.	6	—	—	—	1	2	1	—	—	—	—	—	—	4
	7	—	—	1	3	9	14	9	6	1	—	—	—	43
	8	—	1	6	14	54	38	55	27	11	4	—	—	210
	9	1	3	14	90	199	234	224	72	28	5	3	—	873
	10	2	9	54	199	436	580	568	198	91	27	8	1	2173
	11	1	14	38	234	580	754	777	328	158	41	17	2	2944
	12	—	9	55	224	568	777	734	362	199	50	19	4	3001
	13	—	6	27	72	198	328	362	184	143	38	23	1	1382
	14	—	1	11	28	91	158	199	143	70	24	12	1	738
	15	—	—	4	5	27	41	50	38	24	2	1	1	193
	16	—	—	—	3	8	17	19	23	12	1	6	—	89
	17	—	—	—	—	1	2	4	1	1	1	—	—	10
	Totals	4	43	210	873	2173	2944	3001	1382	738	193	89	10	11660

DD. *Kidderminster. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Stigmata.

		6	7	8	9	10	11	12	13	14	15	16	17	18	19	Totals
	6	—	—	—	—	2	—	2	1	—	2	2	—	—	—	9
	7	—	—	—	—	3	3	1	2	—	—	1	—	—	—	10
	8	—	—	8	20	17	3	2	2	1	—	—	—	—	—	53
	9	—	—	20	26	23	7	6	7	10	5	2	—	—	—	106
Second Sister Plant. Number of Stigmata.	10	2	3	17	23	34	34	39	44	41	17	11	5	1	—	271
	11	—	3	3	7	34	56	101	113	131	58	22	5	6	—	539
	12	2	1	2	6	39	101	236	285	280	158	94	23	8	7	1242
	13	1	2	2	7	44	113	285	362	440	206	129	64	10	6	1671
	14	—	—	1	10	41	131	280	440	522	289	174	109	34	10	2041
	15	2	—	—	5	17	58	158	206	289	210	119	57	20	8	1149
	16	2	1	—	2	11	22	94	129	174	119	92	59	8	2	715
	17	—	—	—	—	5	5	23	64	109	57	59	64	14	3	403
	18	—	—	—	—	1	6	8	10	34	20	8	14	2	—	103
	19	—	—	—	—	—	—	7	6	10	8	2	3	—	—	36
	Totals	9	10	53	106	271	539	1242	1671	2041	1149	715	403	103	36	8348

EE. *Parkstone. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Petals and Petaloid Stamens.

Number of Petals and Petaloid Stamens.		4 petals	4 petals + petaloid stamens	5 petals	5 petals + petaloid stamens	More than 5 petals	More than 5 petals + petaloid stamens	Totals
Second Sister Plant.	4 petals	5684	457	102	70	172	206	6691
	4 petals + petaloid stamens	457	88	20	17	16	47	645
	5 petals	102	20	8	2	9	9	150
	5 petals + petaloid stamens	70	17	2	4	7	9	109
	More than 5 petals	172	16	9	7	8	9	221
	More than 5 petals + petaloid stamens	206	47	9	9	9	24	304
	Totals ...	6691	645	150	109	221	304	8120

FF. *Arncliffe. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Petals and Petaloid Stamens.

Second Sister Plant. Number of Petals and Petaloid Stamens.	4 petals	4 petals +petaloid stamens	Five petals, more than 5 petals, 5 or more with peta-loid stamens	Totals
4 petals	3358	450	213	4021
4 petals + petaloid stamens	450	98	42	590
5 petals, more than 5 petals, 5 or more petals with peta-loid stamens	213	42	32	287
Totals ...	4021	590	287	4898

GG. *Streatham. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Petals and Petaloid Stamens.

Second Sister Plant. Number of Petals and Petaloid Stamens.	4 petals	4 petals +petaloid stamens	5 or more petals	5 or more petals +petaloid stamens	Totals
4 petals	8794	206	373	703	10076
4 petals + petaloid stamens	206	6	17	35	264
5 or more petals ...	373	17	26	47	463
5 or more petals + petaloid stamens	703	35	47	102	887
Totals ...	10076	264	463	887	11690

HH. *Kidderminster. Sister Plants' Apical Flowers.*

First Sister Plant. Number of Petals and Petaloid Stamens.

Second Sister Plant. Number of Petals and Petaloid Stamens.	4 petals	4 petals +petaloid stamens	5 or more petals	5 or more petals +petaloid stamens	Totals
4 petals	5868	268	241	505	6882
4 petals + petaloid stamens	268	48	15	74	405
5 or more petals ...	241	15	8	27	291
5 or more petals + petaloid stamens	505	74	27	90	696
Totals ...	6882	405	291	696	8274

II.   *Parkstone.   Sister Plants' Apical Flowers.*

First Sister Plant.   Colour of Middle Third of Petal.

		r	rp	rw	rpw	p	pw	ppw	vw	w	Totals
Second Sister Plant. Colour of Middle Third of Petal.	r	320	147	8	11	407	36	12	34	16	991
	rp	147	286	15	11	458	81	18	49	14	1079
	rw	8	15	—	1	21	1	—	1	4	51
	rpw	11	11	1	6	44	3	2	6	1	85
	p	407	458	21	44	2262	452	140	319	109	4212
	pw	36	81	1	3	452	221	50	111	38	994
	ppw	12	18	—	2	140	50	52	30	17	321
	vw	34	49	1	6	319	111	30	76	26	652
	w	16	14	4	1	109	38	17	26	26	251
	Totals	991	1079	51	85	4212	994	321	652	251	8636

JJ.   *Arncliffe.   Sister Plants' Apical Flowers.*

First Sister Plant.   Colour of Middle Third of Petal.

		r	rp	p	pw	vw	w	Totals
Second Sister Plant. Colour of Middle Third of Petal.	r	390	155	300	109	5	53	1012
	rp	155	126	217	74	4	38	614
	p	300	217	986	315	10	114	1942
	pw	109	74	315	254	7	84	843
	vw	5	4	10	7	—	4	30
	w	53	38	114	84	4	76	369
	Totals	1012	614	1942	843	30	369	4810

KK.   *Streatham.   Sister Plants' Apical Flowers.*

First Sister Plant.   Colour of Middle Third of Petal.

		r	rp	p	pw	pww	w	Totals
Second Sister Plant. Colour of Middle Third of Petal.	r	792	142	992	89	208	164	2387
	rp	142	36	242	28	35	39	522
	p	992	242	4144	167	516	409	6470
	pw	89	28	167	22	42	38	386
	pww	208	35	516	42	148	87	1036
	w	164	39	409	38	87	116	853
	Totals	2387	522	6470	386	1036	853	11654

LL. *Kidderminster. Sister Plants' Apical Flowers.*

First Sister Plant. Colour of Middle Third of Petal.

Second Sister Plant. Colour of Middle Third of Petal.		r	rp	p	pw	pww	rw	w	Totals
	r	714	251	473	170	49	11	59	1727
	rp	251	308	381	155	47	—	39	1181
	p	473	381	1392	557	181	5	103	3092
	pw	170	155	557	414	104	2	83	1485
	pww	49	47	181	104	46	—	50	477
	rw	11	—	5	2	—	—	—	18
	w	59	39	103	83	50	—	50	384
	Totals	1727	1181	3092	1485	477	18	384	8364

MM. *Parkstone. Sister Plants' Apical Flowers.*

First Sister Plant. Breadth of Margin.

Second Sister Plant. Breadth of Margin.		b	bs	s	sn	n	Totals
	b	46	51	44	33	279	453
	bs	51	44	69	55	431	650
	s	44	69	78	57	456	704
	sn	33	55	57	82	450	677
	n	279	431	456	450	4542	6158
	Totals	453	650	704	677	6158	8642

NN. *Arncliffe. Sister Plants' Apical Flowers.*

First Sister Plant. Breadth of Margin.

Second Sister Plant. Breadth of Margin.		b	bs	s	sn	n	Totals
	b	244	56	99	39	536	974
	bs	56	10	9	21	87	183
	s	99	9	54	18	209	389
	sn	39	21	18	24	161	263
	n	536	87	209	161	2070	3063
	Totals	974	183	389	263	3063	4872

OO. *Streatham. Sister Plants' Apical Flowers.*

First Sister Plant. Breadth of Margin.

Second Sister Plant. Breadth of Margin.		b	bs	s	sn	n	Totals
	b	136	87	205	107	624	1159
	bs	87	42	93	59	340	621
	s	205	93	290	165	885	1638
	sn	107	59	165	112	686	1129
	n	624	340	885	686	4582	7117
	Totals	1159	621	1638	1129	7117	11664

PP. *Kidderminster. Sister Plants' Apical Flowers.*

First Sister Plant. Breadth of Margin.

Second Sister Plant. Breadth of Margin.		*b*	*bs*	*s*	*sn*	*n*	Totals
	b	66	—	84	66	357	573
	bs	—	—	—	—	—	0
	s	84	—	228	116	607	1035
	sn	66	—	116	88	416	686
	n	357	—	607	416	4690	6070
	Totals	573	0	1035	686	6070	8364

QQ. *Parkstone. Sister Plants' Apical Flowers.*

First Sister Plant. Extent of Basal Patch.

Second Sister Plant. Extent of Basal Patch.		*n*	*ns*	*s*	*sd*	*d*	*dl*	*l*	Totals
	n	474	83	108	184	343	266	307	1765
	ns	83	20	39	30	66	48	44	330
	s	108	39	52	60	126	86	72	543
	sd	184	30	60	144	240	160	100	918
	d	343	66	126	240	582	453	293	2103
	dl	266	48	86	160	453	354	236	1603
	l	307	44	72	100	293	236	274	1326
	Totals	1765	330	543	918	2103	1603	1326	8588

RR. *Arncliffe. Sister Plants' Apical Flowers.*

First Sister Plant. Extent of Basal Patch.

Second Sister Plant. Extent of Basal Patch.		*n*	*ns*	*s*	*sd*	*d*	*dl*	*l*	Totals
	n	156	13	28	36	237	140	37	647
	ns	13	2	1	6	20	12	2	56
	s	28	1	8	17	73	52	15	194
	sd	36	6	17	34	125	77	17	312
	d	237	20	73	125	924	466	171	2016
	dl	140	12	52	77	466	410	100	1257
	l	37	2	15	17	171	100	56	398
	Totals	647	56	194	312	2016	1257	398	4880

SS. *Streatham. Sister Plants' Apical Flowers.*

First Sister Plant. Extent of Basal Patch.

	n	ns	s	sd	d	dl	l	Totals
n	186	53	433	128	226	47	186	1259
ns	53	66	263	83	140	25	46	676
s	433	263	1582	527	777	125	494	4201
sd	128	83	527	216	278	38	162	1432
d	226	140	777	278	386	83	292	2182
dl	47	25	125	38	83	18	83	419
l	186	46	494	162	292	83	270	1533
Totals	1259	676	4201	1432	2182	419	1533	11702

(Second Sister Plant. Extent of Basal Patch.)

TT. *Kidderminster. Sister Plants' Apical Flowers.*

First Sister Plant. Extent of Basal Patch.

	n	ns	s	sd	d	dl	l	Totals
n	826	99	124	517	426	1	326	2319
ns	99	22	23	65	59	—	42	310
s	124	23	64	96	61	—	51	419
sd	517	65	96	680	494	6	255	2113
d	426	59	61	494	868	—	197	2105
dl	1	—	—	6	—	—	2	9
l	326	42	51	255	197	2	216	1089
Totals	2319	310	419	2113	2105	9	1089	8364

(Second Sister Plant. Extent of Basal Patch.)

UU. *Parkstone. Sister Plants' Apical Flowers.*

First Sister Plant. Colour at Margin of Petal.

	r	rp	p	rw	pw	vw	w	Totals
r	274	106	426	3	73	21	118	1021
rp	106	90	295	3	85	12	101	692
p	426	295	1860	7	423	150	646	3807
rw	3	3	7	—	1	—	3	17
pw	73	85	423	1	196	45	243	1066
vw	21	12	150	—	45	20	93	341
w	118	101	646	3	243	93	476	1680
Totals	1021	692	3807	17	1066	341	1680	8624

(Second Sister Plant. Colour at Margin of Petal.)

VV. *Crewe. Sister Plants' Apical Flowers.*

First Sister Plant. Form of Capsule.

	Round	Conical	Flat	Totals
Round ...	314	258	155	727
Conical ...	258	342	179	779
Flat ...	155	179	92	426
Totals	727	779	426	1932

(Second Sister Plant. Form of Capsule.)

WW. *Parkstone. Sister Plants' Apical Flowers.*

First Sister Plant. Wrinkling of Petal.

Second Sister Plant. Wrinkling of Petal.	f	w	sw	nw	Totals
f	6	5	30	40	81
w	5	46	164	313	528
sw	30	164	1210	1661	3065
nw	40	313	1661	2906	4920
Totals	81	528	3065	4920	8594

XX. *Arncliffe. Sister Plants' Apical Flowers.*

First Sister Plant. Wrinkling of Petal.

Second Sister Plant. Wrinkling of Petal.	f	w	sw	nw	Totals
f	78	131	252	57	518
w	131	314	596	111	1152
sw	252	596	1310	296	2454
nw	57	111	296	260	724
Totals	518	1152	2454	724	4848

YY. *Streatham. Sister Plants' Apical Flowers.*

First Sister Plant. Wrinkling of Petals.

Second Sister Plant. Wrinkling of Petal.	f	w	sw	nw	Totals
f	14	69	113	149	345
w	69	410	742	936	2157
sw	113	742	1380	1704	3939
nw	149	936	1704	2472	5261
Totals	345	2157	3939	5261	11702

ZZ. *Kidderminster. Sister Plants' Apical Flowers.*

First Sister Plant. Wrinkling of Petals.

Second Sister Plant. Wrinkling of Petal.	f	w	sw	nw	Totals
f	2	28	59	17	106
w	28	444	1013	194	1679
sw	59	1013	3356	779	5207
nw	17	194	779	382	1372
Totals	106	1679	5207	1372	8364

ON THE INHERITANCE OF COAT-COLOUR IN CATTLE.

PART I. SHORTHORN CROSSES AND PURE SHORTHORNS.

By A. BARRINGTON and K. PEARSON.

CONTENTS.

(1) *Introductory.*

As far as we are aware no statistical investigation into the inheritance of coat-colour in cattle has hitherto been published. An attempt will be made in the present memoir to cover some of this ground in a manner similar to that in which investigations have already been made in the case of the coat-colours of the thoroughbred horse[*] and of the greyhound[†]. It is, perhaps, idle to assert that we have throughout desired to give as complete and accurate an account as lay within our power of the mass of material available, and this without prejudice in favour of any theory. . Those acquainted with the spirit which animates the work of biometricians will scarcely doubt this; those who have throughout misrepresented the purport of mathematical investigations into heredity will doubtless continue to do so. At the same time it may not be without service to reiterate what has

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[*] *Phil. Trans.* Vol. 195 A, pp. 92–102 ; *Biometrika*, Vol. II. pp. 229–240.
[†] *Biometrika*, Vol. III. pp. 245–298.

been previously stated as to the scope of biometric work in this field. Our object is to state first the proportion in which sub-classes exist in a " population," to measure next the degree in which these sub-classes are mated at random, and then to determine the extent to which these sub-classes reproduce themselves. We believe that a statistical record of this kind is of fundamental importance whatever physiological theory of heredity be adopted. In treating cattle we have done precisely what was done in the case of thoroughbred horses and grey-hounds, namely, the latest available volumes of the studbook have been taken*, and the colour pedigree of individuals entered therein worked out. In the work on cattle this was done on standard pedigree slips, of which a sample is given on the following page. These were filled in where possible as far as the great-great-grandparents and these schedules, representing a random sample of the existing general population, form the basis of our tables. As in the case of the greyhounds, supplementary investigations were made by collecting special pedi-grees as those of white individuals. These special pedigrees were *not* embraced in the general sample except in so far as they fell into the period there dealt with, but they served to illustrate special points which it seemed desirable to elucidate.

On the basis of the above random sample† we first determined the extent to which random mating occurs in the general population. To what extent is there a fashion or taste in mating cattle? We next enquired how far our tabulated results admitted of a Mendelian interpretation. Let us suppose this question answered in the positive or negative sense, as the case may be, it leaves the tabulated data absolutely where they were as to the further question we have to ask: What is the *average* degree of resemblance between relatives in a population? To what extent do the features of the ancestry impress themselves upon the offspring? This is not a matter of theory, it is simply a statistical enquiry as to the degree of independence of two distributions. It resolves itself mathematically into determining the degree of independence of probability in two "chance" distributions. In order to dispose of all doubts as to quantitative scale and as to "normal" distributions of characters, we have adopted throughout the contingency method of measuring the deviation from independent probability, and in particular that form of it spoken of as mean square contingency. This makes all our results for one race strictly comparable *inter se*. But the colour classifications being different for different species, it is not so legitimate to compare these results together. In order to throw light on this point other methods have been adopted occasionally for comparison and control; these will be referred to in their proper places.

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* In the *second* paper on coat-colour in horses, we began with the earliest studbooks in order to test the influence of the far more frequent greys and blacks.

† The word "random" is used with reference to coat-colour. When one or both parents had appeared several times, no further offspring of such individuals were taken, in order to avoid overweighting certain individuals. This affected the cows more than the bulls, because the former are classified in herds with sometimes close inbreeding. Thus 1000 consecutive cows might possibly have given only 50 to 60 sires.

GENERAL HEREDITY SCHEDULE.

Species: Shorthorns.   No. in Series,   .   Character: Coat-Colour.   Recorder: A. Barrington.

Parents	Male	Female	Grandparents	Male	Female	Gt Grandparents	Male	Female	Gt. Gt. Grandparents	
2 78665 Crusader	red		4 66864 Cornelius	roan		8 60073 Touchstone	r & w		52956	red
									Sunflower	roan
						9 Butter Scotch		white	58189	roan
									Buttercup	r & w
			5 Rosemary 93rd	red	red	10 62235 Bucaneer	red		57992	roan
									Bella 6th	roan
						11 Rosemary 90th	r & w	red	57018	red
									Rosemary 58th	roan
3 Reception 13th	white	roan	6 68388 Commander	roan		12 58582 Cambridge Duke 28th	red		55632	red
									Red Rose 13th	roan
						13 Lady May		roan	54870	roan
									Lady Jane	r & w
			7 Reception 12th		roan	14 65255 Cheque	r & w		62180	roan
									Charity	r & w
						15 Reception 8th		roan	60494	red
									Reception 3rd	roan

Subject.	Male	Female	Siblings	Male	Female	First Cousins.	Type *	Male.	Female.
v 49 p 888	0u	1a roan		0b	1b				
				0c	1c				
Harforth Cymbal				0d	1d				
				0e	1e				
				0f	1f				

Remarks.

* Father's Brother's Offspring = 2_0 Father's Sister's Offspring = 2_1 Mother's Brother's Offspring = 3_0 Mother's Sister's Offspring = 3_1

Any simple or complex Mendelian system, any theory of hereditary determinants, must lead when it is applied to a "population" to its appropriate system of contingency coefficients between relational grades. It may be easy or it may be difficult to work out their theoretical values, but they undoubtedly exist, as soon as we know the nature of the mating. And without knowledge of the amount of mating of sub-groups no determinantal system can be applied to wild populations. If proven a theory of determinants would immensely assist breeders, but it is extremely difficult to see how it could be applied to wild populations and to the Darwinian theory of evolution by inheritance and selection. Hence, if we lay special stress on finding directly the coefficients which fix the intensity of inheritance in populations, it is not because we should in the least undervalue the discovery of a well-demonstrated determinantal theory of inheritance, but because we should then have, before applying it to populations and seeing its full bearing on the problem of evolution, to actually go through the theoretical determination of constants, which can without any such hereditary theory at all be determined from the material at our disposal.

When, however, we have found the degree of resemblance between relatives quantitatively, that is to say, the empirical numbers which for any population mark the divergence from independent probability between any relational grades, we can next ask : Is there any connection between these numbers within the same species ? and are the coefficients for different species, or for different characters in the same species alike ? When we answer these questions we are determining whether heredity is the same or not for all characters and species, and to what extent ancestry and stock are of importance. It may be said that a valid determinantal theory would answer these questions also. So it undoubtedly would, but no such theory is, in our opinion, even approximately established as yet ; and meanwhile there is and will remain every justification for attacking the problem by a method, which starts from no hereditary hypothesis, but merely endeavours to interpret the observed numbers of each class.

So much may be said of the methods adopted in this paper. It is a repetition of what has already been written in this *Journal*, but it is desirable to restate it here in order to emphasize the aim of the present investigation.

(2) *Material.*

When we determined some years ago to consider the inheritance of coat-colour in cattle we directly sought for a simple case which might possibly enable us to convince ourselves of the efficiency of a quite brief Mendelian formula. The Guernsey cattle have a very considerable variety of coat-colour. With shorthorns a briefer classification is sufficient, but still it is wide enough to allow of the possibility of very complex allelomorphs. We were aware that the white shorthorn breeds nearly true, and that the black Galloway does the same. Hence the cross of the white shorthorn and the black Galloway seemed to offer a possibility of testing the simplest Mendelian formula. Unfortunately the resulting blue-grey

cattle, although of great value for trade purposes, do not form the subject of any herdbook record. Still it seemed possible to obtain some simple results by an appeal to large breeders of blue-greys. Their returns form the subject of the first part of the present paper. We next endeavoured to obtain some general idea of the colour inheritance of British cattle, but the colours of the original progenitors of existing types seem very hypothetical, and our researches in this direction, embodied in a later part of this paper, are not very helpful. Attention was then devoted to two types : the shorthorn and the Guernsey. In the case of the former *Coates' Shorthorn Herdbook*, issued by the Shorthorn Society of Great Britain and Ireland, has now reached the fiftieth volume, and we have to heartily thank the Society for placing a copy of it at our disposal for the present investigation. This herdbook is very complete in character and deals with very large numbers of beasts. At least some breeders insert the coat-colour of calves sent to the butcher, but we doubt whether such entries are universal. In the case of the Guernseys, the *English Guernsey Herdbook* has appeared for eighteen years, the pedigrees lead fairly soon back to the *Guernsey Guernsey Studbook*, of which ten volumes had appeared up to date. We have to heartily thank the English Guernsey Society for placing a copy of their herdbook at our disposal. We supplemented it by the purchase of the *Guernsey Guernsey Studbook*. The latter seems to us a less reliable work, as cattle are included with many coat-colours, which would not be admitted now into the English Guernsey record. But a full discussion of this matter is postponed until we come to deal with Guernseys in the second half of this paper. Pedigrees with the statement of coat-colour were filled in for shorthorn and Guernsey cattle from these herdbooks, and to obtain an adequate supply of such pedigrees has demanded a large amount of very laborious work. We shall now take our subdivisions in order :

(3) *Blue-Grey Cattle. Colour Inheritance.*

In this case the blue-grey cattle are as a rule produced by crossing a black polled Galloway cow with a white shorthorn bull. The white shorthorn (with black eyes) and the black cattle breed relatively true, so that if we term one B and the other W we might test a simple Mendelian formula $(BB) \times (WW) = (BW)$, and then further investigate the crosses $(BW) \times (WW)$, $(BW) \times (BW)$ and $(BW) \times (BB)$.

As in the case of Japanese waltzing mice crossed by Albino white mice we do not meet with any simple phenomenon of dominance, the general coat-colour of the offspring being blue-grey. The problem therefore turns on the action of these hybrids. Our first enquiry was as to the nature of the shorthorn bull purchased by the blue-grey cattle breeders. Mr H. L. Fife, of the Raby Estates Office, speaking of experience with the sales of produce from the Raby herd, writes :

"Breeders of blue-grey cattle in the North-West of England, who produce the same by crossing a white shorthorn bull with a black polled cow, are always most particular when buying a white sire to satisfy themselves if possible, that such sire is the produce of two white parents. If he is not, that is to say, if the parent on one side only is white, their theory is that his stock when crossed with black cows do not get the blue-grey colour they like."

This statement led to the framing of the following questions, which were issued in a circular form to the breeders of blue-grey cattle*:

We are told that breeders of blue-grey cattle object to white shorthorn bulls, whose parents were not both pure white as giving bad results.

(i) Can you from your own experience tell us whether you have bred from such a shorthorn bull, and if so whether you found it to give bad colour results and what these results were?

(ii) If a blue-grey cow be crossed with a white shorthorn what is the resulting coat-colour? Could you give us actual instances of this cross? In particular do pure white cattle ever result from this cross?

(iii) If the blue-grey cross be covered by a Galloway bull, what is the resulting coat-colour? Do black as well as blue-grey offspring result? Can you give us instances in your own experience?

(iv) If the blue-grey cross be mated with itself, blue-grey cow with blue-grey bull, what is the resulting coat-colour? Do you know instances of black cattle or pure white cattle resulting from this cross? Are the majority of the offspring blue-grey?

It will be observed that questions (ii), (iii), and (iv) deal respectively with the crosses $(BW) \times (WW)$, $(BW) \times (BB)$ and $(BW) \times (BW)$ and were intended to elicit information with regard to these matings. Question (i) was framed so as to draw forth either facts or impressions of the breeder as to the need for purity in the white ancestry.

These questions produced a number of answers—the experience of all breeders being by no means alike—which we have summarised below. We have most heartily to thank these gentlemen for their kindness and invariable patience in replying to our queries.

There appears to be general assent to the proposition that a pure bred Galloway cow and a white shorthorn bull of white parentage give the desired blue-grey cross.

When the white shorthorn bull is *not* from white parentage, Messrs Hyslop state that besides greys, there appear reds, blacks, particolour black and white, and occasionally a white with dark nose and insides of ears. Mr de Vere Irving prefers to use a shorthorn white bull of white parentage on both sides. He finds on using a white bull of roan parentage, that the offspring are liable to come dark, some almost black in colour, and others very dark blue-grey. Mr G. Hodgson always tries if possible to get a bull from white parents, which he finds makes the progeny the desired *light* blue-grey. If a white bull of white parentage of a satisfactory type is not to be had, then he takes one from roan parents, but this in his experience makes the colour of a darker nature. Mr R. Tinniswood has bred from a pure bred Galloway and a white shorthorn bull not of white parentage and has had occasional coloured calves from this cross. He considers that it is very desirable that the white shorthorn bull should be from white parents to get the best results. Bulls not from white parents mated with pure bred Galloways in the majority of cases give the right colour, but may give reds, roans† or nearly blacks. Mr Tinnis-

* We have to thank Messrs Harrison and Son of Carlisle for kindly providing us with a list of blue-grey cattle breeders.

† The red and roan hair we have examined is exactly alike in appearance to that of red and roan shorthorns.

wood considers that the ancestry of the Galloway cow is even more important than that of the shorthorn bull; and states that the *true pedigree* black Galloway crossed with the white shorthorn of non-white parentage will give a blue-grey, though some of the offspring will be very dark blue-grey. The whole of this evidence seems to show that black Galloway × white shorthorn * does not give invariably the desired light blue-grey blend, but that the deviations from this colour in the opinions of experienced breeders depend on the ancestry of the (BB) and the (WW). The hybrid generation is not uniform unless the parents come of selected stock. Such appears to be the best answer we can get to our first problem.

Our second question relates to the crossing of the hybrid (BW), the blue-grey cattle with (WW) the white shorthorn. This ought to give half blue-grey (BW) and half white shorthorns (WW). The experience of breeders is not uniform, but most probably it is based on white shorthorn bulls with varying colour pedigree.

Mr Tinniswood finds that the cross is invariably a white calf with a black nose and ear linings. He has been careful to use shorthorn bulls which as to ear and nose are perfectly white. Mr Hodgson has frequently bred this cross and finds the results always white, occasionally with a black nose. He says this cross gives white cattle which are not, however, to be reckoned as pure white shorthorns. Thus in the experience of both these breeders (BW) has not occurred as a result of $(BW) \times (WW)$, nor in the experience of at least one of them (WW), but white with black markings. Other breeders, however, have had different experiences. Mr G. Richardson finds all colours to arise from the cross. Messrs Hyslop find like Messrs Tinniswood and Hodgson that white with coloured nose and ears was the resultant coat-colour, but some light blue-grey, some "grizzled" (i.e. red mixed with white hairs), and some black generally at base of coat. Mr de Vere Irving considers the coat-colour uncertain, and cites as illustrations, a blue-grey cow × white shorthorn bull giving a white calf with black nose and blue-grey ears. Another blue-grey cow with the same bull giving last year a blue-grey calf and this year a roan. He has also had *pure* white cattle from this cross. We think we may conclude from these results that the essential feature to which Mendel drew attention, i.e. segregation in the second crosses, certainly does occur, but its nature is not compatible with any simple Mendelian formula. We have not only (WW) and (BW) occurring from $(BW) \times (WW)$, but also red and roan appearing in the hair, as well as white with black markings which did not exist in either the pure bred Galloway or the shorthorn parents. Further, this segregation although possibly more marked than in the hybrid generation is to be found in the hybrid generation itself, if the cow and bull while presenting the external features of pure Galloway and pure shorthorn, have not pure black and pure white ancestry.

We now turn to the cross $(BW) \times (BB)$. This should give (BB) and (BW) in equal proportions. Mr de Vere Irving finds the resulting offspring black or *dark*

* The usual cross is ♀ $(BB) \times$ ♂ (WW), but the reciprocal cross is sometimes made. The produce of this cross is said, however, not to be uniform in colour. It is adopted because shorthorn heifers can be bought on better terms than pure bred Galloway heifers; while the shorthorn heifer as a cow makes a better price than a Galloway for towns' dairying.

blue-grey. He has at present a black heifer from this cross and has personally known instances of the dark blue-greys. Mr Richardson's experience is the same as Mr Irving's, i.e. black and blue-grey. Mr Tinniswood's experience is that the offspring take black, but occasionally, especially if from *light* blue-grey cows, the black is accompanied by white markings. Mr Hodgson finds the coat-colour not very certain, generally black; but from a *light* blue-grey cow he has had blue-greys and roans; from a *dark* blue-grey cow, generally black with only an occasional roan. This is interesting because we have seen that the *light* blue-grey results from the purity of the Galloway cow and the white parentage of the shorthorn bull. It would thus appear that the grandparental ancestry influences the nature of the cross $(BW) \times (BB)$, through the character of the (BW) itself. Messrs Hyslop have had still more varied results, namely, dark blue-grey, some "grizzled" (red and white hairs), black, and white with black nose and ears. In other words if white with black nose and ears be taken to represent the (WW) arising from $(WW) \times (BW)$, it cannot represent the (BW) in $(BB) \times (BW)$.

We now turn to the last problem: what does $(BW) \times (BW)$ give? Messrs Tinniswood and de Vere Irving have never tried the cross. In fact it is clearly not one that blue-grey cattle breeders are likely to make*. Mr Hodgson has had no personal experience of this cross, but has seen instances in which a blue-grey bull being used on blue-grey cows, blue-grey and roan offspring resulted. Mr Richardson has known instances in which this cross being made the offspring were nearly all blue-grey, but the following season with the same cows and bull, red, roan, white, black, with only a few blue-grey calves appeared. Messrs Hyslop state that the cross gives blue-grey, black, red, "grizzled" (red with white hairs) and an occasional white. They know no breeder, however, who makes at present this cross although it used to be followed. So far as this experience goes therefore $(BW) \times (BW)$ appears to give (BW), (BB) and possibly (WW), although it is not white with black markings, such as occurs in the $(BW) \times (WW)$ and $(BW) \times (BB)$ crosses. But on the other hand red hair has appeared when the original pure bred Galloway and shorthorn did not patently contain this colour.

To sum up, we may conclude, we think:

(*a*) That when pure black and pure white parents of races which breed nearly true are crossed, the hybrids are not homogeneous, there is a certain amount of segregation even in this generation, and colour may appear which was latent in the parents. It seems to us a difference in words only, whether we attribute this appearance to a latent determinant, or say that it is reversion to an ancestor. The pedigree is, in fact, the only *à priori* guide the breeder has to the possibility

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* Mr Tinniswood breeds about 100 blue-grey calves a year. As he remarks, it is possible to get blue-greys from crossing a blue-grey cow with either a Galloway or shorthorn bull, but the colour "very seldom occurs." Hence the breeder naturally turns to the white shorthorn and Galloway first cross. Mr Tinniswood has used both crosses, generally the shorthorn bull, but also the *pure* shorthorn white cow and the Galloway bull, a cross he has never known to give red or roan. He has known red or roan from the reciprocal cross, and attributes it to want of pedigree in the Galloway cow.

of such reversion. As he puts it, "it is desirable that the white shorthorn should be of white parentage," and "the Galloway a pure Galloway."

(b) More segregation takes place when the hybrid generation is crossed with either original stock. Red hair appears, the white shorthorn is replaced by white with black nose and ears, and this or pure white appears from the cross $(BW) \times (BB)$. Our hope therefore that a simple Mendelian formula might be tested on this case—the only one in which we knew of two whole colour cattle of different races being habitually crossed—has failed. Our enquiries were, we believe, justified, however, by the apparent simplicity of black × white = blue-grey*. We did not think actual colour statistics in this case worth collecting, for if we must deal with more complex colour categories, it is at once best to turn to breeds like the shorthorn and Guernsey, for which ample colour statistics are already on record. So far as it is possible to make a comparison between rough experience of the present kind on blue-grey cattle and more definite quantitative experience on mice, the colour results of crossing are in many points strikingly like the Oxford experience with Japanese waltzing and albino white mice. It will need a complex allelomorph to describe these colour changes, if indeed the phenomena can be thus described at all.

(4) *Blue-Grey Cattle. Horn Inheritance.*

It seemed worth considering whether a second character, the horns, would admit of a simple Mendelian analysis. The black Galloways are polled cattle†, and the white shorthorns, horned. We can represent these respectively by (PP) and (HH), and we have to enquire as to the following results: $(PP) \times (HH) = (PH)$, $(PH) \times (PP)$, $(PH) \times (HH)$ and $(PH) \times (PH)$.

First, $(PP) \times (HH)$ gives theoretically the blue-grey (PH). Actually Messrs Harrison say (PH) are "chiefly" polled, and Mr de Vere Irving finds that about one in twenty of (PH) is horned. Thus "polled" is dominant, but it is not absolutely dominant. Mr Irving has not found that exceptions are to be associated with want of pedigree in the Galloway, and he has found no difference with regard to horns between ♂ $(PP) \times$ ♀ (HH) and the more usual cross ♀ $(PP) \times$ ♂ (HH). Mr Robert Tinniswood agrees on this point, and finds the result polled if both sire and dam are pure bred. Mr Hodgson finds not more than about 1 in 30 to 40 horned; he considers that this is usually due to impurity in the Galloway pedigree, but horns may occur with exceptionally well bred sire and dam.

$(PH) \times (PP)$. This cross should give (PP). Mr Irving finds the offspring are polled. Mr Richardson says that they are all polled. Mr Tinniswood has not

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* As a matter of fact this blue-grey is, to judge by hair samples, simply a blend of white and black hairs, varying in proportion from almost pure black to a large majority of white.

† Our English informants state that the Galloway is in their experience polled. If the breed now contained (PH)'s, (HH)'s would appear. Boyd Dawkins, *Quart. Journal Geol. Soc.* 1867, p. 177, says that in the 18th century Galloways were not polled, but his evidence for universality of horns seems far from conclusive. Further details as to the crossing of polled and horned breeds will be given in the next number of this Journal.

largely bred this cross but all the calves have been polled. Mr Hodgson on the other hand has found 1 out of 8 to 10 horned.

$(PH) \times (HH)$. This should equal $2(PH) + 2(HH)$ and give half horned and half polled. Messrs Hyslop find the majority horned. Mr Richardson finds three-quarters horned, and Mr Irving that the horned are in a majority of three to one. Mr Irving says that it sometimes happens that the horns do not grow to much size but in the majority of cases they are there, and he does not consider that such cattle can be described as polled. Mr Tinniswood finds the calves bred this way generally horned, perhaps 2 or 3 in twenty would come polled. Mr Hodgson on the other hand finds in this case only 1 in 4 to 6 horned and illustrates by three blue-grey ♀s put to a white shorthorn bull last year giving all polled calves.

$(PH) \times (PH)$. This should give $(PP) + 2(PH) + (HH)$, which being interpreted is polled in a majority of three to one. Few breeders make this cross, and Messrs Irving, Richardson and Tinniswood have had no experience of it. Mr R. Hyslop finds six polled to one horned, and Mr Hodgson says one out of 6 to 8 horned. These numbers are far from the Mendelian ratio.

Thus, as in other cases which it has been our misfortune to come across, we find approximations to Mendelism, but we do not find Mendelism an accurate account of the records provided. The polled (PH) might really be a (PP), it cannot be an (HH), and therefore we could account for less than a half being horned in the cross $(PH) \times (HH)$, but not for more than a half being horned. The only explanation could be that polled in a number of cases are latently horned, i.e. we cannot distinguish (PP) from (PH). This would account for the horned cattle appearing in $(PP) \times (HH)$, but not for the usual absence of horns in $(PH) \times (PP)$, for some reputed (PP)'s would be (PH)'s. Anyhow the further pursuit of a somewhat elusive Mendelism in blue-grey cattle did not seem to us likely to be profitable.

(5) *Colour History of the Shorthorn.*

The classification of shorthorns into broad categories is extremely simple as compared with some other types of domestic cattle. The recognised classes are *Red* (R), this is supposed to contain whole reds; *Red with a little White* (RLW) or red with occasional white spots or with white markings; *Red and White* (RW), i.e. particolour; *Roan* (Ro), a mixture of red and white hairs; and *White* (W). The white are not albinos*, having black eyes and occasionally black noses. Attempts more or less successful to get rid of the black noses have been energetically made by breeders. The existence of black pigment in the nose is probably not always recorded in the herdbook records. We have seen that breeders of blue-grey cattle strive to get shorthorn bulls without black noses, but as there is always black pigment in the eye, the question of absence of black pigment in the nose is perhaps not of first-class importance in the problem of crossing. It appears perfectly certain that eye and nose pigmentation in intercrossing white shorthorns

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* A herd of true albino white cattle (whole white and pink eyes) has been reported to us as existing in Sweden, but we have not succeeded in locating it.

does not now introduce black into the coat-colour, though how far it may appear as red may be open to discussion. Red in the shorthorn appears in different shades roughly classified by breeders as dark and light, and we have corresponding dark and light roan, according to the character of the red hair in the roan. These finer distinctions are not recorded in the herdbook*.

From the standpoint of our present investigation, it would be of very great suggestiveness if we could determine in number and colour the original factors of the modern shorthorn. So far as we have been able to gather from writings on the subject there seems to be a good deal of diversity of opinion among authorities as to the origin of the domestic breeds of British cattle. There are three somewhat divergent main views. First that of Boyd Dawkins[†]. He identifies the *Bos Urus* of the prehistoric and historic periods with the *Bos primigenius* of the Pleistocene period, but he does not consider that either survived in this country as source of the wildpark cattle. He holds that the small dark Welsh and Scotch mountain cattle represent an original Celtic shorthorn, and that the Romans used this Celtic cattle and did not import larger breeds of their own. This whole coloured Celtic shorthorn was driven out not by the Romans, but by the Anglo-Saxons. The Chillingham wild cattle on this view are living representatives of the breed brought by the Anglo-Saxons. They are not survivors of the *Bos Urus* in this country, but imported Anglo-Saxon cattle run wild[‡]. At first sight there does not appear the same reason for the disappearance of the Anglo-Saxon cattle, that there was for the expulsion of the Celtic cattle, and Boyd Dawkins' citations from Welsh Laws may be read in more ways than one, for fines have generally become customary many generations before they are codified. It is also unlikely that all the place-names in Ireland referring to white cattle are later than the Saxon invasion.

Another view has been taken by McKenny Hughes[§]. He considers that the characteristics of the *Urus* nowhere appear in the Romano-British cattle. The Kerry cattle are typical of the Celtic shorthorn, the Chillingham cattle are the nearest representatives of the breed introduced by the Romans[||]. The Highland and Welsh cattle are derived largely from the Roman breed with admixture of the Celtic shorthorn. These are all whole coloured or shaded. The *Longhorns*, frequently particoloured or sheeted, are the offspring of breeds introduced from

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* The range and continuity of the reds seem fairly obvious from the samples of hair colour in our possession.

[†] *Quarterly Journal of the Geological Society*, Vol. XXII. 1866, pp. 391–401. "Fossil British Oxen," *Pleistocene Mammalia, Palaeontographical Society*, Boyd Dawkins and Sandford, 1866–1886, in parts.

[‡] B. A. Report on Herds of Wild Cattle, *Report*, 1887, p. 135, appears to leave the question of importation open. Rütimayer (" Ueber Art und Race des zahmen europäischen Rindes," *Archiv für Anthropologie*, Bd. I. S. 219–250) holds that the English wild cattle, the large horned Hungarian and *Italian* cattle belong to the Primigenius race.

[§] *Archaeologia*, Vol. LV. Part I. pp. 125–158, 1896. "On the more Important Breeds of Cattle which have been recognised in the British Isles, etc."

[||] A comparison of the skulls of Chillingham and Italian bulls with that of the *Bos primigenius* can be easily made at the Natural History Museum, South Kensington.

the Low Countries and Holstein in later mediaeval times. McKenny Hughes says that the Durham and modern shorthorns belong to a much later period and fall outside his enquiry. Thus writings on the subject do not give us very definite results for our present purposes. We further appealed to Professor James Wilson, of the Royal College of Science, Dublin, who is an authority on the matter, and who most kindly provided us with the following brief statement :

(*a*) The first cattle in Britain were of a large type, *Bos primigenius,* which became extinct here in the stone age, although possibly surviving later on the Continent. If they had anything to do with existing British cattle, which is extremely doubtful, the relationship is very remote and is only to be traced through later importations from the Continent.

(*b*) The next type to be noted in Britain is the Celtic shorthorn, *Bos longifrons*, a much smaller animal. Patches of hair have been found in the caves, and these are said to be of a dark colour. Professor Wilson is strongly of opinion that *B. longifrons* was mainly black. It has been said to be red or black.

(*c*) The next invasion, that of the Romans, brought the Roman white cattle, which probably had black markings. The Romans formed joint settlements with the British, and a Romano-British cattle resulted in the neighbourhood of Roman settlements, from crossing the Roman and Celtic types.

(*d*) In the next place we have a Saxon invasion, bringing with it Saxon cattle. There was probably some mixture of breeds on the Celtic and Saxon frontiers, but generally speaking Saxon cattle occupied the parts from which the Romano-British population had been expelled. These cattle were red.

(*e*) Following on the Anglo-Saxon was a Dutch invasion of cattle into the east of England. By Dutch we understand from the "Low Countries." This invasion began in the middle ages and continued till a century ago, and again involved a certain amount of mixture. The Dutch cattle were probably "flecked," the flecking being white with black, red, or brown. The Englishman developing a prejudice against black, we may suppose the reddish mixture to remain, and to have become the basis of roan.

Before, therefore, the beginning of pedigree cattle-breeding, or earlier in the sixteenth century say, we should expect a distribution of cattle races much as follows :

Wales and Scotland, Celtic or black cattle, with some of Roman descent and perhaps some whole reds.

Midland England, very mixed cattle, compounded of the following races in order : Celtic, Roman, Saxon and Dutch. The colours were black, white with black or brown points, red and flecked. Round the edges of the country the breeds were comparatively pure, but in the Midlands, where they all met, they were a mixture ; the Romano-Celtic element probably prevailing.

Southern England, Anglo-Saxon, red.

Eastern England, Dutch cattle, flecked or roan.

The purest British cattle would thus be found in Wales, Scotland (and West Ireland), where black is the prevailing colour.

The Romano-British *were* up the west and middle of England, where they still survived to about a century ago. They were the old longhorn breeds.

The wild white cattle of Chillingham, Chartley, and elsewhere, are Roman cattle run wild owing to the unrest of the Anglo-Saxon, and later and more especially the Danish invasions. Traces of the Roman cross are still to be seen among Scottish, Welsh, and Irish cattle. The wild white cattle have black muzzles and black or brown points; that is their hair is brown, or dark at the ears, round the eyes and muzzle and often at and below the knees.

The Anglo-Saxon red cattle, which were once spread over the south of England, are still to be found in the red cattle of Norfolk, Sussex, Devon, and Hereford.

The Romano-British or longhorns were some black, some white, but mainly flecked, and were largely driven out by the flecked Dutch, although some mixture probably took place.

It will be seen that Professor Wilson's account differs to some extent from both those of Boyd Dawkins and McKenny Hughes.

Upon the ingredients just referred to the breeders had to work when pedigree cattle-breeding, which is scarcely more than a century old, came into vogue. The shorthorn has possibly arisen from four races, the Celtic, a Romano-British, an Anglo-Saxon, and the "Dutch*", and even some of these are mixtures.

Confining our attention to coat-colour, the black Celtic, if it contributed anything, must have been bred out early. But it is doubtful whether there was not a red Celtic, and a red Galloway heifer (1760–1770) appears to have been connected with the beginnings of the breed. Thus the shorthorn red may have had three sources, an Anglo-Saxon red, the red of the Dutch flecking, and the supposed Celtic red. The white may have come through the Romano-British, through an Anglo-Saxon white or possibly through the white in the Dutch. The particolours and the roans are of equally doubtful origin, although it probably is safe to assert that they are due to the breeds of latest importation; and it thus seems fairly impossible to determine *à priori* how many distinct red, roan, particolour, or white types may really exist in the case of the shorthorn. The importance of this statement for any Mendelian interpretation must be obvious. We may have reds which are dominant, recessive or even heterozygous to white or even to other reds, and the search for a Mendelian formula becomes very elusive.

If we turn from the possible ingredients of the shorthorn breed to the history of its origin we find matters still more difficult to disentangle.

There appears, by the will of John Percy of Harum, to have been a breed of shorthorns in Yorkshire as early as 1400, and these were not whole colour. In

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* Their nearest Continental cousins to-day are the German *Fleck-Vieh*.

1640 we again hear of a shorthorn breed; and a native race with short horns, large bodies, and black or red in colour, existed in Holderness and on the banks of the Tees in the middle of the eighteenth century. A new breed of shorthorns was introduced from Holland in the eighteenth century, and much improvement was attributed to the introduction of Dutch bulls, but it is said that these were the offspring of English shorthorn cows sent by James II to William of Orange. In the early shorthorn pedigrees we find some black, flecked, and spotted beasts. However debateable the origin of the shorthorn may be, it appears to have existed as a distinct breed by the middle of the eighteenth century, and possibly 50 or 100 years previously. Its source is probably Yorkshire and Durham with Dutch reinforcements. The early breeders seem to have had much the same colours, red, roan, white, and their mixtures. One noteworthy early breeder introduced a cross which is supposed to have become a potent factor with a *red* Galloway polled cow. The blacks and fleckeds rapidly disappeared. The proportion of colours was, however, somewhat different. Thus from pedigrees before 1850 we find:

Red	R. W.	Roans	Whites	
14·5	42	38	5·5	per cent.

and from recent statistics:

Red	R. L. W.	R. W.	Roan	White	
30	6·5	4·5	54	5	per cent.

Thus the whites have remained about stationary in number, but the whole reds and roans have increased at the expense of the particoloured, and this even if we throw the red with white markings into the particolour.

It cannot be said that this brief examination[*] casts much light on the possible components of the modern shorthorn. We do not know the actual constituents of the early Yorkshire shorthorn. It suggests that the white may have come from Holland as a whole colour and not be a contribution of Saxon or Romano-British factors. We see a definite Dutch contribution and a possible Celtic red through Collings' Galloway cross. We do not know any more certainly whether an Anglo-Saxon red also contributed. Failing this, we are thrown back on actual breeding experiments to disentangle possible Mendelian factors.

(6) *Possible results of the various colour matings and Mendelian interpretations.*

There are fifteen colour crossings possible with our five shorthorn colour categories, and since each crossing might give offspring of one of the five categories, 75 possibilities arise.

[*] See: *Thomas Bates and the Kirklevington Shorthorns*, by C. J. Bates, 1897. Bell's *History of Improved Shorthorn Cattle*, 1871. B. Rudd: *An Account of some of the Stock of Shorthorned Cattle of Ch. and R. Collings*, 1821; *Origin and Pedigree of the Sockburn Shorthorns*, 1822. Lewis F. Allen: *The American Herdbook, to which is prefixed a Concise History of English and American Shorthorns*, 1856, and *History of American Cattle*, 1868.

These categories are arranged in the following table:

TABLE I.

Mating	Possible Offspring				
	R.	R. L. W.	R. W.	Ro.	W.
R. × R.	×	×	×	×	*wp*
R. × R. L. W. ...	×	×	×	×	?
R. × R. W. ...	×	×	×	×	*wp*
R. × Ro.	×	×	×	×	×
R. × W.	×	×	×	×	*wp*
R. L. W. × R. L. W.	×	×	×	×	*wp*
R. L. W. × R. W. ...	×	×	×	×	*wp*
R. L. W. × Ro. ...	×	×	×	×	*wp*
R. L. W. × W. ...	*wp*	?	*wp*	×	*wp*
R. W. × R. W. ...	×	×	×	×	*AHB*
R. W. × Ro. ...	×	×	×	×	*wp*
R. W. × W. ...	×	*wp*	×	×	*wp*
Ro. × Ro.	×	×	×	×	×
Ro. × W.	×	×	×	×	×
W. × W.	*wp*	?	*AHB*	*wp*	×

All the categories marked × were found in the first random sample of 2172 calves taken out of the Herdbook. Some of course are infrequent, but their relative frequency will be found at once by examining Tables II and III. Those marked *wp* came to light in following up the pedigrees of white cattle. They are taken from vols. 37 to 49, in the bulk of cases from vols. 47 to 49; they are not exhaustive, but in most cases several schedules with each type occur in our record. The two cases marked *AHB* were noted in a single volume of the *American Herdbook*. No stress is laid on them.

Thus out of the 75 possible arrangements all but three have been found, and these three are all concerned with the comparatively scarce category of red cattle with white markings. Many of the 75 groups are rare, some excessively rare. It is always possible to assert that the rarest of them are due to misprints or to breeders' mistakes. We have made enquiries of breeders and in some cases additional instances have been provided, but we have kept to the cases actually on Herdbook record*.

Take as an illustrative case $W \times W$. This is a comparatively rare cross; because some breeders think white cattle delicate, others do not breed it because the produce have less sale and at lower prices in the export market. It is either unfashionable or unprofitable. Most breeders assert that $W \times W$ always gives W; it *generally* does, but their experience when you come to actual numbers must be very limited. In fourteen volumes of the Herdbook containing about 57,000 calves, 91 cases only of $W \times W$ were found, though quite possibly

* Mr T. Milne tells us, for example, of an all red heifer turned down with two young bulls, one all red and the other *RLW*, producing from one or other a white calf.

some few were overlooked.   So that the cross is one of extreme rarity.   In these 91 cases the calves were in 86 cases white, in four cases roan*, and in one case red†. The *American Herdbook* examined for other matters produced incidentally a case of $W \times W = RW$.   Thus *Coates' Herdbook* indicates that in $5\frac{1}{2}$ per cent. of cases white does not breed true to white.   We are therefore driven to the conclusion that, if the records are to be trusted, there are latent determinants in white short-horns or in some white shorthorns which under certain circumstances can be called into play.   This view is confirmed by another consideration.   Some wild white cattle, like the late Chartley Herd with its representatives at the Zoological Gardens, have black muzzles and black ear-linings.   They are reputed to drop occasionally black or red calves.   Other herds of wild white cattle have black noses and red inside the ears, and have bred absolutely true to this type for at least a century.   As we have seen, the white shorthorns breed generally true to their type.   The cross wild white cow × shorthorn white bull gives a hybrid pure white cow, but such pure white cows, if crossed with a shorthorn white bull, may produce a considerable percentage of both roan and red as well as white calves.   In other words, while wild white cattle of the type considered breed absolutely and the white shorthorns sensibly true to type, and while the hybrid is pure white, the hybrids' offspring by white shorthorns segregate into colours occurring in the near ancestry of the shorthorn, and possibly but not probably in the distant ancestry of the wild white.   These facts, coupled with the experience that white shorthorns if crossed will, although rarely, give roans and reds seem to show the existence of latent colour determinants in the whites depending upon their ancestral history.   The phenomena described are similar to those noted in albino white mice, which carry in latent form traces of their colour ancestry, traces which can become patent under various crossings.

The simplest Mendelian formula would be based on considering the white as recessive and the red as dominant, and the roan as hybrid, i.e.

$$(RR) \times (WW) = (Ro),$$

and this is *roughly* an expression of some of the facts of the case.   Thus $(WW) \times (WW)$ gives (WW) in all but $5\frac{1}{2}$ per cent. of cases.   In 196 crosses of $(RR) \times (RR)$ we find 156 give (RR), or in about 79·5 per cent. of cases.   But, besides about 3 per cent. of roans, we have 12 per cent. of calves with white markings and 5·5 per cent. of particoloured.   We are thus compelled to consider all red, as we have just con-sidered all white, as either consisting of several subclasses only to be discovered by breeding, some all reds being really heterozygous, or else as containing latent white marking and particolour determinants.   But the difficulty does not stop here, there appear to be authentic cases of $(RR) \times (RR)$ giving (WW).   Thus we find Favourite Duke (R) got out of Clara 3rd (R) not only a normal red bull calf, but a red and white cow calf and the *white* cow calf Clara 5th‡ and the (R) Heydon

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* Bull calves: Unnamed, Vol. xxxvi. p. 277; Shaftesbury, Vol. xxxvii. p. 292.   Cow calves: Crystal Valley, Vol. xliv. p. 273; Seam, Vol. xlviii. p. 809.   Other cases in *American Herdbook*.

† Cow calf, Bright Anna 2nd, Vol. xliii. p. 318.   ‡ *Coates' Herdbook*, Vol. xli. p. 623.

Rose crossed by Earl Clarence 2nd (R) gave the *white* cow calf Minnette *. Such cases may be very rare indeed but, if authentic, reduce the Mendelian formula to a rough empirical statement of a statistical ratio; they are inconsistent with any theory of a pure gamete.

If we pass from the question of purity in the supposed homozygous elements to the first hybrid generation, we find in the Tables II. and III. of our random samples 90 cases of $(RR) \times (WW)$, and the result is (Ro) the assumed hybrid in 85 cases or in 94·5 per cent. of cases, but particoloured (RW) has occurred in two cases, white marking in one case and whole *red* in a third; while travelling beyond the field of our working sample $(RR) \times (WW)$ is actually recorded to have given (WW). Crossing the hybrid generation together, we should expect:

$$(Ro) \times (Ro) = (RR) + 2(Ro) + (WW).$$

Our actual numbers on the random samples are:

$$514\,(Ro) \times (Ro) = 86\,(RR)\dagger + 31\,(RLW) + 35\,(RW) + 278\,(Ro) + 84\,(WW),$$

as against the Mendelian formula:

$$= 128{\cdot}5\,(RR) \qquad\qquad\qquad + 257\,(Ro) + 128{\cdot}5\,(WW).$$

The (Ro) is thus seen to be, like (RR) and (WW), a non-homogeneous group; we are compelled to suppose that some (Ro) have white marking and others particolour determinants latent in them. If we cross the hybrid (Ro) by the original stocks, we have:

$$656\,(Ro) \times (RR) = 243\,(RR) + 46\,(RLW) + 39\,(RW) + 324\,(Ro) + 4\,(WW),$$

as against the Mendelian formula:

$$= 328\,(RR) \qquad\qquad\qquad + 328\,(Ro).$$

The close approximation to the Mendelian number of (Ro) is noteworthy, but the appearance of $4\,(WW)$ is again impossible unless some of the reds are to be treated as heterozygous. The latent white marking and particolour determinants are again manifestly needed.

Lastly, we take the case of $(Ro) \times (WW)$:

$$74\,(Ro) \times (WW) = 1\,(RR) + 1\,(RLW) + 1\,(RW) + 47\,(Ro) + 24\,(WW),$$

as against the Mendelian formula:

$$= \qquad\qquad\qquad\qquad\qquad 37\,(Ro) + 37\,(WW).$$

Here again we are compelled to assert a heterozygous white, or if we treat the individual instances of (RR), (RLW), and (RW) as heterozygotes and club them with the (Ro), the proportion 50 to 24 is very far from the Mendelian equality. Of course the same process could not then be applied to explain the anomalies in the crosses $(Ro) \times (Ro)$ and $(Ro) \times (RR)$. The anomalous cases in these crosses can

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* *Coates' Herdbook*, Vol. xxxviii. p. 231.

† No attempt is made to express anything but (RR) and (WW) in Mendelian form, the constitution of the other individuals being unknown.

be multiplied by seeking for them and, not doing as we are here doing, dealing with the material provided in the pedigrees of a random sample from a few volumes of the *Herdbook*. It would thus seem that no simple Mendelian formula can possibly fit the shorthorn cases. Roughly, such a formula approaches the data in one or two points, but the roughness appears inconsistent with a theory of Mendelism being due to the purity of gametes. It is of course clear that the introduction of a complex allelomorph may improve matters, or the differentiation of whites and reds into different classes, homozygous and heterozygous. Increase in the number of available variables usually does give better fits. But the attempt to differentiate cows by the results of breeding from them must shatter on the paucity of their offspring. A cow will frequently give no more than 8 to 10 calves, and, bred even to the same bull, or to the same type bulls, it may not be till the 9th or 10th calf that she diverges from uniformity in her offspring. Thus one cow may throw 8 white calves and only the 9th be red or roan. This type of experience is not uncommon, and breeders not infrequently attribute it to the effect of age on the cow. To those who believe in individual prepotency—a view hardly consistent with Mendelism—this influence of age may not appear impossible. In the next place, the prejudice against white crosses on the part of breeders, indeed the dislike to white shorthorns in general, renders it almost impossible to effectively test the purity of any (RR) or (WW) bull, although his offspring may be fairly numerous. Further, the bulls are often mated solely within their particular owner's strain, and it is not so evident what would happen if they were mated outside it. This tendency to assortative mating among shorthorn breeders will be discussed more fully in another section. We have seen that there is historically a possibility of two strains of red and two strains of white having been mingled in the shorthorn. Determinants representing parti-colour and white markings can undoubtedly be introduced also—we confess to having made an attempt from this standpoint, which shattered with further examination of Table I.—but the introduction needs a wider practice than we can boast of in inventing Mendelian formulae, and until we are more convinced than we are at present of the soundness of such formulae we should prefer to leave the invention to those who have had it. *Coates' Herdbook* presents a wide range of material, and whatever we may think of the categories selected, the record has been made by persons in absolute ignorance of recent controversies about heredity. It is therefore really impartial material for Mendelians to unravel.

For the remainder of this paper we shall confine ourselves to the biometric treatment of the problem, which, as we have already noted, is independent of the existence or non-existence of any Mendelian formula. We shall compare the results with other investigations of a like kind and endeavour to interpret the results in biometric language.

(7) *Assortative Mating in the Shorthorn Population.*

Three sets of matings were worked out, namely : (*a*) the very recent matings of the parents of calves entered on the pedigree schedules, (*b*) the matings of the

great-great-grandparents of the calves on these schedules, and (c) a special series of matings taken from the early volumes of the *Herdbook*. Thus we have measures of the extent to which breeders now, 15 or 20 years ago, and perhaps 50 years ago, have mated like with like. The Tables each contain about 825 matings, and are given as Tables A, B, C of the Appendix. The general measure of the deviation from random mating was obtained by calculating the mean square contingency coefficient. The results were as follows:

Table of Assortative Mating Coefficients.

Recent Times :	·2030
15—20 Years ago :	·1912
50 Years ago :	·1219

It will thus be seen that the tendency of breeders to mate like with like has been very sensible for many years, and has now reached a value comparable with the unconscious assortative mating in the case of man [*], or with that which breeders of fancy mice consciously or unconsciously produce[†]. This selective action ought to be taken into account when we endeavour to estimate the resemblance of parent to offspring.

(8) *On Direct Inheritance. First Degree.*

The usual four tables, sires and dams with bull and cow calves, were formed. These tables will be found as D, E, F, and G of the Appendix. They were worked out by mean square contingency, and, for the sake of comparison and control, also by mean contingency and by fourfold tables. In order to provide material, on which it might be possible to consider Mendelian formulae, Tables II. and III. are provided, giving both sire and dam for over 1000 bull and 1000 cow calves respectively. These tables have been already used in the previous section. They differ from the mating Table A, in that the same cow and bull are repeated not more than three times, while in the mating tables they are entered as often as they occur on the schedules.

A word must be said here about the contingency method as applied to these tables. The coefficient of mean square contingency is an absolute measure of the dependence of the offspring upon the parents, as long as we stick to the five groups now dealt with, but it would most probably be increased in value if we subdivided these groups—for example, divided the reds and roans into light and dark reds and roans. It has been pointed out elsewhere that this subdivision, without we immensely increase the number of individuals dealt with, must be limited, or we shall obtain very large contributions to the contingency from isolated units, which cannot of course be subdivided. Experience in comparing tables reduced by different methods seems to indicate that 25- to 49-fold contingency tables approach closely in average value to the results reached by 4-fold table methods[‡]. It would have been a great advantage in the case

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[*] *Biometrika*, Vol. iii. pp. 373 and 487.   [†] *Biometrika*, Vol. iii. p. 20.
[‡] *Biometrika*, Vol. iii. p. 253.

of the shorthorn tables to have been able to compare the 25-fold table results with those for 36- and 49-fold tables; and thus determine how far we were approximating to a limit. This would be the more important if we had any method of differentiating the large classes of reds and roans, which we have seen may quite possibly have sprung from two original races. As no sub-classification is provided by the *Herdbook*, this cannot be attempted, and we are left with the

TABLE II.

Distribution to Sires and Dams of ♂ Offspring.

Coat-Colour of Sire.

	Offspring	Red	Red, Little White	Red and White	Roan	White	Totals
Red	R. ...	75	10	5	68	1	159
	R. L. W....	10	4	3	13	—	30
	R. W. ...	5	3	—	4	1	13
	Ro. ...	4	1	1	91	27	124
	W. ...	—	—	—	2	—	2
	Group Total	94	18	9	178	29	328
Red, Little White	R. ...	6	2	—	8	—	16
	R. L. W....	9	1	1	5	—	16
	R. W. ...	3	1	2	3	—	9
	Ro. ...	—	1	—	15	6	22
	W. ...	—	—	—	—	—	—
	Group Total	18	5	3	31	6	63
Red and White	R. ...	14	2	3	10	1	30
	R. L. W....	8	1	1	8	—	18
	R. W. ...	5	2	9	10	—	26
	Ro. ...	3	2	—	40	9	54
	W. ...	—	—	—	—	—	—
	Group Total	30	7	13	68	10	128
Roan	R. ...	56	7	6	45	—	114
	R. L. W....	12	5	—	18	—	35
	R. W. ...	14	3	—	13	—	30
	Ro. ...	79	25	14	142	7	267
	W. ...	—	—	—	53	5	58
	Group Total	161	40	20	271	12	504
White	R. ...	—	—	—	1	—	1
	R. L. W....	1	—	—	1	—	2
	R. W. ...	—	—	—	—	—	—
	Ro. ...	26	4	3	18	—	51
	W. ...	—	—	—	8	—	8
	Group Total	27	4	3	28	—	62
	Grand Total	330	74	48	576	57	1085

Coat-Colour of Dam.

TABLE III.

Distribution to Sires and Dams of ♀ Offspring.

Coat-Colour of Sire.

	Offspring	Red	Red, Little White	Red and White	Roan	White	Totals
Red	R. ...	81	13	5	58	—	157
	R. L. W....	13	8	3	7	—	31
	R. W. ...	6	—	3	11	—	20
	Ro. ...	2	—	4	76	13	95
	W. ...	—	—	—	2	—	2
	Group Total	102	21	15	154	13	305
Red, Little White	R. ...	12	6	—	14	—	32
	R. L. W....	4	5	—	5	—	14
	R. W. ...	2	3	—	5	—	10
	Ro. ...	1	—	1	28	4	34
	W. ...	—	—	—	—	—	—
	Group Total	19	14	1	52	4	90
Red and White	R. ...	17	2	3	10	—	32
	R. L. W....	4	3	—	7	—	14
	R. W. ...	18	8	9	11	—	46
	Ro. ...	1	3	1	41	9	55
	W. ...	—	—	—	—	—	—
	Group Total	40	16	13	69	9	147
Roan	R. ...	61	12	8	41	—	122
	R. L. W....	14	4	3	13	—	34
	R. W. ...	10	6	5	22	—	43
	Ro. ...	78	17	17	136	7	255
	W. ...	—	—	—	31	4	35
	Group Total	163	39	33	243	11	489
White	R. ...	—	—	—	—	—	—
	R. L. W....	—	—	—	—	—	—
	R. W. ...	2	—	1	1	—	4
	Ro. ...	19	3	5	15	—	42
	W. ...	—	—	—	7	3	10
	Group Total	21	3	6	23	3	56
	Grand Total	345	93	68	541	40	1087

Coat-Colour of Dam.

somewhat overweighted groups of red and roan—amounting to nearly $\frac{1}{3}$ and $\frac{1}{2}$ of the total offspring—to deal with. These large sub-groups are likely to reduce the mean square contingency coefficients, and we have no means of really testing how far we have already approached a limiting value of the mean square contingency coefficient.

In dealing with the material by the 4-fold table method, we considered that the best division we could make was between beasts with some whole red and

beasts with no whole red, the latter class including roans and whites.   Table IV.
gives our results:

TABLE IV.

Parental Inheritance in Shorthorns.

Nature of Percentage	Mean Square Contingency	Mean Contingency	Fourfold Table
Sire and Bull Calf ...	·37	·43	·46
Sire and Cow Calf ...	·40	·50	·52
Dam and Bull Calf ...	·40	·44	·44
Dam and Cow Calf ...	·42	·46	·43
Mean	·40	·46	·46

The result seems to us to indicate that the values by mean square contingency
have come out somewhat too small.   The general effect of an assortative mating
if there were no selection of parents would be to increase the apparent parental
correlation.   If r were the real correlation, ρ the assortative mating, and $\mathfrak{r}$ the
observed parental correlation, we should have

$$\mathfrak{r} = r \, \frac{1 + \rho}{\sqrt{1 + 2\rho r^2}}.$$

If $r = \cdot 5$ and $\rho = \cdot 2$, we should expect the correlation altered by about $\frac{1}{10}$
increment.   But this assumes that there has been no selection of parents,
whereas in the case of the bulls at least there is obviously very considerable
selection.   The tables show that the red whites of both kinds are rejected, and
that red and roan bulls are the most fashionable.   But we know that any
stringency in parental selection is more effective in reducing parental correlation
than assortative mating can be in raising it[*].   The amount of this cannot be
determined in the present case, but it very probably contributes to the reduction
observed in the above table below the values determined for horse and grey-
hound.

The general conclusion that we can draw is that the parental resemblance
to offspring in shorthorn cattle exceeds ·4, and may quite reasonably equal ·46,
or even a higher value.   It lies therefore well within the range of values ·4 to ·5
found already for man, horse, and dog.

(9)   *On Direct Inheritance.   Second and Higher Degrees.*

We now turn to the degree of resemblance between the offspring and more
distant ancestry.   The following results were obtained[†]:

[*]   "Mathematical Contributions to the Theory of Evolution," *Phil. Trans.* Vol. 200 A, p. 39.

[†]   The tabulated material will be found in the Appendix as Tables H, I, J, K, L, M, N, and O.   No
individual grandparent was taken more than six times, and on the average no individual occurs more
than three times.

TABLE V.

Grandparental Inheritance in Shorthorns.

		Sex change
Paternal Grandsire and Bull Calf ...	·19	0
Paternal Grandsire and Cow Calf ...	·25	1
Maternal Grandsire and Bull Calf ...	·18	2
Maternal Grandsire and Cow Calf ...	·17	1
Paternal Grandam and Bull Calf ...	·15	1
Paternal Grandam and Cow Calf ...	·16	2
Maternal Grandam and Bull Calf ...	·24	1
Maternal Grandam and Cow Calf ...	·25	0
Mean =	·20	

It will be seen at once that this is lower than the results obtained for man, horse, and dog; i.e. about ·2 instead of ·3. This is what we might expect, as the parental correlations found for the shorthorn by mean square contingency are about ·4 instead of the ·5 for pigmentation in man, horse, and dog. We are not prepared to assert, however, that it connotes an actual lessening of the intensity of inheritance, because other methods in the parental case seem to indicate that the resemblance is really greater than is shown by mean square contingency with only a classification into five very unequal groups.

The values are clearly somewhat irregular. If we group them according to the number of sex changes we find:

Grandparental resemblance with no sex change = ·22
,, ,, one ,, = ·20
,, ,, two sex changes = ·17

It would thus appear that, as in the case of eye-colour in man, there is a weakening of the intensity of resemblance with each change of sex. But if this be more than an *appearance*, it must be screened by other factors in the case of parentage, for in that case the values are ·40 for no change of sex and ·40 for one change of sex. This result is remarkably consonant with what we have previously observed in the case of greyhounds* where we found no sensible influence of change of sex in the parental resemblance, but an apparent influence on the grandparental relation. In eye-colour in the case of man it was found throughout all relationships†. We failed to demonstrate a change of sex influence in the case of coat-colour in horses‡, and it seems worth considering the matter further in shorthorns. Accordingly in attempting to estimate the great-grandparental influence, when we gave up the excessive labour of forming the 16 great-grandparental tables which would have meant dealing with upwards of 16,000 entries, and a far

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* *Biometrika*, Vol. III. p. 258.

† Pearson: *Phil. Trans.* Vol. 195 A, pp. 114–117. Lutz: *Biometrika*, Vol. II. pp. 237–240.

‡ *Ibid.* Vol. II. p. 232.

larger number than this of pedigree great-grandparental relations *, we have simply classified into great-grandparents and (i) male, (ii) female descendants, with 0, 1, 2, and 3 changes of sex. This required the preparation of 8 instead of the full 16 tables. If we write the pedigree in the following manner:

$$8\,\male = 9\,\female \qquad 10\,\male = 11\,\female \qquad 12\,\male = 13\,\female \qquad 14\,\male = 15\,\female$$

$$4\,\male \qquad = \qquad 5\,\female \qquad\qquad 6\,\male \qquad = \qquad 7\,\female$$

$$2\,\male \qquad\qquad = \qquad\qquad 3\,\female$$

$$0\,\male \qquad\qquad\qquad 1\,\female$$
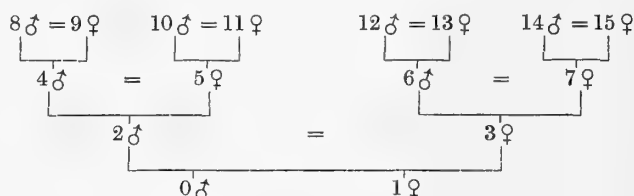
then:

8 with 0	gave Table P of	great-grandsire and ♂ offspring,	no sex change
15 with 1	„ Q „	grandam and ♀	„	„	„
8, 12, 14 with 1	„ R „	grandsire and ♀	„	one	„
9, 11, 15 with 0	„ S „	grandam and ♂	„	„	„
10, 12, 14 with 0	„ T „	grandsire and ♂	„	two changes
9, 11, 13 with 1	„ U „	grandam and ♀	„	„	„
10 with 1	„ V „	grandsire and ♀	„	three „
13 with 0	„ W „	grandam and ♂	„	„	„

The results obtained, still by the method of mean square contingency, are given in the table below:

TABLE VI.

Great-Grandparental Inheritance in Shorthorns.

	Sex change	Resemblance
Great-Grandsires and ♂ Offspring ...	0	·20
„ „ „ „	2	·12
„ „ ♀ „	1	·18
„ „ „ „	3	·17
Great-Grandams and ♂ Offspring ...	1	·15
„ „ „ „	3	·15
„ „ ♀ „	0	·26
„ „ „ „	2	·12
		Mean = ·17

These results are irregular, and not very conclusive; probably large enough numbers were not taken in several of the cases, but the tabulation of great-grandparents in large numbers is by no means an easy task, as in a pedigree stock high ancestors tend to be repeated many times. But in both grandsires and grandams, if we do not club together ♂ and ♀ offspring we find that no change of sex is

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* The process throughout has been the same in all the ancestral tables; the produce is taken once only, and the same ancestor occurs on an average not more than three times, although individuals occur much more frequently, because as we go back there is much repeated blood owing to the fashion in bulls, and it would be almost impossible labour to reach 1000 independent great-great-grandsires. This method was adopted to avoid overweighting the tables with any particular individuals. The same principle was also used in the horse tables.

more potent than two changes (and than either one or three). There does not, however, seem to be much marked difference between one and three changes. On the whole it seems probable that a change of sex does produce some slight lessening in the ancestral resemblance beyond the parents; it is marked in eye-colour in man, and just appears in the coat-colours of greyhound and shorthorn, although if it exists in coat-colour of horses it has been screened by the probable errors of the results.

The mean result for eye-colour in man for great-grandparental resemblance is ·19, and for horse's coat-colour ·20, both results sensibly higher again than that for shorthorns.

Still one further ancestral resemblance has been worked out, namely, that between great-great-grandparents and the offspring. But the complete set of 32 tables has been replaced by four dealing only with each sex in the ancestry and the offspring regardless of the exact line of descent. The results are given below*:

TABLE VII. *Great-Great-Grandparent and Offspring.*

Great-Great-Grandsire and ♂ Offspring ...	·09
Great-Great-Grandsire and ♀ Offspring ...	·14
Great-Great-Grandam and ♀ Offspring ...	·13
Great-Great-Grandam and ♂ Offspring ...	·13
	Mean = ·12

The only comparable result is the combined table for all sexes provided by Dr Alice Lee† in the case of coat-colour in horses, and this gave for the great-great-grandparent and offspring the value ·15, thus agreeing with the previous cases in giving a higher value than the shorthorn results do.

We can now put together our results for ancestry in shorthorns and place alongside them those for other pigmentation researches:

TABLE VIII. *Ancestral Resemblance.*

Species	Man‡	Horse‡	Basset§ Hound	Greyhound ‖	Shorthorn	Mean Result
Character	Eye-Colour	Coat-Colour	Coat-Colour	Coat-Colour	Coat-Colour	Pigmentation
Method employed	Fourfold Table	Fourfold Table	Fourfold Table	36-fold Contingency	25-fold Contingency	Various
Parent	·49	·52	·53	.53	·40	·49
Grandparent	·32	·30	·22	·33	·20	·27
Great-Grandparent ...	·19	·19	—	—	·17	·18
Great-Great-Grandparent	—	·15	—	—	·12	·13

* Given as Tables X, Y, Z, Ω, in the Appendix.   † *Biometrika*, Vol. ii. p. 235.   ‡ *Ibid.* Vol. ii. p. 222.
§ *R. S. Proc.*, Vol. lxvi. p. 157, omitting the cases of inheritance through the sire, as there can be little doubt that the sires are unreliable.   ‖ *Biometrika*, Vol iii. pp. 254–8.

But we do not consider these results as they stand as *strictly* comparable. We believe the lower values in the case of the shorthorns are not due to a lesser intensity of inheritance, but to the fact that the only possible contingency table is not finely enough divided. This we hold is indicated by the fourfold and mean contingency methods giving sensibly higher values than the mean square contingency method (see p. 448).

It appeared desirable to determine the best geometrical series: a, ar, ar^2, ar^3, to fit the shorthorn results. Accordingly we made:

$$(\cdot 40 - a)^2 + (\cdot 20 - ar)^2 + (\cdot 17 - ar^2)^2 + (\cdot 12 - ar^3)^2$$

a minimum. This led to the least square type equations:

$$\cdot 40 + \cdot 20r + \cdot 17r^2 + \cdot 12r^3 = a\,(1 + r^2 + r^4 + r^6),$$
$$\cdot 20 + \cdot 34r + \cdot 36r^2 \qquad = a\,(r + 2r^3 + 3r^5).$$

Eliminating a, r is given by:

$$\cdot 17r^7 + \cdot 28r^6 + 1\cdot 20r^5 - \cdot 04r^4 + \cdot 63r^3 - \cdot 36r^2 + \cdot 06r - \cdot 20 = 0.$$

The appropriate root of this is $r = \cdot 64$ nearly, giving $a = \cdot 382$, and the best fitting geometrical series is $\cdot 38$, $\cdot 24$, $\cdot 16$, $\cdot 10$, as against the observed $\cdot 40$, $\cdot 20$, $\cdot 17$, $\cdot 12$. But the remarkable result of the investigation is that the factor of reduction for the geometrical series in the case of eye-colour in man was found [*] to be $\cdot 62$, and for coat-colour in horses $\cdot 66$[†], and the reducing factor for shorthorns is $\cdot 64$, exactly their mean. The reducing factor for greyhounds is seen to be about $\cdot 62$[‡]. Thus disregarding absolute values we see that the manner in which ancestral resemblance decreases with ascent is very closely the same for such different species as man, horse, greyhound, and shorthorn, and for most purposes may be taken equal to the round number 2/3. If we assume $r = 2/3$, we find the best value of a for the shorthorn is almost exactly $\cdot 37$, giving the series $\cdot 37$, $\cdot 25$, $\cdot 16$, and $\cdot 11$, against the observed $\cdot 40$, $\cdot 20$, $\cdot 17$, and $\cdot 12$. It would thus appear that the general effect of applying the 25-fold mean contingency method has not been to lower the relative intensity of the successive grades of ancestry, but to weaken in about the same proportions the absolute values. The factor of reduction is about 38/50, or we should have for shorthorns the best fitting series,

$$\cdot 50, \qquad \cdot 32, \qquad \cdot 21, \qquad \cdot 13,$$

as against an observed series,

$$\cdot 53, \qquad \cdot 26, \qquad \cdot 22, \qquad \cdot 16.$$

Either or both of these are in quite reasonable agreement with what has been observed for man and horse.

Generally we may conclude that ancestral influence in the shorthorn diminishes at the same rate sensibly as in man, horse, and dog, and that absolute values are

[*] *Biometrika*, Vol. II. p. 222.

[†] *Ibid.* Vol. II. p. 222.

[‡] The mean of Dr Warren's rather irregular results for *Hyalopterus* and *Daphnia* is again $\cdot 61$. *Biometrika*, Vol. I. p. 139.

probably closely alike in the four cases, although the categories in the case of the shorthorn render the comparison harder.

(10) *On Collateral Inheritance.*

The work of tabling pairs of whole siblings is somewhat lengthy in the case of shorthorns, the breeding life of a cow is short, and there is a good deal of changing in the case of the bull used. Accordingly we have contented ourselves with the cases of whole brothers and whole sisters*. It is much easier to table cases of half siblings, and six tables of half siblings—brother-brother, sister-sister, brother-sister, the bull parent being common, and again the cow parent being common—have been constructed†. These half-sibling tables are of peculiar interest, because there is a good deal of obscurity at present about the degrees of likeness we should expect to find between half siblings. *A priori* we should expect the degree of likeness of half siblings to be half that of whole siblings, but this relationship has not been verified on any of the material hitherto considered: half siblings are relatively more alike than whole siblings, the addition of another common parent does not double the resemblance.

In Table IX below we have put together the results with regard to whole and half siblings as far as pigmentation results have yet been worked out.

Several conclusions may be drawn from this table. In the first place we see that half siblings through the bull are more alike than half siblings through the cow, and this is so for all three combinations. It would thus appear that the bull is more influential than the cow, but there is no prepotency of the bull over the cow in the parental resemblances. We should thus have the paradox that the bull caused the offspring to be more alike than the cow does, but not by causing them to be more like himself! Notwithstanding that the whole-sibling resemblances in shorthorns are of the same general value as those for eye-colour in man or coat-colour in Basset Hounds or Greyhounds of different litters, our opinion is that they are too low. Our experience with the parental tables leads us to believe that the grade of the resemblance is nearer that for hair-colour in man or coat-colour in horses. But we should not expect the reduction to be as great in this as in the parental case, owing to the special emphasis breeders lay on certain classes of parents. It is quite clear that neither in horses nor in shorthorns is the degree of resemblance for half siblings half that between whole siblings. The ratio of resemblance of maternal half siblings (the only case worked out for horses) to whole siblings is for horses ·57, and for shorthorns is ·56. These results are closely in accord, and our impression that a constant ratio exists between half and whole sibling resemblance, and that this ratio is greater than ·5, is not shaken by the values in the case of Basset Hounds. We think that a very profitable line of investigation would deal with the relation of half to whole brethren in a variety of characters, and, if possible, with measurable characters as well as pigments. So far as we can see in this, as in other features of collateral inheritance, short-horns range themselves well within the limits already found for other species.

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* Tables α and β of the Appendix.   † Tables γ, δ, ϵ, ζ, η, θ, of the Appendix.

TABLE IX.

Collateral Inheritance.

Species	Man			Horse	Shorthorn	Basset Hound	Greyhound		Mean
	Eye-Colour		Hair-Colour	Coat-Colour	Coat-Colour	Coat-Colour	Coat-Colour, Studbook Returns		Pigmentation
Character	I	II							
							36-fold Contingency		
Reduction Method	4-fold Table	4-fold Table	4-fold Table	4-fold Table	25-fold Contingency	4-fold Table	Same Litter	Different Litter	Various
Whole Brothers	·52	·54	·62	·62	·52	·53	·52	·52	·55
Whole Sisters ...	·45	·52	·57	·69	·54		·60	·56	·56
Whole Brother and Sisters	·46	·53	·55	·58	—		·56	·51	·53
Half Brothers through Father	—	—	—	—	·34	All half siblings through Dam ·22*	—	—	·34
Half Brothers through Mother	—	—	—	·36	·30		—	—	·33
Half Sisters through Father	—	—	—	—	·36		—	—	·36
Half Sisters through Mother	—	—	—	·43	·29		—	—	·36
Half Sister and Brother through Father	—	—	—	—	·32		—	—	·32
Half Sister and Brother through Mother	—	—	—	·28	·30		—	—	·29
Mean result for whole siblings	·48	·53	·58	·63	·53	·53	·56	·53	·55
Mean result for half siblings	—	—	—	·36	·32	·22*	—	—	·33

(11) General Conclusions.

(a) With the exception of blue-grey cattle we are hardly likely to meet with a simpler colour combination in cattle than that to be found in the case of the shorthorn. If we consider the case of the cross of the polled black Galloway and the white shorthorn, then whether we deal with coat-colour or horns we find certain phases which suggest Mendelian ratios, but no simple Mendelian formula applies *rigidly* ; we find ourselves neglecting sensible percentages of occurrences incompatible with a theory of the pure gamete. The hybrids of two races which at any rate breed approximately true, like the white shorthorn and white wild cattle, throw a considerable range of colours when they are mated. Thus to describe the colour action of shorthorn crosses will need a differentiation of the primary colour categories into homozygous and heterozygous groups, or the introduction of a

* Not included in the general means of half siblings, as of doubtful validity.

somewhat complex system of latent characters, because practically every colour group will be found—with larger or smaller frequencies—to reproduce every other colour group. The only alternative is to reject the records in bulk as patently in error, because they do not fit rigidly a simple Mendelian scheme.

(b) When we turn to the statistical treatment of the data we are at once met by a further difficulty. The method of contingency enables us to free ourselves from any question of colour order or scale, and thus we are not troubled with any hypothesis as to a normal or Gaussian distribution. But while thus taking its place as the rational statistical process for dealing with colour data, we are dependent on breeders' colour classifications which are not made with a view to statistical treatment, and provide the statistician, as in this case of shorthorns, with groupings like those of red and roan not fine enough for his purposes. He obtains measures of intensity of resemblance comparable within the species—he can investigate the relative intensity of ancestral and collateral likenesses, but it is not so easy to compare the results for one species with a second. It would be a great advantage if red and roan could be recorded in light and dark shades, for the range in these colours is very wide and needs differentiation. With a 36- or 49-fold table we should undoubtedly get results better comparable with those for horses and greyhounds.

(c) Notwithstanding these difficulties we actually find, however, that the shorthorns fit quite reasonably into the general range of results for man, horse, and dog. The "decay" of ancestral resemblance was found to be ·64, a value lying between that found for man and dog, and expressible in round numbers by saying that the relationship decreases 2/3 at each upward grade. Thus the geometrical series feature of the ancestral law is maintained in the case of short-horns. The parental relationship lies between ·4 and ·5, and is probably nearer the latter than the former number; the whole sibling resemblance is a little over ·5 at a minimum, and appears to be very similar in range of value for that found for man and dog. There is a substantial assortative mating produced by breeders mating like with like, but while this would raise the apparent values of the con-tingency coefficients, the fact that red and roan are especially fashionable tends by its selective action to produce more than an equivalent lowering. Generally the shorthorn data strengthen our view that—whether it be Mendelian, or some other yet to be found—there is a single basal law of inheritance, manifesting itself notwithstanding assortative mating, selection, and environment in the clustering of the inheritance constants for all species of populations about the value ·45 to ·5 for parental inheritance, and about ·5 to ·55 for fraternal inheritance, with a geometrical series of a ratio roughly about 2/3 for ancestral grades. Such an ancestral law will, we are convinced, be found to flow from the physiological basal law if it be discovered. Meanwhile each fresh series as it is worked out biometrically tends to strengthen the view that apparent variations in the in-tensity of hereditary resemblance are not real variations in nature. The hereditary process is one, producing in a population definite degrees of average resemblance

between grades of kinship, and the apparent variation in intensity of inheritance is due to environmental influence, the selective action of breeders, assortative mating, and the difficulty of applying a uniform method of reduction to categories so varying in their extent and frequency as those we meet with in dealing with the records of quantitatively undetermined characters like pigmentation.

APPENDIX.

TABLES OF SHORTHORN MATING AND INHERITANCE.

I. ASSORTATIVE MATING PRODUCED BY BREEDERS' SELECTIONS.

A. *Modern Matings.*

Bull.

Cow.	R.	R. L. W.	R. W.	Ro.	W.	Totals
R. ...	69	18	7	129	23	246
R. L. W. ...	17	13	5	36	5	76
R. W. ...	12	7	4	43	10	86
Ro. ...	100	29	15	216	9	369
W. ...	16	4	5	20	3	48
Totals	224	71	36	444	50	825

B. *Matings of 15 to 20 Years ago.*

Bull.

Cow.	R.	R. L. W.	R. W.	Ro.	W.	Totals
R. ...	63	10	9	105	3	190
R. L. W. ...	11	4	1	19	5	40
R. W. ...	39	8	13	62	4	126
Ro. ...	142	30	40	188	18	418
W. ...	29	3	7	21	—	60
Totals	284	55	70	395	30	834

C. *Matings of 50 Years ago.*

Bull.

Cow.	R.	R. L. W.*	R. W.	Ro.	W.	Totals
R. ...	10	—	20	54	11	95
R. L. W. ...	—	—	—	—	—	—
R. W. ...	17	—	47	137	38	239
Ro. ...	38	—	64	292	41	435
W. ...	3	—	15	34	7	59
Totals	68	—	146	517	97	828

* Formerly recorded under R. W.

II. PARENTS AND OFFSPRING.

D. *Sires and Bull Calves.*

Sires.

Bull Calves.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	151	21	14	132	2	320
	R. L. W. ...	40	11	5	45	—	101
	R. W. ...	27	9	11	30	1	78
	Ro. ...	112	33	18	306	49	518
	W. ...	—	—	—	63	5	68
	Totals	330	74	48	576	57	1085

E. *Sires and Cow Calves.*

Sires.

Cow Calves.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	171	33	16	123	—	343
	R. L. W. ...	35	20	6	32	—	93
	R. W. ...	38	17	18	50	—	123
	Ro. ...	101	23	28	296	33	481
	W. ...	—	—	—	40	7	47
	Totals	345	93	68	541	40	1087

F. *Dams and Bull Calves.*

Dams.

Bull Calves.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	159	16	30	114	1	320
	R. L. W. ...	30	16	18	35	2	101
	R. W. ...	13	9	26	30	—	78
	Ro. ...	124	22	54	267	51	518
	W. ...	2	—	—	58	8	68
	Totals	328	63	128	504	62	1085

G. *Dams and Cow Calves.*

Dams.

Cow Calves.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	157	32	32	122	—	343
	R. L. W. ...	31	14	14	34	—	93
	R. W. ...	20	10	46	43	4	123
	Ro. ...	95	34	55	255	42	481
	W. ...	2	—	—	35	10	47
	Totals	305	90	147	489	56	1087

III. GRANDPARENTS AND OFFSPRING.

H. *Paternal Grandsire and Male Offspring.*

Paternal Grandsire.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	124	21	9	129	13	296
	R. L. W. ...	32	7	4	50	3	96
	R. W. ...	22	4	6	50	2	84
	Ro. ...	143	19	24	276	32	494
	W. ...	9	2	1	36	1	49
	Totals	330	53	44	541	51	1019

I. *Paternal Grandsire and Female Offspring.*

Paternal Grandsire.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	160	19	14	115	7	315
	R. L. W. ...	28	8	4	54	1	95
	R. W. ...	39	7	11	42	2	101
	Ro. ...	124	30	26	261	15	456
	W. ...	14	1	4	34	4	57
	Totals	365	65	59	506	29	1024

J. *Maternal Grandsire and Male Offspring.*

Maternal Grandsire.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	151	31	20	135	5	342
	R. L. W. ...	39	6	10	49	2	106
	R. W. ...	21	9	6	48	2	86
	Ro. ...	180	34	38	285	26	563
	W. ...	19	1	2	34	2	58
	Totals	410	81	76	551	37	1155

K. *Maternal Grandsire and Female Offspring.*

Maternal Grandsire.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	126	28	23	161	13	351
	R. L. W. ...	39	8	7	37	4	95
	R. W. ...	29	10	15	40	1	95
	Ro. ...	117	39	43	245	20	464
	W. ...	9	5	5	39	3	61
	Totals	320	90	93	522	41	1066

L. *Paternal Grandam and Male Offspring.*

Paternal Grandam.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	116	19	49	186	18	388
	R. L. W. ...	27	5	15	50	4	101
	R. W. ...	21	9	26	56	5	117
	Ro. ...	119	20	61	277	44	521
	W. ...	10	3	4	33	6	56
	Totals	293	56	155	602	77	1183

M. *Paternal Grandam and Female Offspring.*

Paternal Grandam.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	123	13	29	176	18	359
	R. L. W. ...	36	3	16	47	3	105
	R. W. ...	22	4	12	51	5	94
	Ro. ...	145	17	53	306	40	561
	W. ...	14	—	4	28	7	53
	Totals	340	37	114	608	73	1172

N. *Maternal Grandam and Male Offspring.*

Maternal Grandam.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	131	32	43	125	11	342
	R. L. W. ...	29	6	18	34	3	90
	R. W. ...	16	7	11	42	2	78
	Ro. ...	127	33	72	287	32	551
	W. ...	7	2	2	46	6	63
	Totals	310	80	146	534	54	1124

O. *Maternal Grandam and Female Offspring.*

Maternal Grandam.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	133	22	46	130	10	341
	R. L. W. ...	28	8	8	33	3	80
	R. W. ...	25	6	26	43	8	108
	Ro. ...	94	32	67	235	32	460
	W. ...	6	4	—	32	5	47
	Totals	286	72	147	473	58	1036

IV.　GREAT-GRANDPARENTS AND OFFSPRING.

P.　*Great-Grandsire and Male Offspring.　No sex change.*

Great-Grandsire.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	82	4	6	89	5	186
	R. L. W....	25	3	2	28	1	59
	R. W. ...	14	0	4	30	2	50
	Ro. ...	115	6	8	154	8	291
	W. ...	6	4	1	19	1	31
	Totals	242	17	21	320	17	617

Q.　*Great-Grandam and Female Offspring.　No sex change.*

Great-Grandam.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	52	10	23	65	13	163
	R. L. W....	8	7	5	17	1	38
	R. W. ...	7	—	13	21	1	42
	Ro. ...	48	11	35	103	18	215
	W. ...	1	1	4	15	1	22
	Totals	116	29	80	221	34	480

R.　*Great-Grandsire and Female Offspring.　One sex change.*

Great-Grandsire.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	220	30	32	207	10	499
	R. L. W....	57	11	9	70	5	152
	R. W. ...	47	7	10	82	4	150
	Ro. ...	228	50	43	468	25	814
	W. ...	21	6	10	60	6	103
	Totals	573	104	104	887	50	1718

S.　*Great-Grandam and Male Offspring.　One sex change.*

Great-Grandam.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	187	30	84	262	22	585
	R. L. W....	54	1	21	81	17	174
	R. W. ...	36	1	14	72	12	135
	Ro. ...	239	46	105	477	71	938
	W. ...	24	4	10	59	5	102
	Totals	540	82	234	951	127	1934

T. *Great-Grandsire and Male Offspring. Two sex changes*

Great-Grandsire.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	263	36	33	230	15	577
	R. L. W....	60	11	11	71	5	158
	R. W. ...	47	10	8	78	4	147
	Ro. ...	287	41	56	400	29	813
	W. ...	30	8	5	42	4	89
	Totals	687	106	113	821	57	1784

U. *Great-Grandam and Female Offspring. Two sex changes.*

Great-Grandam.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	172	27	73	282	32	586
	R. L. W....	48	7	18	81	8	162
	R. W. ...	42	9	38	79	7	175
	Ro. ...	199	34	111	386	66	796
	W. ...	25	3	21	52	3	104
	Totals	486	80	261	880	116	1823

V. *Great-Grandsire and Female Offspring. Three sex changes.*

Great-Grandsire.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	69	17	17	70	2	175
	R. L. W....	17	1	3	21	2	44
	R. W. ...	18	2	5	21	3	49
	Ro. ...	101	13	17	112	14	257
	W. ...	16	—	3	17	1	37
	Totals	221	33	45	241	22	562

W. *Great-Grandam and Male Offspring. Three sex changes.*

Great-Grandam.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	65	7	31	100	17	220
	R. L. W....	12	5	11	29	3	60
	R. W. ...	10	4	14	23	3	54
	Ro. ...	87	12	39	163	22	323
	W. ...	6	2	4	20	3	35
	Totals	180	30	99	335	48	692

V. GREAT-GREAT-GRANDPARENTS AND OFFSPRING.

X. *Great-Great-Grandsire and Male Offspring.*

Great-Great-Grandsire.

♂ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	266	26	47	248	22	609
	R. L. W....	64	3	9	55	5	136
	R. W. ...	66	11	21	82	8	188
	Ro. ...	315	32	55	374	30	806
	W. ...	39	4	8	41	3	95
	Totals	750	76	140	800	68	1834

Y. *Great-Great-Grandsire and Female Offspring.*

Great-Great-Grandsire.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	215	20	29	177	27	468
	R. L. W....	43	3	11	61	7	125
	R. W. ...	35	4	8	39	5	91
	Ro. ...	250	16	63	318	39	686
	W. ...	26	4	8	59	6	103
	Totals	569	47	119	654	84	1473

Z. *Great-Great-Grandam and Male Offspring.*

Great-Great-Grandam.

♂ Offspring		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	236	25	105	364	40	770
	R. L. W....	64	6	7	79	13	169
	R. W. ...	46	5	36	116	16	219
	Ro. ...	256	33	133	505	57	984
	W. ...	29	9	23	50	8	119
	Totals	631	78	304	1114	134	2261

Ω. *Great-Great-Grandam and Female Offspring.*

Great-Great-Grandam.

♀ Offspring.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	169	19	68	258	27	541
	R. L. W....	36	3	19	69	4	131
	R. W. ...	19	5	17	58	15	114
	Ro. ...	214	25	98	437	54	828
	W. ...	24	3	18	53	11	109
	Totals	462	55	220	875	111	1723

VI. COLLATERAL INHERITANCE. WHOLE AND HALF-SIBLINGS.

α. *Whole Brothers.*

First Brother.

Second Brother.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	290	29	69	155	27	570
	R. L. W. ...	29	44	8	22	1	104
	R. W. ...	69	8	108	109	12	306
	Ro. ...	155	22	109	494	75	855
	W. ...	27	1	12	75	50	165
	Totals	570	104	306	855	165	2000

β. *Whole Sisters.*

First Sister.

Second Sister.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	302	26	48	155	18	549
	R. L. W. ...	26	40	9	23	2	100
	R. W. ...	48	9	112	98	17	284
	Ro. ...	155	23	98	554	81	911
	W. ...	18	2	17	81	38	156
	Totals	549	100	284	911	156	2000

γ. *Half Brothers through Sire.*

First Half Brother.

Second Half Brother.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	348	59	42	245	25	719
	R. L. W. ...	59	54	19	49	10	191
	R. W. ...	42	19	26	59	5	151
	Ro. ...	245	49	59	446	58	857
	W. ...	25	10	5	58	24	122
	Totals	719	191	151	857	122	2040

δ. *Half Sisters through Sire.*

First Half Sister.

Second Half Sister.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	266	41	41	246	17	611
	R. L. W. ...	41	42	19	37	9	148
	R. W. ...	41	19	40	65	20	185
	Ro. ...	246	37	65	518	67	933
	W. ...	17	9	20	67	24	137
	Totals	611	148	185	933	137	2014

ε. *Half Brother and Sister through Sire.*

Half Brother.

Half Sister.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	144	26	33	99	6	308
	R. L. W. ...	17	22	4	24	6	73
	R. W. ...	38	7	14	32	7	98
	Ro. ...	136	25	33	215	41	450
	W. ...	15	4	1	45	6	71
	Totals	350	84	85	415	66	1000

ζ. *Half Brothers through Dam.*

First Half Brother.

Second Half Brother.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	294	32	83	185	22	616
	R. L. W. ...	32	12	18	41	14	117
	R. W. ...	83	18	54	95	21	271
	Ro. ...	185	41	95	428	80	829
	W. ...	22	14	21	80	30	167
	Totals	616	117	271	829	167	2000

η. *Half Sisters through Dam.*

First Half Sister.

Second Half Sister.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	246	29	66	215	26	582
	R. L. W. ...	29	22	7	42	10	110
	R. W. ...	66	7	74	107	16	270
	Ro. ...	215	42	107	416	91	871
	W. ...	26	10	16	91	24	167
	Totals	582	110	270	871	167	2000

θ. *Half Brother and Sister through Dam.*

Half Brother.

Half Sister.		R.	R. L. W.	R. W.	Ro.	W.	Totals
	R. ...	136	17	31	114	12	310
	R. L. W. ...	18	9	—	21	6	54
	R. W. ...	26	9	31	38	12	116
	Ro. ...	121	26	64	190	36	437
	W. ...	17	2	5	41	18	83
	Totals	318	63	131	404	84	1000

HEREDITARY DEAFNESS.

A DISCUSSION OF THE DATA COLLECTED BY
DR E. A. FAY IN AMERICA.

By EDGAR SCHUSTER, M.A., Francis Galton Research Fellow in
National Eugenics in the University of London.

THE present paper is an attempt* to apply the newer methods of statistics, introduced by Mr Francis Galton and developed by Prof. Karl Pearson, to a valuable collection of hereditary data that have hitherto only been discussed by the older ones.

The object is to obtain results which admit of direct comparison with others derived from quite other collections. These results take the form of " correlation coefficients," which give a numerical measure of the intensity of heredity.

The best material for this purpose would have been a random sample of the population, of which the following particulars were known, namely the facts which relate to the presence or absence of deafness, and of its nature when present, among their parents, children, brothers and sisters, etc.; and not only this but also the total number of their relatives in those degrees in which the number is variable.

Deafness of the degree with which we are dealing is fortunately of such rarity that a sample of this kind, in order to be useful, would have to contain at least a million persons, so that practically speaking it is outside the range of possible attainment for a private individual.

In default of this one is reduced to processes involving many possibilities of error, as one has to supply by estimate the deficiencies in such information as is accessible, in order to obtain artificial samples for making correlation tables.

It might be objected that the facts taken have already been stated in a form that anyone can understand, and are now presented in a way that is only intelli-

gible to persons with a certain amount of special knowledge, and moreover, owing to the various errors which may have crept in during the process, gives uncertain and misleading information even to them. In answer one must lay stress on the extreme importance of bringing the facts relating to the inheritance of rare characters such as congenital deafness into line with those relating to common ones, such for instance as tallness or shortness of stature. Without such comparisons it is impossible to make real advance towards knowledge of the general laws of heredity, so that even an imperfect attempt at making them is better than none at all. The results need mislead no one, as every assumption and every known source of error is pointed out; and as statistical methods do not consume the material which they use, the same data can be employed over and over again to test any new theory which may seem worthy of investigation.

Material. The material on which this paper is based is derived entirely from E. A. Fay's "Marriages of the Deaf in America" (Volta Bureau, Washington, U.S.A., 1898, pp. vii and 527). The collection of the facts contained in this book was undertaken, as a labour of love, by Dr E. A. Fay, the editor of *The American Annals of the Deaf*, the expenses of the investigation being borne by the Volta Bureau, which had been endowed, partly for this purpose, by Dr A. Graham Bell. How much labour must have been needed in order to obtain this immense and valuable mass of data can be realised from the fact that nearly every marriage of a deaf person, which had taken place in America and Canada from 1850 to 1894, and a large number which took place before that period are here recorded. There are in all, included in the book, records of the marriages of 4471 pairs of persons, of whom at least one in each pair was known to be of a degree of deafness defined on p. 7 of the introduction in the following words :—

"The term 'deaf,' as used in this monograph, invariably refers to the class of persons for whose benefit our 'schools for the deaf' are intended—persons so deaf from birth, childhood or youth that they cannot be educated in common schools, persons who are more frequently but less accurately classed as 'the deaf and dumb' or 'deaf-mutes.' It includes all persons who are recorded in school reports, census reports, marriage records, etc., as 'deaf and dumb,' 'deaf-mutes,' 'totally deaf,' 'very deaf,' 'deaf,' or 'very hard of hearing,' and all who have attended the schools for the deaf. Persons recorded as 'slightly deaf,' 'partially deaf,' 'deaf in one ear,' 'deaf in adult life,' 'hard of hearing,' etc., are not included, unless they have attended schools for the deaf."

Pages 138–499 consist of a tabular record of the 4471 marriages, the details given concerning each being as follows :—Date of information, date of marriage, total number of children, number deaf, number hearing, number unknown whether deaf or hearing. After this come the following particulars concerning both the husband and the wife; whether they were deaf or hearing or whether this was unknown, the year of birth, the total number of the family in which they came (i.e. their brothers and sisters and themselves); how many of these were deaf, how many hearing, and about how many this was not reported; whether they had any

deaf relatives excluding brothers, sisters and children ; whether the deafness was congenital or if not at what age it occurred, and to what cause it was attributed ; the schools at which they were educated, and lastly there is a broad space left for remarks. The remarks consist mostly of indicating whether the husband or wife was related to, or had been married to, or was subsequently married to, any other of the husbands or wives, whose marriages are recorded in the tables. Where any of these particulars could not be obtained, a dotted line is ruled across the space in the table in which it should have been set down. The first part of the book consists of an analysis of the tables, but for various reasons I have not made any use of this part.

Deafness. Deafness may be divided into two categories, congenital and "acquired" or "adventitious." Congenital deafness appears for the most part to be due to faults in the structure of the membranous labyrinth of the inner ear ; but the eighth nerve may also be affected, and in some cases the deafness results from defects in the central nervous system, which may coexist with perfectly formed ears. It is usually associated with an anomalous condition of the functions of balance, and much experimental work has been done recently in Germany on this point (by Pollak, Alexander, Kreidl, Hammerschlag, etc.). It may possibly be compared to the structural peculiarities of the semicircular canals of Japanese waltzing mice.

Acquired deafness is due generally to changes in the middle and inner ears produced by inflammation, which may be idiopathic or set up by various diseases, such as meningitis, scarlet fever, etc.

Should a child be born with the faculty of hearing not developed beyond a certain pitch, he will be unable to learn the art of speech in the ordinary manner, that is to say by imitating the sounds formed by the speech of others ; and should he lose his hearing before, or but shortly after, learning to talk, the effect will be the same. He may, however, be taught to speak in a fairly intelligible manner by special methods, which consist in causing him to imitate not the sounds themselves, but the different muscular movements that produce them. But in order that this may be done with success, there needs to be patience and skill on the part of the teacher and fair mental ability on the part of the pupil.

In making the correlation tables contained in this paper, it has been impossible to separate congenital from acquired deafness, because one is only given information on this point for one set of people, as will be explained later, and not for their brothers or sisters or children. Also it is almost certain that a large number of congenital deaf-mutes are included in the adventitious class, as this is apparently considered a matter of less shame than to be born deaf. If a child is found to be deaf at the age when it should begin to speak, it is very difficult to say definitely whether it has been born deaf, or whether the deafness was produced by some disease in its infancy, so that when there is any possible pretext for attributing it to the latter cause, this is done. As the possession or non-possession of deaf

relatives is sometimes used to assist classification into the congenital and acquired classes, it follows that those who are included in the former class cannot be used as a fair sample of true congenital deaf-mutes for the purpose of the study of the inheritance of deafness.

Methods and Results.

(1)   *Correlation between Father and Children and Mother and Children.*

The first step taken was to tabulate afresh the offspring born from the various classes of marriages enumerated below, in doing which I have only used such families as had their total number of offspring definitely stated.   Table I. shows

TABLE I.

Showing number of marriages and number of deaf and hearing offspring produced from them, in which either the father or the mother or both parents were deaf.

	Class of Marriage	Number of Marriages	Number of Children		
			Deaf	Hearing	Doubtful
1	Father born deaf, Mother born deaf	311	184	480	46
2	Father born deaf, Mother not born deaf	552	93	1078	84
3	Mother born deaf, Father not born deaf	562	96	936	86
4	Father deaf but not known to be born deaf, Mother ditto ...	1142	68	1696	217
5	Father deaf but not known to be born deaf, Mother born deaf	478	72	767	82
6	Mother deaf but not known to be born deaf, Father born deaf	442	66	831	70
7	Father deaf but not known to be born deaf, Mother not deaf	234	47	449	21
8	Mother deaf but not known to be born deaf, Father not deaf	153	15	292	18
9	Total of marriages in which Father was deaf	2717	464	4470	450
10	Total of marriages in which Mother was deaf	2610	429	4235	437

the result of this process.   The first class of marriage consists of those in which both the husband and wife (the father and mother of the family) were stated to be congenitally deaf.   The second consists of those in which the husband was congenitally deaf but the wife was either adventitiously deaf, not deaf at all, or where doubt existed as to nature of her deafness, or as to whether she was deaf or not. Class 3 is the converse of this.   In class 4 both the husband and wife were deaf, but they were either adventitiously deaf or at any rate were not certainly known to be congenitally deaf.   Class 5 : the husband's deafness was similar in nature to that of class 4, but the wife was born deaf, and class 6 is the converse of class 5. In classes 7 and 8 the husband in the former, and the wife in the latter, were deaf in the same manner as those in class 4, while the wife in the first case, and the husband in the last, were not deaf at all.   Class 9 sums up all the cases in which the husband was deaf at all, and class 10 does the same for the wives.

It is interesting to note the high proportion (·2771 of the number about whom it was known whether they were deaf or hearing) of deaf children in class 1

as compared to class 4 (·0386), while classes 2 and 3 are intermediate between the two with ·0794 and ·0930 respectively. I do not however intend going into this point, as it is dealt with by Fay in some detail. I have not made use of Fay's tables principally because no distinction of the sexes was made, fathers and mothers were grouped together, and I thought it advisable to keep them separate to see whether the paternal correlation was different from the maternal.

The next step was to tabulate all those cases in which either the father or the mother of the husbands and wives of the marriages recorded was deaf. For the result of this process see Table II. The total number of husbands and wives in

TABLE II.

Showing the whole number of husbands and wives about whom definite infor-
mation is given as to whether or no they had any deaf relatives other than
children and brothers and sisters, also what number of these had either one
or both parents deaf.

	Congenital				Adventitious			
	Total Number	Father Deaf	Mother Deaf	Both Parents Deaf	Total Number	Father Deaf	Mother Deaf	Both Parents Deaf
Husbands ...	648	11	7	37	1147	4	3	13
Wives ...	672	12	13	36	1184	5	3	14

this table means the total numbers about whom definite information was given as to whether or not they had deaf relatives other than children or brothers and sisters. The numbers in the columns "Father deaf," "Mother deaf," "Both parents deaf," possibly do not represent the whole numbers about whom this statement could have been made truthfully; but they must be looked upon as giving minimum values for them.

It was from Tables I. and II. that the paternal and maternal correlation tables were constructed. As it was only possible to divide the material into two classes, namely deaf and hearing, the ordinary form of correlation table could not be used, and the fourfold method had to be adopted. This method involves an assumption, which in the present instance it is impossible to test, that the distribution of the material with regard to the variable character dealt with is normal. Where this is the case, or even approximately the case, it has been shown that one obtains practically the same results from this form of table as one does from the longer one.

The actual process of constructing the correlation table was as follows: firstly, from Table I. one learns that 2717 deaf fathers (classing those that were born deaf and those that became deaf after birth together) have 464 deaf children and 4470 hearing children, also 450 children about whom it was not known

whether they were deaf or hearing. There is no reason to suppose that among the doubtful ones the deaf did not bear the same proportion to the hearing as among the others; that is to say that among 450 of them there would be 42 deaf and 408 hearing; therefore by adding these to the others we have 506 deaf and 4878 hearing. Weighting the fathers with the number of their children (i.e. including them once for each child that they have), there are then 4878 deaf fathers with hearing children, and 506 with deaf. In order to complete the table it is necessary to know how many hearing fathers with hearing children, and how many with deaf children, would occur in a random sample of the population containing the deaf fathers mentioned above. Now from Table II. we know that a total of 3658 deaf people have in 132 cases deaf fathers, and in the remaining 3519 cases hearing fathers; and we have to include in the table we are constructing 506 deaf people with deaf fathers, therefore we may take it that the number of deaf people with hearing fathers which correspond with this number is 506/132 multiplied by 3519, which is equal to 13,489.

There only remains in order to complete the table to determine how many hearing people with hearing fathers should be included in the sample. We know from the American census of 1890 that 659 deaf people occur in every million of the general population. We are dealing here with 506 plus 4878 = 5384 deaf fathers, therefore if we assume that deaf fathers bear the same numerical relation to hearing fathers, as deaf persons in general bear to hearing persons, then the 5384 deaf fathers occur among a total number of 5384/659 multiplied by a million, which is equal to 8,169,954 fathers, and as 5384 of these are deaf there remain 8,164,570 who are hearing. Of these 13,489 have deaf offspring, therefore 8,151,081 have hearing offspring, and thus Table III. may be completed.

TABLE III.

Fathers.

Children.		Deaf	Hearing
	Hearing ...	4878	8,151,959
	Deaf... ...	506	13,489

$h = 2 \cdot 926730$
$H = \cdot 005507$
$k = 3 \cdot 212070$
$K = \cdot 002294$
$r = \cdot 51$

TABLE IV.

Fathers.

Children.		Deaf	Hearing
	Hearing ...	4878	21,222,727
	Deaf... ...	506	13,489

$h = 3 \cdot 477260$
$H = \cdot 000945$
$k = 3 \cdot 212070$
$K = \cdot 002294$
$r = \cdot 57$

There is also another way of completing the table. If instead of assuming that deaf fathers bear the same numerical relation to hearing fathers, as deaf persons in

general do to hearing persons, we work on the supposition that this relation is borne by deaf children to hearing children; then as we are dealing with 13,489 plus 506 = 13,995 deaf children, these must occur among a total number of 21,236,722 children, of whom 21,222,727 are hearing, and as 4878 have deaf fathers, 21,217,849 would have normal fathers, and in this way Table IV. may be constructed.

TABLE V.

Fathers.

	Deaf	Hearing
Hearing ...	4878	8,159,692
Deaf... ...	506	4878

(Children.)

$h = k = 3 \cdot 212070$
$H = K = \cdot 002294$
$r = \cdot 62$

TABLE VI.

Fathers.

	Deaf	Hearing
Hearing ...	3519	5,533,042
Deaf... ...	132	3519

(Children.)

$h = k = 3 \cdot 212070$
$H = K = \cdot 002294$
$r = \cdot 46$

Tables III. and IV. thus combine the information derived from Tables I. and II. In Table II. the assumption is made that the proportion of deaf to hearing fathers is that of the general population, and in Table IV. a similar assumption is made for the children. Tables V. and VI. assume that both with regard to fathers and children this is the case. If this assumption be made then the table must be a symmetrical one, and as one cannot get a symmetrical table by combining the two sources of information, one has to construct a separate table out of each. In Table V. the information from Table I. only is used, and in Table VI. that from Table II.

Tables VII., VIII., IX. and X. were made for mothers in exactly the same way as III., IV., V. and VI. were for the fathers. In Table XI. the values of the correlation coefficients for each of these eight tables are set down. It will be

TABLE VII.

Mothers.

	Deaf	Hearing
Hearing ...	4632	7,722,294
Deaf... ...	469	13,121

(Children.)

$h = 2 \cdot 919020$
$H = \cdot 005632$
$k = 3 \cdot 212070$
$K = \cdot 002294$
$r = \cdot 51$

TABLE VIII.

Mothers.

Children.		Deaf	Hearing
Hearing ...		4632	20,604,933
Deaf... ...		469	13,121

$h = 3\cdot483478$
$H = \cdot000925$
$k = 3\cdot212070$
$K = \cdot002294$
$r = \cdot56$

TABLE IX.

Mothers.

Children.		Deaf	Hearing
Hearing ...		4632	7,730,783
Deaf... ...		469	4632

$h = k = 3\cdot212070$
$H = K = \cdot002294$
$r = \cdot62$

TABLE X.

Mothers.

Children.		Deaf	Hearing
Hearing ...		3525	5,533,036
Deaf... ...		126	3525

$h = k = 3\cdot212070$
$H = K = \cdot002294$
$r = \cdot45$

TABLE XI.

Method of construction of Table	Value of r	
	Paternal	Maternal
Tables I. and II. both used, distribution of deafness among parents assumed to be that of general population	0·51	0·51
Tables I. and II. both used, distribution of deafness among children assumed to be that of general population	0·57	0·56
Table I. only used, distribution of deafness both among parents and children assumed to be that of general population	0·62	0·62
Table II. only used, distribution of deafness both among parents and children assumed to be that of general population	0·46	0·45
Mean value of r	0·54	0·535

seen that the values of r vary between 0·45 and 0·62, with a mean value for paternal correlation of 0·54, and for maternal of 0·535.

Now we have two sources of inaccuracy, the tendencies of which are known, in arriving at these values. Firstly, the inclusion of persons whose deafness was due to causes acting on them from without. Although these are probably far less numerous than the whole number entered by Fay as "acquired" or "adventitious" cases, yet there are certainly enough of them present to considerably reduce the value of r in all the tables if, as it is generally supposed, this form of deafness is not inherited; it is however quite possible that the physical condition, which renders the owner of it liable to become deaf in this way, is inherited. Secondly, the percentage of deaf-mutes given in the census returns is in all probability too low. If a higher percentage had been allowed for in making the tables the effect would have been to reduce the value of r; as it was not allowed for one must consider that the latter error has to a certain extent balanced the former, though exactly to what extent is quite uncertain.

Another factor to be taken into account in considering the paternal and maternal correlation coefficients is that of "Assortative Mating" of the parents. The tendency for the deaf to marry the deaf is an exceedingly strong one, and that for obvious reasons which need not be dwelt on here. In Table I. a total of 2954 marriages is recorded, and in 2373 of these both husband and wife were deaf, in 344 the husband was deaf and the wife hearing, and in 237 the reverse was the case.

In order to determine exactly the coefficient of assortative mating, that is to say the correlation between husband and wife, I constructed two correlation tables from these data; one of these assumed that the distribution of deafness among husbands was normal, the other that this was the case for wives. Owing to the very large value of r in both cases, it was not found possible to determine it exactly, as this would have involved the solution of an equation containing very much higher powers of x than x to the twentieth. The tables are therefore not included here, but the value of r is certainly considerably higher than ·9.

(2) Fraternal Correlation.

As has been mentioned previously Fay has endeavoured to give in his tables the size of the families in which the 4471 pairs of husband and wife are contained, and also the number of deaf-mutes contained in them. In his statement of this, brothers are not distinguished from sisters, but all are included and numbered together. In several cases the numbers are not known or not stated definitely. Excluding these we find that the remaining deaf husbands and wives are included in 3837 families, the distribution of which with regard to size is shown in the last column of Table XII. It is also shown graphically in Figure I., and is there fitted with the skew curve (Prof. Pearson's General Probability Curve of Type I.), which seems to represent the observations with fair accuracy.

Of each of these families every possible pair of deaf and deaf, of deaf and hearing, and of hearing and hearing brothers and sisters was made. To take a

FIG. I. Curve showing distribution of size of 3837 families in America, each of which contains at least one married deaf-mute.
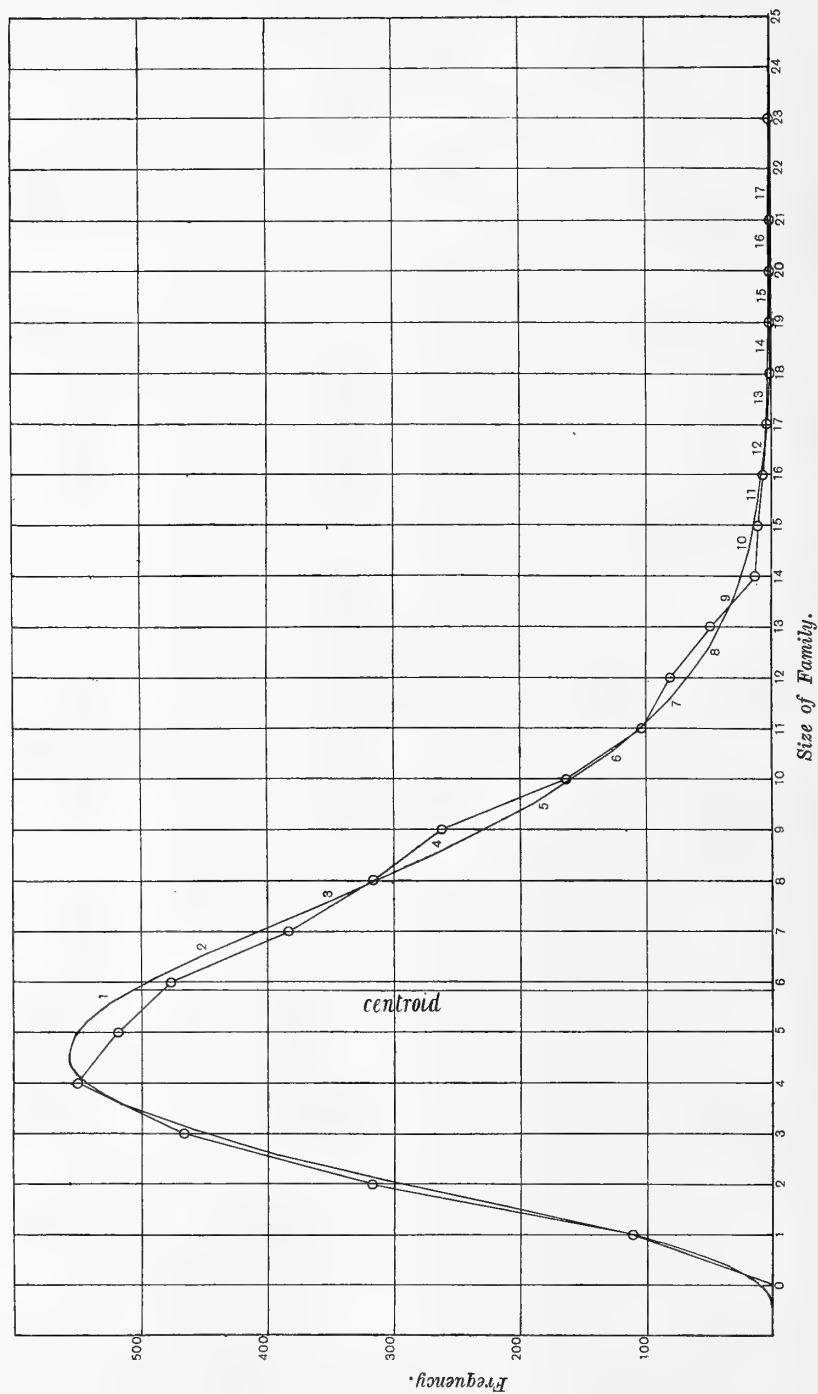
Equation for Curve of Type I.: $y = 555 \cdot 79 \left(1 + \dfrac{x}{61 \cdot 9957}\right)^{39 \cdot 9766} \left(1 - \dfrac{x}{5 \cdot 2895}\right)^{3 \cdot 4108}$.

Mean size of Family $= 5 \cdot 8328$.

Origin of curve at $4 \cdot 5817$.

TABLE XII.

Comparing distribution of size of families which contain at least one congenitally deaf person, with that of those containing at least one person said to have acquired deafness.

Number in Family	Nature of Deafness			Totals
	Congenital	Acquired	Doubtful	
1	32	73	6	111
2	83	218	18	317
3	118	337	17	466
4	180	365	16	550
5	168	344	17	518
6	144	323	18	476
7	117	260	16	383
8	97	215	12	316
9	88	176	11	262
10	58	103	8	163
11	46	55	5	104
12	31	50	1	81
13	18	35	—	49
14	5	11	—	14
15	5	6	—	11
16	1	5	1	7
17	1	3	—	4
18	—	1	—	1
19	—	1	—	1
20	—	1	—	1
21	—	1	—	1
22	—	—	—	—
23	1	—	—	1
Totals	1193	2583	146	3837

N.B. 85 families are included both in the congenital and in the acquired class, as they contain at least one member of each of these two classes.

concrete example of this; consider a family of 12 persons containing 6 deaf members and 6 hearing. Taking the deaf members first each of them can be paired with each of the other deaf ones, thus making 30 pairs of deaf and deaf; they can also be paired with the hearing members, making 36 pairs of deaf and hearing; similarly the 6 hearing persons can be paired with the 6 deaf, making 36 pairs of hearing and deaf, and with one another, making 30 pairs of hearing and hearing. The results can be written in tabular form as in Table XIII. Out of a family of 12, 132 pairs can be made, and, speaking generally, out of a family of n members $n(n-1)$ pairs can be made. Table XIV. is the sum of tables such as XIII. made for each family.

Now there are altogether 5423 deaf persons in the families used, and the distinctive point of these families is that they contain at least one married deaf

TABLE XIII.

*Showing pairs which can be made out of a family of 12 persons,
6 deaf and 6 hearing.*

	Deaf	Hearing
Hearing ...	36	30
Deaf... ...	30	36

TABLE XIV.

Showing total number of pairs made out of the 3837 families used.

	Deaf	Hearing
Hearing ...	23,772	88,572
Deaf... ...	5226	23,772

member. They should therefore be compared with hearing people contained in families which have at least one married member. If they bear the same numerical relation to these that deaf people in general do to hearing people in general, then they occur in a total number of 5423 multiplied by 1,000,000/659 persons. Assuming that the mean size of families, which contain a married member but do not contain a deaf one, is the same as that of families which contain a deaf married one, and that the distribution of size is the same; then, as from 5423 deaf persons 23,772 plus 5226 (equals 28,998) pairs can be made with their brothers and sisters, the total number of pairs which can be made from $\frac{5423}{659} \times 1,000,000$ persons is $\frac{5423}{659} \times 1,000,000 \times \frac{28998}{5423} = 44,003,035$. Deducting from this number all the pairs that contain deaf-mutes there are left 43,950,262 pairs of hearing and hearing brothers and sisters, and by substituting this number for the number of pairs of hearing and hearing in Table XIV., Table XV. is obtained.

TABLE XV.

Fraternal Correlation Table constructed from Table XIV.

	Deaf	Hearing
Hearing ...	23,772	43,950,262
Deaf... ...	5226	23,772

$h = k = 3{\cdot}212070$
$H = K = {\cdot}002294$
$r = {\cdot}74$

With regard to the assumption that families which do not contain a deaf member are the same in point of size as those that do, I endeavoured to obtain evidence from the data at my disposal. It seems quite clear that, at any rate, a considerable proportion of those persons who are said to have acquired deafness must in reality have no sort of inborn tendency in this direction; the families which contain them can therefore be taken as more nearly approaching a sample of families whose only common character is that they contain at least one married member, than those which contain a congenital deaf-mute. If then it is found that the mean size and the variation in size in both of these classes is approximately the same, it could be argued that this is to a certain extent evidence that the assumption is a reasonable one. The families were accordingly divided into these two classes, and the first two columns in Table XII. show the distribution of size for each class. Table XVI. gives a comparison of the mean size of family, and of

TABLE XVI.

Showing difference in mean size of families containing at least one congenital deaf-mute and of those containing at least one member who has acquired deafness; also the difference in the standard deviations.

		Mean	Standard Deviation
Congenital	...	6·0821	2·9862
		± ·0583	± ·0412
Acquired	...	5·7704	2·9070
		± ·0386	± ·0273
Difference	...	·3117	·0792
		± ·0699	± ·0495

the standard deviation. It will be seen that the mean size of families containing at least one congenital deaf-mute is larger than those containing one of the "acquired" variety by an amount which is rather more than four times as great as its probable error. There is thus a certain amount of reason for believing that families containing a deaf-mute member are in reality somewhat larger than those that do not. If this had been allowed for in making the correlation table the number of pairs of hearing and hearing brothers and sisters would not have been so great, and there would have been a consequent reduction in the value of r. The difference in the value of the standard deviation is not twice as great as its probable error, and is therefore probably not significant.

The value of r for fraternal correlation was found from Table XV. to be approximately ·74, which is a rather higher value than is normal, though not very much higher than that for the amount of red in coats of greyhounds of the same litter, which was found to be ·700 (*Biometrika*, Vol. II. p. 390).

Table XVII. explains itself. The mean number of deaf members per family in each case is not insisted upon, owing to the probable gross inaccuracies of the classification into cases of congenital and acquired deafness.

TABLE XVII.

Showing total number of deaf persons in families containing at least one con-
genitally deaf person, also in those containing at least one person who has
acquired deafness.

Number of Deaf Persons in Family	Nature of Deafness			Totals
	Congenital	Acquired	Doubtful	
1	524	2260	101	2885
2	354	221	22	559
3	189	73	13	242
4	82	22	5	99
5	25	4	4	31
6	8	1	—	9
7	8	1	1	9
8	1	1	—	1
9	2	—	—	2
Totals	1193	2583	146	3837

N.B. 85 families are included both in the congenital and in the acquired class, as they contain at least one member in each of these two classes.

Conclusions.

In conclusion it need hardly be pointed out that the results obtained can only be regarded as provisional, as the data on which they are based were not collected with a view to the construction of correlation tables, and are therefore not particularly well suited to this purpose.

With this reservation it may, however, be asserted that striking confirmation has been obtained of previous work on widely different characters, at any rate with regard to the correlation between father and children, and mother and children.

Quite recently our knowledge of this subject has been greatly enlarged and lucidly summarized by Prof. Pearson and Dr Lee in their paper, " On the Laws of Inheritance in Man " (*Biometrika*, Vol. II. p. 355); and it will be unnecessary to repeat here the excellent comparative tables that are given there. It will be sufficient to mention one or two examples to show for what a broad range of characters parental inheritance has been measured, and to quote the general conclusion of the authors on this point.

While it has been found in the present paper that the mean of the results obtained by four different methods for the inheritance of deafness is ·54 for fathers and children and ·535 for mothers and children (*vide* Table XI.), the mean value of parental inheritance of stature in man is ·506, of span, ·459, and of eye colour, ·495; similarly that of coat-colour in horses is ·522, in Basset hounds ·524, and in

greyhounds ·507 (*Biometrika*, Vol. ii. p. 379, Table V.). The general conclusion is expressed as follows :—" The coefficient of parental heredity varies to some extent from character to character in man, having a mean value of about ·46. This value is however in quite close agreement with the results obtained for other species, and we may roughly say that parental heredity in the species hitherto dealt with is close to ·5." (*Biometrika*, Vol. ii. p. 396 (v.).)

The value obtained for fraternal correlation is less satisfactory, but if it is not directly confirmatory of, it is in no way inconsistent with previous determinations for other characters, for the range of variation of fraternal correlation seems to be much greater than that of parental. Thus ·74 here arrived at is much larger than the mean value for other characters in man (·5), but it does not differ much from the values obtained for coat-colour in horses and dogs, or for certain characters in waterfleas and aphides, all of which will be found duly set forth in the paper referred to above.

One more conclusion can be drawn, and that is the urgent necessity for the collection of satisfactory data, on which to work at this and other problems in heredity. The Interdepartmental Committee on Physical Deterioration, in their report, published in 1904, strongly recommend that an Anthropometric Survey should be undertaken by the Government at the earliest possible moment (*vide* Section 54). Should this recommendation ultimately be adopted, there is no reason why the scope of this survey should not be somewhat extended so as to include the collection of data bearing on heredity; but till then one must make what use one can of any material that one can get.

Postscript. Since writing the above I have been able to obtain values for paternal, maternal, and fraternal correlation from information collected in England. For the first two the following material was used :

(1) A record of the offspring of marriages in which one or both parents were deaf from the register of the National Association in Aid of the Deaf and Dumb. Of 161 marriages there recorded both parents were deaf in 127, while in 26 the husband was deaf and the wife was either hearing or not stated to be deaf, and in 8 the reverse was the case. Thus there were in all 153 marriages in which the husband was deaf, of which 125 had proved fertile by the time of the information. The total number of children was 435, of which 52 occurring in 25 families were deaf. There were 135 marriages in which the wife was deaf. 109 of these had resulted in a total number of 379 children, of which 50 occurring in 25 families were deaf. The only part of this used in the tables was that in families with a deaf father there were 383 hearing children and 52 deaf, and in families with a deaf mother there were 329 hearing children and 50 deaf.

(2) The rest of the material used is contained in Table XVIII. The number of deaf parents given there is probably much understated. Both Dr Kerr, to whose kindness I owe the information concerning the L.C.C. schools, and Mr Moore, who sent me that relating to Margate, warned me of this.

TABLE XVIII.

Source of Information	Total Number of Children	With Father Deaf only	With Mother Deaf only	With both Parents Deaf
Birmingham. Royal Institution for the Instruction of Deaf and Dumb children	679	1	6	1
Exeter. Royal West of England Institution for the Deaf and Dumb	66	1	1	—
London. 10 L. C. C. Schools	413	1	—	4
Margate. Royal Asylum for the Deaf and Dumb ...	1302	—	—	27
Newcastle-on-Tyne. Northern Counties Institution for the Deaf and Dumb	154	—	2	4
Stoke-on-Trent. North Staffordshire Joint School Authority's Blind and Deaf School	170	2	1	2
Totals	2784	5	10	38

TABLE XIX.

Fathers.

	Deaf	Hearing
Hearing ...	383	924,329
Deaf... ...	52	3315

$h = 3.3087$
$H = .001674$
$k = 2.6849$
$K = .010913$
$r = .45$

TABLE XX.

Fathers.

	Deaf	Hearing
Hearing ...	383	7,179,796
Deaf... ...	52	3315

$h = 3.3087$
$H = .001674$
$k = 3.8436$
$K = .000247$
$r = .58$

Children. (label at left of both tables)

Tables XIX. and XX. give the paternal correlation coefficient. They were constructed from the material described above in exactly the same way as Tables III. and IV. were from Fay's data, except that the rate per million of deaf persons used was that of the 1901 English census, namely 468·71. The mean value obtained from them is ·515, while that obtained from Tables III. and IV. is ·54.

Tables XXI. and XXII. correspond exactly with Tables VII. and VIII., and like them they give a mean value for maternal correlation of ·535.

The material from which the fraternal correlation coefficients were deduced is shown in Table XXIII. It consists of 459 families from the register of the

TABLE XXI.
Mothers.

Children.		Deaf	Hearing
	Hearing ...	329	805,373
	Deaf... ...	50	2850

$h = 3{\cdot}3087$
$H = {\cdot}001674$
$k = 2{\cdot}6887$
$K = {\cdot}010743$
$r = {\cdot}47$

TABLE XXII.
Mothers.

Children.		Deaf	Hearing
	Hearing ...	329	6,183,965
	Deaf... ...	50	2850

$h = 3{\cdot}3087$
$H = {\cdot}001674$
$k = 3{\cdot}8408$
$K = {\cdot}000250$
$r = {\cdot}60$

TABLE XXIII.
Showing Distribution of Deaf in Families used for Fraternal Correlation.

Number of Persons in each Family	National Association in Aid of the Deaf and Dumb									Exeter and Manchester				
	Number of Deaf in Family								Totals	Number of Deaf in Family				Totals
	1	2	3	4	5	6	7	8		1	2	3	4	
1	10	—	—	—	—	—	—	—	10	4	—	—	—	4
2	23	2	—	—	—	—	—	—	25	17	1	—	—	18
3	47	6	—	—	—	—	—	—	53	20	4	—	—	24
4	56	7	2	—	—	—	—	—	65	23	4	1	1	29
5	48	9	1	3	—	—	—	—	61	24	1	1	—	26
6	45	8	5	4	—	—	—	—	62	10	2	—	—	12
7	33	3	1	3	—	1	—	—	41	11	1	—	1	13
8	35	3	5	—	1	—	—	—	44	5	—	1	—	6
9	25	5	1	1	1	—	—	—	33	11	1	—	—	12
10	17	5	2	1	—	—	1	—	26	4	—	2	—	6
11	16	—	3	—	—	—	—	—	19	1	—	—	—	1
12	3	—	—	—	—	—	—	—	3	—	—	—	—	—
13	5	—	1	1	—	—	—	—	7	2	—	—	—	2
14	2	2	—	—	—	—	—	—	4	—	—	1	—	1
15	—	—	—	2	—	—	—	—	2	—	—	—	—	—
16	1	—	—	—	—	—	—	—	1	1	—	—	—	1
17	1	—	—	—	—	—	—	—	1	—	—	—	—	—
18	2	—	—	—	—	—	—	—	2	—	—	—	—	—
Totals	369	50	21	15	2	1	1	—	459	133	14	6	2	155

National Association : Mean size of family ... ... $6{\cdot}1939 \pm {\cdot}0954$
Standard deviation ... ... $3{\cdot}0299 \pm {\cdot}0674$
Mean number of deaf per family $1{\cdot}3399$

Exeter and Manchester : Mean size of family ... ... $5{\cdot}1871 \pm {\cdot}1489$
Standard deviation ... ... $2{\cdot}7493 \pm {\cdot}1053$
Mean number of deaf per family $1{\cdot}2064$

National Association, and a group of 155 families of which 64 came from the institution at Exeter and 91 from that at Manchester. From the 459 families Table XXIV. was made, and from the 155 Table XXV., both in exactly the same way as Table XV.

<div align="center">

TABLE XXIV.

National Association in Aid of the Deaf and Dumb.

</div>

		Deaf	Hearing
Hearing	...	2880	7,243,407
Deaf...	...	518	2880

$h = k = 3.3087$
$H = K = .001674$
$r = .72$

<div align="center">

TABLE XXV.

Exeter and Manchester.

</div>

		Deaf	Hearing
Hearing	...	727	1,737,273
Deaf...	...	88	727

$h = k = 3.3087$
$H = K = .001674$
$r = .68$

The values of r were found to be ·72 and ·68 respectively, whereas ·74 was given by Table XV. It may thus be said that with regard to parental and fraternal correlation the English material gives results which agree very well with the American, if one takes into consideration the roughness both of the material and of the methods employed.

The mean size of the families from the National Association Register is $6.1939 \pm .0954$; that of the Exeter and Manchester families is $5.1871 \pm .1489$. The reason for this difference is that in the former case the families are probably all complete, whereas in the latter they are certainly incomplete in many cases, the number being taken at the time of the entry of one of their deaf members, generally aged 8 or 9, into one of the two institutions.

ON THE CORRELATION BETWEEN VACCINATION AND SMALLPOX IN THE LONDON EPIDEMIC, 1901-2.

By F. M. TURNER, M.D.

AT the suggestion of Prof. Pearson I have undertaken the calculation of the smallpox statistics of the Metropolitan Asylums Board Hospitals for the epidemic of 1901–2, in continuation of the two papers already published on the same subject by Macdonell in this Journal ; and the results are given in this paper. I also include a discussion of several points, which have occurred to me in the course of the calculations, which seem to me to have considerable importance in altering the values of the coefficients. Of these the most important is a suggestion as to the probable frequency distribution of severity among smallpox cases, which I call a "curtailed" normal distribution, and a discussion is given of the mathematical formulae appropriate to such a distribution. All the data were obtained from the *Annual Reports of the Metropolitan Asylums Board* for the years 1901 and 1902.

Taking a fourfold division into vaccinated and unvaccinated and into recoveries and deaths, as in the above-mentioned papers, we get the following tables for the calculation of r. They only differ in the treatment adopted for the doubtful cases, which in Table I. are classed with the vaccinated, in Table III. with the unvaccinated, and in Table II. have been omitted altogether.

TABLE I.

All Cases, 1901—2. Vaccination and Severity.

	Recoveries	Deaths	Totals
Unvaccinated	1525	753	2278
Vaccinated or doubtful ...	6505	876	7381
Totals ...	8030	1629	9659

$$r = -\,\cdot4246 \pm \cdot0049.$$

TABLE II.

All Cases except those in which Vaccination was Doubtful, 1901—2.
Vaccination and Severity.

	Recoveries	Deaths	Totals
Unvaccinated ...	1525	753	2278
Vaccinated ...	6240	705	6945
Totals ...	7765	1458	9223

$r = -\cdot 4741.$

TABLE III.   *All Cases, 1901—2.   Vaccination and Severity.*

	Recoveries	Deaths	Totals
Unvaccinated or doubtful ...	1790	924	2714
Vaccinated	6240	705	6945
Totals	8030	1629	9659

$r = -\cdot 4922.$

Of these three values I prefer that of Table I., as I believe that a majority of the doubtful class were really vaccinated. This class includes not only those cases stated to have been vaccinated, in whom the scar was certainly absent, but also those in which the presence or absence of the scar could not be affirmed with certainty on account of the abundance of the smallpox eruption. This latter class is much more numerous than the former and naturally has a high death-rate. Consequently the position assigned to the doubtful class in the table has an influence on the value of r out of proportion to the size of the class.

The reasons for believing the majority of the doubtful cases to have been vaccinated are:

(*a*)   In those cases where direct observation fails, it seems reasonable to attach value to the statement that the patient had been vaccinated, as likely to be right in the majority of instances.

(*b*)   Smallpox affects vaccinated and unvaccinated persons at very different ages; not only is the prevailing age different in the two groups, but also the two groups only overlap to a moderate extent. The age distribution among the doubtful cases coincides most closely with that of the vaccinated class, as is seen in the adjoining table.

TABLE IV.

Age incidence among the Vaccinated, Unvaccinated and Doubtfully Vaccinated.

Age	0–9	10–19	20–29	30–39	40–49	50–59	Over 60	Totals
Vaccinated ...	134	1163	2517	1755	893	320	163	6945
Doubtful cases ...	33	73	98	77	62	52	41	436
Unvaccinated ...	1274	619	244	90	31	13	7	2278

(c) On the other side it is often argued that the death-rate among the doubtful class is high, and corresponds closely with that among the unvaccinated. Hence it is assumed that the majority of "doubtful" cases were probably unvaccinated. I do not agree with the conclusion, for the high death-rate (higher than among the admittedly unvaccinated) may be due to the fact that in cases with very severe eruption it is impossible to be sure that a scar is present.

It is noticeable that the above values are lower than any of those found by Macdonell for previous epidemics, as well as for those he obtained for the same epidemic in London. His figures for the latter were obtained before the *M. A. B. Report* for 1902 was published, and are subject to the drawback of including an undue proportion of deaths. I cannot explain this better than by quoting the article in the *Times* from which his figures were drawn. "Returns during the "progress of an outbreak have a serious drawback, which must be carefully borne "in mind. It is obvious that only completed cases can be dealt with, as the "result of those still under treatment is uncertain; but since death usually takes "place at a comparatively early stage of the illness, the fatal cases are always in "advance of the recoveries.......The case mortality therefore always appears very "much higher during an outbreak than it really is. In the present instance it "is probably more than twice as high as it should be."

The following two tables are copied from his paper:

TABLE A.

London, Cases admitted to M. A. B. Hospitals, and completed to Recovery or Death, up to November 30th, 1901*.

	Recoveries	Deaths	Totals
Unvaccinated ...	30	60	90
Vaccinated ...	195	45	240
Totals	225	105	330

$r = -\cdot6605 \pm \cdot0406.$

TABLE B.

London, Cases admitted to M. A. B. Hospitals, and completed to Recovery or Death, up to the end of 1901.

	Recoveries	Deaths	Totals
Unvaccinated ...	96	98	194
Vaccinated ...	652	108	760
Totals	748	206	954

$r = -\cdot5779 \pm \cdot0311.$

* I have altered the sign of r, and the arrangement of the table to make it comparable with my own tables.

From these two tables and a further table of Islington cases, brought up to the end of the epidemic, he concludes that the correlation diminished as the epidemic progressed.

I have calculated the correlation separately for the two years 1901 and 1902.

TABLE V.

London, all Cases admitted to M. A. B. Hospitals during the year 1901.

	Recoveries	Deaths	Totals
Unvaccinated	253	119	372
Vaccinated or doubtful ...	1198	173	1371
Totals	1451	292	1743

$r = -\cdot3851.$

TABLE VI.

London, all Cases admitted to M. A. B. Hospitals during the year 1902.

	Recoveries	Deaths	Totals
Unvaccinated	1272	634	1906
Vaccinated or doubtful ...	5307	703	6010
Totals	6579	1337	7916

$r = -\cdot4330.$

I therefore conclude that the correlation did not diminish with the progress of the epidemic. Table V. is strictly comparable with Table B in period, and differs from it chiefly in taking account of the ultimate result of all those cases remaining in hospital at the end of the year. Both my tables include the doubtful cases which are omitted in Macdonell's. The effect is to lower the values of r, but not as can be seen by comparing Tables I. and II. to an extent sufficient to account for the difference.

I have also calculated the coefficients of correlation between severity of disease and number, area and foveation of scars from the figures for the vaccinated cases only; severity of disease being estimated as in the previous tables by the fact of death or recovery.

TABLE VII.

Vaccinated cases only, 1901—2. *Area of Vaccination Scars and Severity.*

	Recoveries	Deaths	Totals
Area $<\frac{1}{2}$ inch ...	1402	293	1695
Area $>\frac{1}{2}$ inch ...	4784	379	5163
Totals ...	6186	672	6858

$r = -\cdot2997.$

TABLE VIII.

Vaccinated cases only, 1901—2. Number of Vaccination Scars and Severity.

	Recoveries	Deaths	Totals
Less than 4 scars ...	3449	506	3955
4 scars or more ...	2761	169	2930
Totals	6210	675	6885

$r = - \cdot 2631.$

TABLE IX.

Vaccinated cases only, 1901—2. Foveation of Scars and Severity.

	Recoveries	Deaths	Totals
Scars less than half foveated ...	5203	602	5805
Scars more than half foveated...	980	58	1038
Totals	6183	660	6843

$r = - \cdot 1664.$

These figures agree well with those given by Macdonell for the Glasgow epidemic of 1900–1, less closely for the epidemic of 1892–5 in the same town. The slight differences in the total numbers observed in the three tables are due to the measurement of area or the counting of the number of scars having presented special difficulty in certain cases.

An important practical point to settle, if possible, is whether these three correlations are independent or two of them secondary to the third. It seems probable *à priori* that the protection afforded by vaccination is proportional to the area of the vaccine vesicles, which may be regarded as laboratories for the production of an antitoxin. If this were so the correlation between number of scars and severity would be secondary to that between number and area of scars.

TABLE X.

Vaccinated cases only, 1901—2. Number and Area of Scars.

	Area $< \frac{1}{2}$ inch	Area $> \frac{1}{2}$ inch	Totals
Less than 4 scars ...	1526	2398	3924
4 scars or more ...	165	2761	2926
Totals	1691	5159	6850

$r = + \cdot 6648.$

Denoting by r_{AS}, r_{NS}, r_{AN}, the total correlations between area of scars, number of scars and severity of disease taken in pairs, and ρ_{AS}, ρ_{NS}, the partial correlations

between area and number of scars with severity of disease, we have, when (i) area is constant and (ii) number is constant respectively,

$$r_{NS} = -\cdot2631, \quad r_{AS} = -\cdot2997, \quad r_{AN} = +\cdot6648.$$

$$\rho_{NS} = \frac{r_{NS} - r_{AS}r_{AN}}{\sqrt{(1 - r^2_{AS})(1 - r^2_{AN})}} = -\cdot0897,$$

$$\rho_{AS} = \frac{r_{AS} - r_{NS}r_{AN}}{\sqrt{(1 - r^2_{NS})(1 - r^2_{AN})}} = -\cdot1733.$$

Dealing in the same manner with area and foveation of scar, we have the following table:

TABLE XI.

Vaccinated cases only, 1901—2.   Area and Foveation of Scars.

	Area $<\frac{1}{2}$ inch	Area $>\frac{1}{2}$ inch	Totals
Scars less than half foveated ...	1530	4255	5785
Scars more than half foveated...	145	893	1038
Totals	1675	5148	6823

$$r = +\cdot2386.$$

And
$$r_{AS} = -\cdot2997, \quad r_{FS} = -\cdot1664, \quad r_{AF} = +\cdot2386,$$

$$\rho_{FS} = \frac{r_{FS} - r_{AS}r_{AF}}{\sqrt{(1 - r^2_{AS})(1 - r^2_{AF})}} = -\cdot1012,$$

$$\rho_{AS} = \frac{r_{AS} - r_{FS}r_{AF}}{\sqrt{(1 - r^2_{FS})(1 - r^2_{AF})}} = -\cdot2719.$$

It seems therefore not improbable that among cases with equal area of vaccination marks, there is no difference in the amount of protection afforded by different numbers of scars.   The apparent protection afforded by foveation of the scars, though not very great in itself, can with less probability be attributed to secondary association of foveation with large scars.

It would be a good thing if these two points could be definitely settled, for a great deal of time is spent in smallpox hospitals in observing and recording these data, some of which may possibly be unnecessary.

The fact that ρ_{NS} as above calculated is not equal to 0 may be due to the fact that some scars grow considerably between infancy and adult life.   This of course would tend to lessen r_{AS} (the area being measured many years after the operation), but would leave r_{NS} unaltered unless, as is probably not common, one or more scars disappeared entirely.   The above formulae also apply only to linear correlation, while the correlation in these cases may be skew.

I have paid much attention to the subject of age in these statistics, as the subject seems to me important from two points of view. It is known that the protection afforded by vaccination against attack is not permanent, but diminishes with lapse of time. A quantitative estimate of the amount of protection soon after vaccination and after the lapse of different intervals of time would be a useful guide as to the proper period at which to perform revaccination. Since no statistics are available as to the protection against attack, it seems reasonable to enquire into the time-decrement of the protection against death and take this as a guide. The second reason for enquiry as to age of the patients is the heterogeneity of the figures given above as regards age. If we take the correlation coefficient between vaccination and severity to be a measure of the protection afforded by the operation, it seems reasonable to demand that the two groups, vaccinated and unvaccinated, should be similar in all respects, so far as that is possible of attainment. But this is far from the case in respect to age in the above tables. Table IV. shews that the ages of the vaccinated and unvaccinated differed greatly, and this would tend to alter the value of the correlation. Using the language of multiple correlation there is a very high correlation, among the patients treated in hospital, between age and vaccination. That being so, we cannot affirm à priori a close correspondence in magnitude, nor even in sign, between the total correlation r_{rs} between vaccination and severity, and the partial correlation ρ_{rs} between the same factors.

To ascertain the time-decrement of protection I have divided up the cases into five age groups and worked out the value of r in each. Nearly all the cases who were vaccinated at all were vaccinated in infancy, and the number of these who had been revaccinated was small. The assumption that the age of the patient measures the number of years that had elapsed between the last operation of vaccination and the attack of smallpox is in most cases correct.

The following shews the number of revaccinated cases admitted:

TABLE XII.

Number of Revaccinated Persons Admitted in 1901 and 1902.

Interval in Years since Revaccination.

	0—10	10—20	20 and over	Uncertain	Totals
1901	12	24	45	1	82
1902	34	79	161	2	276
Totals	46	103	206	3	358

The figures for 1902 are not complete in that no record was kept of revaccination at Gore Farm Hospital. As the cases admitted to that hospital numbered 565 only, as against 7316 to the two hospitals where the record was kept, the omitted cases cannot have been numerous.

A complete list of the 276 cases recorded in 1902 is published in the report. From this I have compiled the following details as to age:

TABLE XIII.

Ages of Revaccinated Persons Admitted in 1902.

	Under 15	15—20	20—25	25—30	30—40	40—50	50—60	Over 60	Totals
No. of cases	3	—	13	21	72	103	44	20	276
No. of deaths	—	—	1	—	9	7	8	2	27

From these figures it will be seen that Tables XVI. and XVII. below only contain 3 revaccinated cases, and the high value of *r* at this period is not due to prevalent revaccination.

TABLE XIV.

Patients under 10 years of age, 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated	832	442	1274
Vaccinated or doubtful ...	159	8	167
Totals	991	450	1441

$r = - ·5567.$

TABLE XV.

Patients under 10 years of age, 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated or doubtful ...	859	448	1307
Vaccinated	132	2	134
Totals	991	450	1441

$r = - ·6636.$

TABLE XVI.

Patients from 10 to 19 years of age, 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated	469	150	619
Vaccinated or doubtful ...	1198	38	1236
Totals	1667	188	1855

$r = - ·6414.$

TABLE XVII.

Patients from 10 *to* 19 *years of age,* 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated or doubtful ...	527	165	692
Vaccinated	1140	23	1163
Totals	1667	188	1855

$r = - \cdot 6933.$

TABLE XVIII.

Patients from 20 *to* 29 *years of age,* 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated	158	86	244
Vaccinated or doubtful ...	2428	187	2615
Totals	2586	273	2859

$r = - \cdot 5364.$

TABLE XIX.

Patients from 20 *to* 29 *years of age,* 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated or doubtful ...	216	126	342
Vaccinated	2370	147	2517
Totals	2586	273	2859

$r = - \cdot 6212.$

TABLE XX.

Patients from 30 *to* 39 *years of age,* 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated	49	41	90
Vaccinated or doubtful ...	1548	284	1832
Totals	1597	325	1922

$r = - \cdot 4093.$

TABLE XXI.

Patients from 30 *to* 39 *years of age,* 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated or doubtful ...	90	77	167
Vaccinated	1507	248	1755
Totals	1597	325	1922

$r = - \cdot 4790.$

TABLE XXII.

Patients 40 years of age and over, 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated	17	34	51
Vaccinated or doubtful ...	1172	359	1531
Totals	1189	393	1582

$r = - \cdot 4771.$

TABLE XXIII.

Patients 40 years of age and over, 1901—2.

	Recoveries	Deaths	Totals
Unvaccinated or doubtful ...	98	108	206
Vaccinated	1091	285	1376
Totals	1189	393	1582

$r = - \cdot 4440.$

Of the two methods of dealing with the doubtful cases I have given my reasons above for preferring their inclusion with the vaccinated rather than the unvaccinated. As between Tables XIV. and XV., however, I think it possible that a majority of the "doubtful cases" under 10 were unvaccinated, in which case the value given by Table XV. would be the more accurate.

On comparing the values of either series, however, we find that the value of r does not vary very much in the three periods under 30 years of age, whereas after the age of 30 there is a moderate fall in value. The maximum occurs in the period from 10 to 20. This result is in striking contrast to the opinion of the members of the medical profession as to the duration of immunity against attack; which is that the protection afforded by vaccination is highest for the first few years after the operation and then falls off rapidly, so that at 10 or 12 years of age revaccination becomes desirable. So long as no statistics are available of the number of persons exposed to risk of infection by smallpox, it is impossible to calculate by mathematical methods the protection afforded by vaccination against attack. But the above opinion as to the duration of vaccination-immunity is really arrived at by methods identical in principle to the above fourfold tables, that is by recording the numbers attacked of vaccinated and unvaccinated persons at different ages, and contrasting these with the very roughly known proportion existing between vaccinated and unvaccinated persons in the community at large. Inside a smallpox hospital there is found a very high correlation between age and vaccination, say ·9 or ·8 (see Tables XXIV., XXVI. and XXVIII.). If there existed an equally high correlation among the whole community between age and vaccination, we might assume that vaccination either failed to protect, or protected with equal force throughout life. But if there is only slight correlation between

these two conditions among the population outside, which is most probably the case, the protection gained by the vaccinated in infancy must diminish as age advances.

We seem therefore driven to the conclusion that protection against attack and protection against death, though both produced by the same operation, disappear at widely different rates. In a later part of this paper, however, I put forward a suggestion which I think accounts for the discrepancy in the figures, and allows us to suppose that the protections against attack and against death produced by vaccination are the same, not only in cause, but also quantitatively as measured by correlation coefficients.

The effect of the age of the patients upon the value of r in Table I. is perhaps best shewn by comparing it with the values derived from Tables XIV.—XXIII.

The mean value of r from the five tables, in which the doubtful cases have been included with the vaccinated, is $-\cdot5234$, against $-\cdot4246$ of Table I. And the mean value from the other five tables is $-\cdot5795$ against $-\cdot4922$ of Table III. The effect has therefore been to diminish r. It is interesting, however, to use the formulae for multiple correlation to test the result.

Taking the division of age at 10 years, we have the following table:

TABLE XXIV.

All Cases, 1901—2*.

	Under *10*	Over *10*	Totals
Unvaccinated ...	1274	1004	2278
Vaccinated ...	134	6811	6945
Totals ...	1408	7815	9223

$r = + \cdot912.$

TABLE XXV.

All Cases, 1901—2.

	Recoveries	Deaths	Totals
Under *10* ...	991	450	1441
Over *10* ...	7039	1179	8218
Totals ...	8030	1629	9659

$r = - \cdot3150.$

* I have inadvertently omitted the doubtful cases in calculating this table. As these high correlations are laborious to work out I have not thought it necessary to recalculate the value with the doubtful cases included. In all the tables of age and vaccination I have carried the series in r to twelve terms, and the terms beyond would certainly influence the third figure, probably the second also. The Tables XIV. and XV. also give series in r which converge slowly. In these two cases also I have stopped at twelve terms.

From Tables I., XXIV. and XXV. we get

$$r_{VS} = -\cdot4246, \quad r_{AV} = +\cdot912, \quad r_{AS} = -\cdot3150,$$

$$\rho_{VS} = \frac{r_{VS} - r_{AV}r_{AS}}{\sqrt{(1 - r^2_{AV})(1 - r^2_{AS})}} = -\cdot3530.$$

If we take the age division at 20 or 30 years we get the four Tables XXVI.—XXIX.

TABLE XXVI.

All Cases, 1901—2.

	Under *20*	Over *20*	Totals
Unvaccinated...	1893	385	2278
Vaccinated or doubtful ...	1403	5978	7381
Totals ...	3296	6363	9659

$r = +\cdot821.$

TABLE XXVII.

All Cases, 1901—2.

	Recoveries	Deaths	Totals
Under *20* ...	2658	638	3296
Over *20* ...	5372	991	6363
Totals ...	8030	1629	9659

$r = -\cdot0904.$

Whence we deduce

$$r_{VS} = -\cdot4246, \quad r_{AV} = +\cdot821, \quad r_{AS} = -\cdot0904,$$

$$\rho_{VS} = -\cdot6178.$$

TABLE XXVIII.

All Cases, 1901—2.

	Under *30*	Over *30*	Totals
Unvaccinated	2137	141	2278
Vaccinated or doubtful ...	4018	3363	7381
Totals ...	6155	3504	9659

$r = +\cdot671.$

TABLE XXIX. *All Cases,* 1901—2.

		Recoveries	Deaths	Totals
Under *30*	...	5244	911	6155
Over *30*	...	2786	718	3504
Totals	...	8030	1629	9659

$$r = + \cdot 1360.$$

Whence we deduce

$$r_{VS} = - \cdot 4246, \quad r_{AV} = + \cdot 671, \quad r_{AS} = + \cdot 1360, \quad \rho_{VS} = - \cdot 7022.$$

The discrepancy between the above three values of ρ_{VS} is largely due to the want of linear correlation between age and severity of disease as measured by death or recovery. Another possible cause has occurred to me, which may have contributed to this result; and as this cause would also account for the small value shewn above for the diminution of vaccination protection with time, I am inclined to lay stress upon it.

Let us take any individual out of a population exposed to smallpox. Three things may happen to him, either he may escape the disease altogether, he may contract smallpox in a mild form and eventually recover, or he may contract it in a severe form and die. Into which of the three classes any individual falls, depends on his power of resistance to the disease or his immunity, and on the amount of virulence of the infection to which he is exposed. In some diseases, as diphtheria and tetanus, we are acquainted with immunity produced by a chemical substance, and which may therefore be quantitatively measured. In diphtheria and tetanus the above three classes correspond to definite quantitative amounts of antitoxin. In smallpox we have not this knowledge, but the analogy makes it justifiable to regard immunity as a quantity, though not as yet measurable. The immunity may be natural, or acquired by vaccination, or the result of the two causes combined.

If in such a population we desire to examine the correlation between severity of disease, which is the observed fact rendering the immunity visible to us[*], with any other factor as vaccination, age, &c., we may make a sixfold table, thus:

TABLE C.
State as regards Disease.

State as regards Vaccination		No Disease	Disease and Recovery	Disease and Death	Totals
Unvaccinated	...	*a*	*b*	*c*	*s*
Vaccinated	...	*d*	*e*	*f*	*t*
Totals	...	*g*	*l*	*m*	*N*

[*] The influence of exposure to different degrees of virulence of infection is probably also an important factor, but is ignored, because at present beyond our observation. In dealing with a large population there will probably be no marked differences in this respect between the vaccinated and the unvaccinated classes.

where
$$g = a + d, \quad l = b + e, \quad m = c + f,$$
$$s = a + b + c, \quad t = d + e + f,$$
$$N = s + t = g + l + m.$$

This sixfold table can be turned into a fourfold table by adding the middle column to either the first or second. If the distribution of the two characters among the population is a normal one, the same value of r will be obtained by applying the "fourfold table" formulae to either of these. This, however, will not be the same as the value calculated by the same formulae from the fourfold table obtained by omission of the first column, thus:

TABLE D.

	Recovered	Died	Totals
Unvaccinated ...	b	c	u
Vaccinated ...	e	f	v
Totals ...	l	m	n

where
$$u = b + c, \quad v = e + f, \quad n = l + m = u + v.$$

We have no accurate evidence on the distribution of immunity among the population at large, but the following reasons point to the assumption of normal distribution being more correct for the former sixfold than the latter fourfold table.

(1)   The same agent, vaccination, is known to protect against attack and also against death. On the former supposition this fact is intelligibly represented in the table, for if an array representing a group of persons all vaccinated to an equal degree were to be vaccinated further, the severity of disease in each individual case would be reduced. It follows that some who would have contracted a fatal attack would now recover, i.e. some cases would be transferred from column 3 to column 2. Similarly others would be transferred from column 2 to column 1, and would not suffer from smallpox after exposure. In the fourfold table the first transfer is similarly represented, but the second is not. Whatever unit may ultimately be used to measure severity, the hypothesis of normal distribution applied to the latter table cannot explain a reduction of severity to 0 or a negative quantity.

(2)   Among vaccinated persons the severe cases are much less frequent than the mild. The hypothesis of normal distribution amongst those affected with smallpox requires that in the total, and also in each array, the cases shall cluster round a certain mean severity and that cases of very great or very slight severity shall be extremely rare. If, however, normal distribution of immunity applies to the whole population, we can readily see how grades of slight severity can be the commonest.

Thus in Fig. 1 the abscissae represent degrees of immunity, which in the case of those who contract the infection are represented by the degree of severity
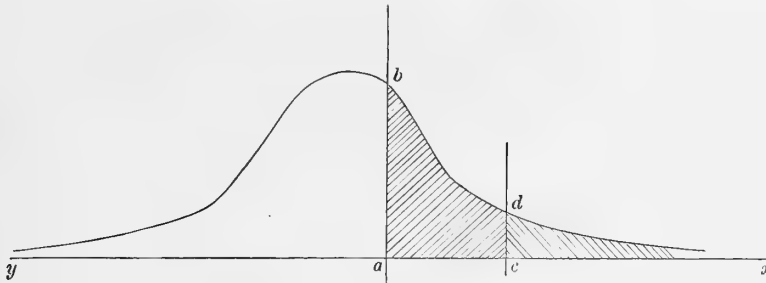
FIG. 1.

of the resulting disease. In those who escape infection the degree of immunity is greater than that required to prevent the disease appearing. It may be much more than this, but the excess is latent, and we have no means of observing it. The ordinates represent frequencies, the line ab divides those who escape infection from those who do not; the line cd divides the deaths from the recoveries.

The area $abdc$ represents the number of recovered cases, and xcd the number of deaths. It is obvious that, if those who escape infection exceed half of those who are exposed, the frequencies of the different degrees of severity diminish without exception, as the degree increases. Probably the actual frequency curve among the vaccinated is of this type.

Assuming for the above reasons that the normal distribution applies to Table C we see that the same cannot hold also for Table D, though all the above calculations have been based on the latter assumption. It is therefore important to examine what is the effect of neglecting one column in a sixfold normal table and treating the remaining figures as if they conformed to a normal distribution.

Such a table I shall call a "curtailed" normal distribution, and the value of r obtained from the full table I shall call the true value. That which is obtained from the curtailed table by the use of any formula I shall call the "apparent" value and shall denote by r'.

As a first example I take the table on p. 214 of *Biometrika*, Vol. I., which is shewn by Macdonell to be a close fit to the normal correlation table. Drawing a line under "head breadth 15·0" I have kept all figures above the line and discarded all those below. The resulting table is therefore a curtailed correlation table, analogous to our smallpox tables drawn from the inmates of a smallpox hospital, all those who remained unaffected by the disease having escaped observation. A curtailed table in measurable characters would result if we collected statistics of height from soldiers, a population from whom all below a certain standard height had been already rejected.

Using product moments, I have found in the above case $r' = \cdot2446$, whereas the true value $r = \cdot4016 \pm \cdot010$.

Using fourfold tables the value of r' varies very much according to the point chosen for division. Thus I have calculated the following three tables:

TABLE XXX.

Head Length.

Head Breadth.		19·0 or less	Over 19·0	Totals
	Less than 15·0 ...	701	599	1300
	15·0	109	119	228
	Totals ...	810	718	1528

$r' = ·0837.$

TABLE XXXI.

Head Length.

Head Breadth.		19·0 or less	Over 19·0	Totals
	Less than 14·9 ...	619	458	1077
	14·9 or 15·0 ...	191	260	451
	Totals ...	810	718	1528

$r' = ·2266.$

TABLE XXXII.

Head Length.

Head Breadth.		19·0 or less	Over 19·0	Totals
	Less than 14·8 ...	526	348	874
	14·8 to 15·0 ...	284	370	654
	Totals ...	810	718	1528

$r' = ·2595.$

It is seen that in every case r' is less than r.

If we use product moments a mathematical formula can be obtained.

Let
$$z = \frac{N}{2\pi \sqrt{1 - r^2}\, \sigma_x \sigma_y} \cdot e^{-\frac{1}{2(1-r^2)}\left(\frac{x^2}{\sigma_x^2} - \frac{2rxy}{\sigma_x \sigma_y} + \frac{y^2}{\sigma_y^2}\right)} \quad \ldots\ldots\ldots\ldots(\text{I.})$$

represent the equation to the correlation surface, and $x = h' = h\sigma_x$ be the equation to the bounding line.

Let $\bar{x}$, $\bar{y}$ be the coordinates of the centroid, r' the correlation, both the centroid and the correlation applying to the curtailed surface only.

Then
$$r' = \frac{\int_h^\infty \int_{-\infty}^{+\infty} (x - \bar{x})(y - \bar{y})\, z\, dx\, dy}{\left\{\int_h^\infty \int_{-\infty}^{+\infty} (x - \bar{x})^2 z\, dx\, dy \times \int_h^\infty \int_{-\infty}^{+} (y - \bar{y})^2 z\, dx\, dy\right\}^{\frac{1}{2}}} \quad \ldots\ldots\ldots(\text{II.})$$

and
$$\bar{x} = \frac{\displaystyle\int_h^\infty \int_{-\infty}^{+\infty} zx\,dx\,dy}{\displaystyle\int_h^\infty \int_{-\infty}^{+\infty} z\,dx\,dy} \qquad \dots\dots\dots\dots\dots\dots\dots\dots(\text{III.}).$$

Since the centroid of every y array lies on the regression line, the centroid of the figure must also lie upon this line. Consequently

$$\bar{y} = \bar{x}\,.\,r\,.\,\frac{\sigma_y}{\sigma_x} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots(\text{IV.}).$$

All the integrals in (II.) and (III.) can be reduced to the probability integral or directly integrated.

Putting
$$e^{-\frac{1}{2}h'^2} = \sqrt{2\pi}p \quad \text{and} \quad \int_{h'}^\infty e^{-\frac{1}{2}x^2}\,dx = \sqrt{2\pi}q,$$

we have
$$\int_h^\infty \int_{-\infty}^{+\infty} z\,dx\,dy = \frac{N}{\sqrt{2\pi}\,\sigma_x}\int_h^\infty e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}\,dx$$
$$= Nq,$$

and
$$\int_h^\infty \int_{-\infty}^{+\infty} xz\,dx\,dy = \frac{N}{\sqrt{2\pi}\,\sigma_x}\int_h^\infty e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}x\,dx$$
$$= Np\,.\,\sigma_x,$$

$$\therefore\ \bar{x} = \frac{p}{q}\,\sigma_x\dots\dots \qquad \dots\dots\dots\dots\dots\dots\dots(\text{V.}).$$

From (IV.)
$$\bar{y} = r\,.\,\frac{p}{q}\,\sigma_y\dots\dots\dots\dots\dots\dots\dots\dots\dots(\text{VI.}).$$

Substituting the values from (V.) and (VI.) we obtain the following values for the three integrals in (II.):

$$\int_h^\infty \int_{-\infty}^{+\infty} (x-\bar{x})^2 z\,dx\,dy = N\sigma_x^2\,\frac{hpq + q^2 - p^2}{q},$$

$$\int_h^\infty \int_{-\infty}^{+\infty} (y-\bar{y})^2 z\,dx\,dy = N\sigma_y^2\,\frac{rhpq - r^2p^2 + q^2}{q},$$

$$\int_h^\infty \int_{-\infty}^{+\infty} (x-\bar{x})(y-\bar{y})\,z\,dx\,dy = Nr\sigma_x\sigma_y\,\frac{hpq + q^2 - p^2}{q}.$$

Substituting these values in (II.) we obtain:

$$r'^2 = \frac{r^2(hpq + q^2 - p^2)}{r^2(hpq - p^2) + q^2} \qquad \dots\dots\dots\dots\dots\dots(\text{VII.}),$$

or
$$r^2 = \frac{r'^2 q^2}{q^2 + (1 - r'^2)(hpq - p^2)} \qquad \dots\dots\dots\dots\dots(\text{VIII.}).$$

From (V.) we see that the distance between the centroids of the curtailed and the whole figures, resolved along the axis of x, is $\dfrac{p}{q}\sigma_x$. The distance of the limit of the curtailed figure from the latter centroid resolved in the same way is $h' = h\sigma_x$; so that $\dfrac{p}{q} - h$ is the expression for the distance of the curtailed centroid from the limiting straight line. It follows that $\dfrac{p}{q} - h$ is always positive.

Since p and q are essentially positive and $r^2 < 1$ we see from (VIII.) that it is impossible for r' to be numerically greater than r. If, however, $r = 0$ or 1, then r' also $= 0$ or 1.

The same can be shewn by differentiating $p - h$.

The equation (VIII.) can be written thus:

$$r^2 = \frac{r'^2}{1 + (1 - r'^2)\dfrac{hpq - p^2}{q^2}} \quad\dots\dots\dots\dots\dots\dots\dots\dots(\text{IX.}).$$

In the case of smallpox correlations r' is the calculated apparent correlation. r is unknown, and cannot be calculated since h is also unknown. But it will be seen that when r' is constant r is greater when

$$\frac{p^2 - hpq}{q^2} = \left(\frac{p}{q}\right)^2 - h\left(\frac{p}{q}\right)$$

is less.

Calling this quantity z, the following gives values of z for a range of values of h:

h	z		h	z
$-2\cdot0$	$+\cdot119$		$+0\cdot1$	$+\cdot658$
$1\cdot5$	$\cdot227$		$0\cdot2$	$\cdot662$
$1\cdot0$	$\cdot371$		$0\cdot3$	$\cdot700$
$0\cdot5$	$\cdot514$		$0\cdot4$	$\cdot715$
$0\cdot4$	$\cdot540$		$0\cdot5$	$\cdot729$
$0\cdot3$	$\cdot565$		$1\cdot0$	$\cdot796$
$0\cdot2$	$\cdot592$		$1\cdot5$	$\cdot848$
$0\cdot1$	$\cdot613$		$2\cdot0$	$\cdot854$
$-0\cdot0$	$+\cdot637$		$+3\cdot0$	$1\cdot30$

It will be seen that, as h increases, z and consequently the discrepancy between r and r' also increase[*].

[*] A simple geometrical interpretation of the quantity z is obtained thus. Let C, Fig. 2, represent the centroid of the whole figure, CX, CY the axes, PN the dividing line, and O the centroid of the curtailed figure. Then $CN = h\sigma_x$. Draw OM perpendicular to CX.
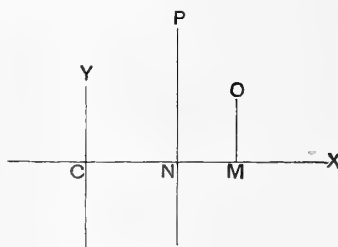
Fig. 2.

$$CM = \frac{p}{q}\cdot\sigma_x,$$

$$z = \left(\frac{p}{q}\right)^2 - h\left(\frac{p}{q}\right),$$

$$\therefore z \times \sigma_x^2 = CM^2 - CM\cdot CN = CM\cdot MN.$$

In the numerical example worked out above

$$r = ·4016 \pm ·010,$$

$$q = \frac{1528}{3000} = ·509\ 333$$

from the tables, $\qquad\qquad h = -·023\ 398,$

$$p = +·399\ 0257,$$

whence $\qquad\qquad r' = ·2570.$

This agrees closely enough with the value ·2446 obtained by direct calculation considering the probable error of r.

In a curtailed normal distribution the use of fourfold tables and formulae is not strictly applicable, i.e. the result obtained from them is not the true correlation of the figure, for the formulae only apply to truly normal figures. Since, however, the formulae give a numerical result to whatever fourfold table they are applied, it is useful to discover the relation between the value so obtained, and the correlation coefficient of the whole figure.

I have not succeeded in finding a solution of the general problem. I have, however, calculated a fair number of curtailed normal tables by the fourfold formulae either chosen from the tables in Macdonell's paper on "Criminal Anthropometry," or formed by first using the fourfold formulae to get the frequency of a normal table with the value of $r = ·5$ at various points of division ; and then using these frequencies to form curtailed tables and calculating r'. It is rash to generalise from a few instances, but I think the following statements are true.

(1) The apparent correlation r' is always less than the true correlation r. The difference increases as the unobserved portion g increases, or as h decreases.

(2) In the same distribution, when r and h are constant, r' varies with the point of division. If this is close to the bounding line r' becomes very small.

An interesting problem, which may prove of practical application, is to solve a table of the form of Table C., when a and d are unknown, but $g = a + d$ is known. This may be done by the tedious process of trial and error. Give any value to a and from the sixfold tables deduce two fourfold tables. If the normal distribution holds good, the values of r obtained from the two tables will be identical.

As an example let us suppose that the number of persons exposed to infection in London was double the number who developed the disease. From this we get the following sixfold table.

TABLE E.

All Exposed to Infection.

	Not Diseased	Affected with Smallpox		Totals
		Recovered	Died	
Unvaccinated ...	a	1525	753	
Vaccinated ...	d	6505	876	
Totals ...	9659	8030	1629	19318

If $a = 450$ we get the two following fourfold tables:

	Not Diseased	Diseased	Totals
Unvaccinated ...	450	2278	2728
Vaccinated ...	9209	7381	16590
Totals ...	9659	9659	19318

$$r_1 = -\cdot5373.$$

	Alive	Dead	Totals
Unvaccinated ...	1975	753	2728
Vaccinated ...	15714	876	16590
Totals ...	17689	1629	19318

$$r_2 = -\cdot5429.$$

If $a = 400$ we find

$$r_1 = -\cdot5611, \qquad r_2 = -\cdot5496.$$

Whence by interpolation we get the correct solution:

$$a = 434, \qquad d = 9225,$$
$$r = -\cdot5451.$$

If, however, the number exposed to risk is not much larger than the number who contract the disease the values of r and r' more nearly coincide. Thus if the number exposed who do not contract the disease is equal to only one-tenth of those who do, we have $g = 966$, and I have calculated $a = 39$, $r = -\cdot4492$. On the other hand if the number of persons exposed to infection was several times larger than that of those who developed the disease, the true value of r considerably exceeded $-\cdot54$.

TABLE F.

	Not Diseased	Diseased		Totals
		Recoveries	Deaths	
Unvaccinated ...	39	1525	753	2317
Vaccinated ...	927	6505	876	8308
Totals ...	966	8030	1629	10625

$$r = -\cdot4492.$$

I think the above figures afford some explanation of the apparent small diminution of vaccination protection as age advances. If my contention be correct that a normal distribution in immunity may more correctly be assumed for the whole population exposed to attack, than for that part only which actually contracts the disease, and appears in smallpox hospital statistics, then the values given above (Tables II. to IV. and XIV.—XXIII.) are apparent values of r corresponding to curtailed distributions.

If any time-diminution occurs, then the number of persons sufficiently protected to escape the disease altogether will be much larger at the ages 0–10 than at higher ages, say 20—30. Hence, using the above notation, in Table XIV. the value of g is much greater than in Table XIX. Hence $r - r'$ will be greater in the former case than in the latter. Although, therefore, the values of r' in the two cases are nearly equal, the values of r probably differ considerably.

The above investigation of curtailed distributions also gives a possible explanation of the small correlation between severity of disease and number, size and foveation [i.e. depth] of the scars.

For we have seen that if in any normal distribution, if one of the factors is divided into three classes, then leaving out either of the extreme classes from consideration will reduce the sixfold to a fourfold table. The resulting apparent correlation, however, will be less than the true value. Now in arranging our Tables I.—III. we have recognised only two degrees of vaccination, " vaccinated" and "unvaccinated." But if we split the vaccinated group into two, " four scars or more" and "less than four scars," we may arrange the whole as a sixfold table. Hence Table VIII. is seen to be a curtailed form of Table I.; and similar arguments apply to Tables VII. and IX. Macdonell, who discovered the lower correlation in these instances, remarks* : "It is obvious that in dealing with the last four tables we have descended to a much lower plane of correlation, and the results may possibly somewhat modify medical opinion as to the degree of significance of foveation, number of scars and scar area." If, however, the normal distribution held good for Table I., then a " lower plane of correlation " would necessarily follow in Tables VII.—IX., not as a medical, but as a mathematical phenomenon.

* *Biometrika*, Vol. I. p. 383.

Summary.

1. The correlation coefficient between severity of disease and previous vaccination in the late London epidemic was lower than in any of the previously investigated cases.

2. There seem stronger reasons for assuming normal distribution for the whole population exposed to risk of infection, than for the population attacked by the disease.

3. If the former assumption is correct the smallpox correlation tables of Pearson, Macdonell, and of this paper, correspond only to a portion of the normal correlation surface.

4. Values of r calculated from such "curtailed" normal distributions are lower than the true value, whether product moments are used or fourfold formulae, or contingency*. As to the first method a complete investigation is given, as to the other two I have failed to find a mathematical proof, and base the statement upon calculated instances, unfortunately not numerous.

5. Values of r have been calculated for each age group from the figures of the London epidemic. These appear to shew that the protection of vaccination lasts without perceptible diminution for about 30 years. This result is opposed to the opinion of most of those who have had experience of smallpox. My hypothesis given under (2) would explain the discrepancy, for the real values of the correlation at early ages would be considerably higher than the apparent values drawn from smallpox cases only.

6. The correlations between area of scars and severity, number of scars and severity, foveation of scars and severity in the London epidemic agree closely with those calculated by Macdonell from Glasgow statistics. They are all considerably lower than the correlation between vaccination and severity. The difference would be expected, if the normal distribution held for the total population admitted to hospital. For the tables from which the three former correlations have been calculated are "curtailed" by the omission of the unvaccinated group.

7. The figures from the London epidemic do not enable us to decide whether the correlations of severity with number and foveation of scars are secondary to that between severity and area of scars; though they may be so.

8. The gross correlation between vaccination and severity is lower than the mean net correlation at various ages, owing to the very high correlation between age and vaccination among inmates of smallpox hospitals.

* No examples of contingency formulae are given in the paper, as none of the tables were adapted to this method.

MISCELLANEA.

I. On the Distribution of Severity of Attack in Cases of Smallpox.

By KARL PEARSON, F.R.S.

Dr Turner finds that the protection provided by vaccination does not diminish as rapidly with age as he would expect it to do.   His expectation arises from the fact that current medical opinion considers that the immunity provided by vaccination diminishes rather rapidly with the increase of the period which has elapsed since vaccination.   I do not propose to consider whether protection against death when disease is incurred is really of the same character as immunity against an attack.   But I should wish to point out that Dr Brownlee has reached a very similar conclusion to Dr Macdonell on this point by a very different process, and further that it follows, if we simply use the coefficient of association, which makes no appeal to normality of distribution.   On the other hand, while I am distinctly interested in Dr Turner's theory of a "curtailed normal curve," I am compelled to say that I consider it an extremely improbable hypothesis in the present application.   The group of persons who catch smallpox are a *selected* group of the general population, selected because (i) they have come in contact with the disease, (ii) they were at the time of such contact not sufficiently immune against it.   This sufficiency of immunity must depend not only on prior vaccination, but on a host of other causes, the virulence of the poison they encountered, their particular state of health at the time in question, their conduct before and after the risk was run, etc., etc.   It is impossible to suppose a rigid line drawn at a certain grade and say all below this grade escape this disease, all above will contract it.   I cannot understand how those 'selected' to incur the disease differ from any other naturally selected group with which we are acquainted.   Now biometricians are dealing with selected groups every time they measure the variation of a character in a species, but no such truncated normal distributions have yet exhibited themselves*.

Dr Turner says "A curtailed table in measurable characters would result if we collected statistics of height in soldiers, a population from whom all below a certain standard height had already been rejected" (p. 497 above).   Now this is a case which can be well tested.   For example, height standards exist for both the American and Italian armies.   Yet what do we find ?   That the distribution of the statures of the accepted recruits in both countries, so far from forming a curtailed normal distribution is in some cases as close to a complete Gaussian

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* American trotting horses give the nearest approach, there being a time limit to entry in the record of trotters.   But this is a perfectly arbitrary line drawn across the trotting population by the hand of man, and not a selection due to a complex of natural causes,

curve as anything with which we are acquainted *. The fact is that there are so many other factors on which selection depends that we get no "curtailing" of the distribution at all. I should imagine that it would be precisely the same with those who are selected to incur small-pox. The selection depends on so very much else than a certain absolutely fixed grade of immunity. This point is of such importance that I think it worth illustrating by an example. It is one which Professor Weldon has most kindly provided me with out of the extensive reductions he has made of Italian conscripts and recruits. In the Province of Verona in the five years between 1875 and 1879 there were 16,203 conscripts† and from these were selected 3810 recruits‡. The following table contains the two distributions:

Stature in Centimetres	124—125	126—127	128—129	130—131	132—133	134—135	136—137	138—139	140—141	142—143	144—145	146—147	148—149	150—151	152—153	154—155	156—157	158—159	160—161
Conscripts	4	—	2	1	1	1	—	2	7	7	22	33	55	117	225	626	725	1027	1503
Selected Recruits	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16	148	239	354

Stature in Centimetres	162—163	164—165	166—167	168—169	170—171	172—173	174—175	176—177	178—179	180—181	182—183	184—185	186—187	188—189	190—191	192—193	Totals
Conscripts	1869	2065	2125	1703	1525	1058	699	384	205	131	45	24	7	4	1	—	16203
Selected Recruits	471	542	538	425	414	256	170	133	55	35	9	3	—	2	—	—	3810

In the diagram the two distributions with their corresponding curves are given, and we see that in the selected recruits there is not the slightest approach to a curtailed normal curve. In fact, if we examine the fundamental constants of the distribution § we find:

<div align="center">

For the Conscripts   *For the Recruits*

$\sqrt{\beta_1} = \cdot 1658 \pm \cdot 0130$ $\sqrt{\beta_1} = \cdot 3409 \pm \cdot 0268$

$\beta_2 = 3\cdot 7516 \pm \cdot 0260$ $\beta_2 = 2\cdot 7989 \pm \cdot 0536.$

</div>

The probable errors are those which would arise if the distribution were truly normal¶. The deviations from symmetry are for the two cases 12·6 and 12·7 times their probable errors respectively. It is impossible therefore to say more than that the two curves are both markedly skew and deviate equally from normality in this respect. In the next place the deviations from meso-kurtosis are in the two cases 28·9 and 3·8 times their probable errors; in other words the curve for the conscripts diverges indefinitely more from the normal curve than that for the selected recruits. This is only one instance out of many which emphasise the same points, i.e. that there is no approach in the selected curve to curtailment and it differs on the whole less from normality than is the case with the unselected material.

* See, for example, Baxter's statistics for U.S. recruits discussed, *Phil. Trans.* Vol. 186, A, p. 385.

† R. Livi, "Classificazione delle stature......dei coscritti delle leve di terra negli anni 1875–79." *Annali di Statistica*, Vol. VIII. 1883, pp. 144—9.

‡ R. Livi, *Antropometria Militare*, Parte I., p. 260, Roma, 1898.

§ *Phil. Trans.* Vol. 186, A, p. 368. *Biometrika*, Vol. IV., pp. 174—7.

¶ *Phil. Trans.* Vol. 198, A, p. 278.

Comparison of Frequency Distributions of Verona Conscripts and Selected Recruits.

Scale of Frequency.

Stature in cms.

Conscripts.

Selected Recruits.
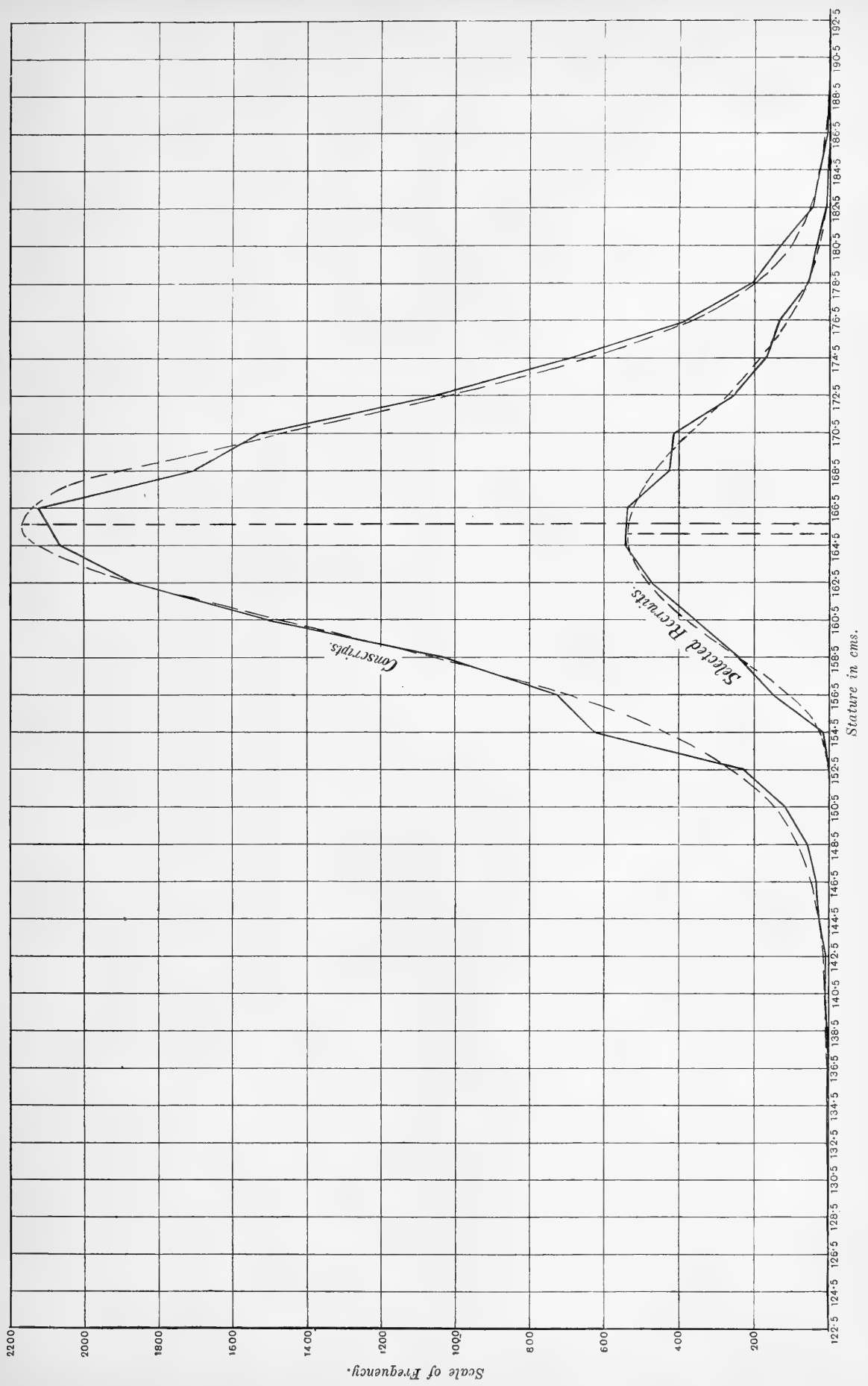
I have next endeavoured to form an estimate of the distribution of severity in the cases of smallpox, and the only method of doing this seemed to be to form a frequency distribution for the intervals which elapse between either (a) onset or (b) eruption and the first bath. This may be taken in a rough sort of way as a measure of the severity of the attack. ·Unluckily any character which depends on time usually gives a skew frequency distribution and the present case is no exception to the rule. But I do not think that the resulting distributions can be in any way described as curtailed normal curves. Dr John Brownlee kindly provided me with the particulars of between 800 and 900 cases. I could distinguish no sensible differences in the male and female distributions. Further, the vaccinated cases formed a very large proportion of the total, for example 779, as against 55 unvaccinated and 21 doubtful. Hence, taking into account that what Dr Turner is dealing with is the distribution of cases in all classes, vaccinated or unvaccinated, I have clubbed all groups together to get my distribution of severity. There were 57 deaths, which must of course be excluded, from a time to bath test of severity, they occurred with rather an erratic distribution at a mean interval of 10·4 days after onset or 7·6 days after beginning of the eruption. Thus the interval between onset and eruption in the fatal cases is about 2·8 days, while in the recoveries it is 3·1 days, so that there may, when more material is forthcoming, be found to be a sensible difference in this interval for the two classes of cases.

The following table gives the distribution of severity of attack as measured by the two tests of days (i) from onset to bath and (ii) from eruption to bath.

Days	4—5	6—7	8—9	10—11	12—13	14—15	16—17	18—19	20—21	22—23	24—25	26—27	28—29	30—31	32—33	34—35	36—37	38—39	40—41	42—43	Totals
Onset to Bath ...	2	13	40	131	192	152	99	73	40	24	13	17	10	6	4	6	2	1	—	1	826
Eruption to Bath	10	55	164	174	156	96	71	39	26	13	21	6	8	2	7	3	3	1	—	—	855

Neither of these distributions can be considered as a curtailed normal curve. They must, I think, be looked upon as significantly skew distributions of the usual type such as in practice almost invariably arise, when time is the variate to which we plot frequency. They do not appear to me to give any support to Dr Turner's view expressed on p. 496 above as (2), or to the suggestion on p. 497, that the mildest cases are more frequent than those more severe ; the different degrees of severity do not diminish without exception as the degree increases, and since the bulk of our cases are all vaccinated, it is not possible to suppose that the actual frequency curve among the vaccinated is of the type suggested by Dr Turner. The modal frequency corresponds to a sensible degree of severity of attack, i.e. to about 13 days from onset to first bath, while the mildest cases correspond to only 4 days. It seems therefore quite impossible to suppose, as Dr Turner does on p. 502, that the severity frequency distribution is half a normal curve of all exposed to risk. The distribution of the attacked very considerably passes the mode, and on the assumption made by Dr Turner, those exposed to risk and escaping must be a very small fraction indeed of the total population exposed to risk. Indeed the above distributions show that the severity of the attack rises from zero to a maximum, and then falls, in the usual skew frequency fashion, at a slower rate to zero. The constants of the above distributions have been worked out ; it will suffice here to give, however :

$$\text{Onset to Bath} \qquad \beta_1 = 2\cdot3229, \quad \beta_2 = 6\cdot2466.$$

$$\text{Eruption to Bath} \qquad \beta_1 = 2\cdot606, \quad \beta_2 = 6\cdot3664.$$

Thus both distributions are sensibly non-Gaussian.

It may be said that this skewness might itself account for the results which Dr Turner considers opposed to current medical opinion and which he would account for by a curtailed normal distribution. Possibly it may, but I hold that this is very unlikely, and for the following reasons :

(i) The severity is here measured by a *time* test, which invariably gives a skew distribution; but if severity could be measured in some other quantitative manner, for example by intensity of eruption, mean rise of temperature or in some such fashion, we should most probably reach a more normal distribution.

(ii) The effect of even the large amount of skewness indicated above on the correlation found by a fourfold division is, I think, not likely to be very large. It is too often assumed that the distribution must be *very closely* normal to give a good result by the fourfold table method. As a matter of fact, if the fourfold division falls on the "long tailed side" of the mode, as it does in these vaccination statistics, we get close values to the actual correlation even by the fourfold table method. To illustrate this I took the only correlation which was available on the data, namely the correlation between onset to eruption and eruption to bath. This correlation was found by the ordinary product moment process, which is independent of any hypothesis of normal distribution and came out $= -\cdot174 \pm \cdot023$. In other words, if severity of the disease be measured by a long period from eruption to bath, then a short period between onset and eruption is associated with severity. This is in accordance with the previous indication that the fatal cases have a short period from onset to eruption.

I now formed a fourfold table from my exceedingly skew distribution, dividing between 14 and 15 days from eruption to bath and between 3 and 4 days from onset to eruption. This gave :

Onset to Eruption.

		3 days and less	*4* days and more	Totals
Eruption to Bath.	*14* days and less ...	395	235	630
	15 days and more ...	138	51	189
	Totals ...	533	286	819

whence the correlation $-\cdot166$ was deduced. It is clear that the skewness has not sensibly influenced the value of the correlation as determined from this fourfold table.

The divergence of the fourfold dividing lines from the median is certainly not quite as great in this case as in the vaccination problem, but it is difficult to go nearer the tails and get anything like a reliable result. Dividing between 4 and 5 days from onset to eruption, and 16 and 17 days from eruption to bath, I get :

Onset to Eruption.

		4 days and less	*5* days and more	Totals
Eruption to Bath.	*16* days and less ...	595	104	699
	17 days and more ...	112	8	120
	Totals ...	707	112	819

In this case the probable error of the frequency 8 is no less than 2, but a change of 2 in the value of this frequency would change the value of r by about 20 p.c. Such a division is

therefore very unreliable*. We find, however, that the correlation $= - \cdot 238$ with a probable error of about $\pm \cdot 05$. In other words within the limits defined by the probable errors, we get a correlation sensibly equal to the actual value $- \cdot 174$. I think accordingly that we may assume that the skewness of our severity test would not materially alter the condition of affairs.

I conclude as follows :

(i)   There is no reason *à priori* for supposing that the distribution of severity of attack in smallpox follows a curtailed normal curve. Such curves are contrary to any existing experience of the distribution of frequency in *selected* groups. In particular they do not occur in the case of soldiers suggested by Dr Turner.

(ii)   The only tests I have been able to apply to the frequency distribution of severity is that of length of period from onset or eruption to bath. These both give a continuous and not a curtailed curve. The curves are skew, as we are accustomed almost invariably to find them when time is the variate.

(iii)   This skewness, due to the time as variate, may not really be characteristic of the distribution of smallpox severity ; this could only be determined if we chose a variate other than time to measure severity. But if it were characteristic, it does not appear that a fourfold division taken so far towards the tail as occurs when we divide severity at death would seriously affect the result. It is shewn that within the limits of probable error the product moment method of calculating the coefficient of correlation and the fourfold division method lead to sensibly the same results, even when we have a skewness in the distribution as great as is indicated in the eruption to bath test of severity.

II.   On the Mean Duration of Life of Individuals Dying within a Year after Birth†.

By RAYMOND PEARL, Ph.D.

The usual custom in tabulating census returns of mortality at different ages is to use a five year base unit for ages above 5, and below that age a one-year unit. This method of tabulation makes the finding of the moments of the frequency distribution somewhat less simple than would be the case if all the base elements were equal. Furthermore, the age distribution of the heavy mortality of the first year of life is not given at all. It becomes a very important matter to know, at least approximately, this first year distribution when one attempts to find the moments for the whole material, because the frequency in this element must be centred at some point before one can proceed with the calculations. The mean age at death of those dying under one year must be known. In connection with some work on vital statistics which is being carried on in the Zoological Laboratory of the University of Michigan, it became necessary to have as exact a determination of the centering point of this first year mortality as it was possible to get. It is the purpose of this note to present the results obtained.

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* Dr Turner's Tables XIV. and XV. contain only 8 and 2 cases in their fourth quadrants and the probable errors of the resulting coefficients are very considerable. I think Dr Brownlee's figures (*Biometrika*, Vol. IV. p. 325 ; *Journal of Hygiene*, Vol. V. pp. 533—4) may be safely taken as giving a reasonable measure of the reduction with time of vaccination protection.

† Contributions from the Zoological Laboratory, University of Michigan, No. 88.

TABLE I. *Deaths (excluding still-born) at the designated age per* 1000 *legitimate or illegitimate children born.*

Age in days and months	LEGITIMATE					
	1877—1881 average		1882—1890 average		1891—1895 average	
	♂	♀	♂	♀	♂	♀
0 to 1 day	9·32	7·16	10·27	7·89	10·83	8·12
1 „ 2 days	5·41	4·12	5·60	4·30	5·52	4·15
2 „ 3 „	3·52	2·80	3·60	2·77	3·45	2·67
3 „ 4 „	2·33	1·87	2·37	1·85	2·24	1·73
4 „ 5 „	1·93	1·51	1·87	1·60	1·73	1·38
5 „ 6 „	2·32	1·76	2·13	1·64	1·86	1·40
6 „ 7 „	2·87	2·22	2·54	1·89	2·13	1·61
7 „ 8 „	2·71	2·08	2·37	1·88	2·03	1·59
8 „ 9 „	2·31	1·84	2·05	1·62	1·76	1·41
9 „ 10 „	1·86	1·51	1·75	1·38	1·56	1·24
10 „ 11 „	1·90	1·54	1·81	1·46	1·70	1·33
11 „ 12 „	1·89	1·47	1·82	1·43	1·67	1·39
12 „ 13 „	1·96	1·61	1·91	1·54	1·83	1·46
13 „ 14 „	1·93	1·60	1·86	1·54	1·77	1·49
14 „ 1 month*	21·73	18·90	21·97	18·98	22·14	18·80
1 „ 2 months*	22·59	19·76	22·70	19·79	23·20	19·78
2 „ 3 „	18·58	15·86	18·92	16·44	19·72	16·62
3 „ 4 „	15·96	13·59	16·45	14·18	16·90	14·40
4 „ 5 „	13·30	11·26	13·99	11·93	14·15	12·13
5 „ 6 „	11·51	9·85	12·07	10·46	12·24	10·55
6 „ 7 „	10·61	9·05	11·05	9·60	11·01	9·53
7 „ 8 „	9·30	8·33	9·91	8·87	9·72	8·77
8 „ 9 „	8·74	7·96	9·22	8·47	9·18	8·20
9 „ 10 „	8·29	7·83	8·69	8·20	8·30	7·85
10 „ 11 „	7·51	7·22	7·80	7·51	7·44	6·89
11 „ 12 „	6·94	6·92	7·17	6·98	6·69	6·49
	197·32	169·62	201·89	174·20	200·77	170·98

Age in days and months	ILLEGITIMATE					
0 to 1 day	11·48	9·37	12·18	9·62	12·82	10·27
1 „ 2 days	8·10	6·37	8·05	6·23	8·06	6·24
2 „ 3 „	5·28	4·06	5·50	4·32	5·54	4·25
3 „ 4 „	3·54	2·96	3·79	3·14	3·69	2·76
4 „ 5 „	3·32	2·77	3·37	2·85	3·08	2·48
5 „ 6 „	4·09	2·97	3·65	2·85	3·35	2·48
6 „ 7 „	4·54	3·58	4·03	3·29	3·51	2·65
7 „ 8 „	4·06	3·43	3·59	3·17	3·26	2·98
8 „ 9 „	3·62	3·12	3·47	2·64	3·08	2·41
9 „ 10 „	3·28	2·60	3·10	2·54	2·91	2·43
10 „ 11 „	3·53	2·67	3·32	2·67	3·04	2·51
11 „ 12 „	3·30	2·67	3·32	2·94	3·35	2·71
12 „ 13 „	3·43	3·11	3·50	2·85	3·48	2·86
13 „ 14 „	3·33	3·12	3·37	2·84	3·75	2·95
14 „ 1 month*	44·83	39·59	46·19	40·05	48·11	42·00
1 „ 2 months*	53·39	46·48	54·03	47·77	55·03	48·96
2 „ 3 „	40·95	37·06	40·93	37·86	43·36	38·63
3 „ 4 „	31·98	29·74	33·01	30·15	33·84	30·66
4 „ 5 „	24·21	23·04	25·77	23·81	26·49	25·12
5 „ 6 „	19·61	17·97	20·12	19·22	20·49	19·96
6 „ 7 „	16·03	15·38	17·04	16·45	17·45	16·84
7 „ 8 „	12·91	13·00	13·93	13·54	14·04	13·36
8 „ 9 „	11·39	11·11	11·77	12·02	11·51	11·98
9 „ 10 „	9·35	10·00	10·55	10·49	10·33	10·06
10 „ 11 „	8·50	8·63	8·56	8·87	8·27	8·29
11 „ 12 „	7·11	7·37	7·51	8·11	7·39	7·08
	345·16	312·17	353·65	320·29	359·23	322·92

* It is assumed that these are calendar months.

Now it is evident that the larger the number of frequency elements for the mortality of the first year for which we can get data, the more accurate will be the determination of the mean, for the reason that the rate of infantile mortality changes rapidly with lapse of time after birth. It is fairly easy to get data giving the number of deaths occurring in each month of the first year, but a still finer division is desirable. Especially is this true for the first month of life. Roughly from 25 to 45 per cent. of all the deaths falling within the first year occur in the first month. Of these the larger proportion occur early in the month. Fortunately the German statistics, which are in many respects among the best vital statistics available, give a rather detailed age grouping of the deaths under one year. I have used in this work the Prussian statistics* for the years 1877 to 1895, split up into three groups as follows: 1877–81, 1882–90, 1891–95. The data are given in the form of *death-rates per* 1000 *born* of the same class, in the given period. Separate returns are given for male and female, and legitimate and illegitimate mortality. Up to the fourteenth day after birth the mortality for each day is recorded. The mortality of the remainder of the first month is grouped together in a single class, and from the beginning of the second month to the end of the first year after birth the rates are given by months. Still-births are, of course, excluded. A glance at Table I. will make clear the way in which the returns are sub-divided.

It is evident that the grouping here is sufficiently fine to make possible a very accurate determination of the mean age at death. The material was dealt with in the following way: the rates were treated as frequencies, except in the determination of probable errors where, of course, the absolute number of deaths was used. A standard month of 30 days was assumed: then with a unit of 30 days the first and second moment coefficients about an arbitrary axis were determined. From these the position of the mean and the value of the second moment about it were easily found. Only the "rough" second moment was calculated, as it was deemed sufficiently accurate for present purposes, and furthermore it was difficult to determine the proper corrective terms to apply in this case. In the calculations each frequency element was for practical convenience, centred at the mid-point of its range. The error made by so doing is negligible.

The results are shown in Table II.

TABLE II.

Mean Duration of Life in Days of Individuals Dying in the First Year after Birth.

		1877—81		1882—90		1891—95	
		Mean	S. D.	Mean	S. D.	Mean	S. D.
Legitimate	♂	111·44	103·92	113·33	101·13	112·54	100·26
,,	♀	116·75	106·13	118·65	105·97	117·91	104·50
Illegitimate	♂	96·81	91·80	98·64	92·47	98·08	91·41
,,	♀	102·86	94·19	104·94	94·89	103·64	92·92

On account of a lack of necessary data it was not possible to determine the probable errors of all these constants. The general order of magnitude of the probable errors, however, may be seen from a consideration of a single case. For the period 1882–90 data were available from

* *Statistisches Handbuch für den preussischen Staat*, Bde. i. ii. and iii. Berlin.

which the probable errors could be determined. I find for the probable error of the mean legitimate male mortality 1882–90 a value of ± ·0676 day, and for the mean legitimate female mortality ± ·0784 day. For the illegitimates the probable errors would be of course somewhat higher, although as nearly as I am able to judge from a rough estimate the probable errors of the means for this group would not exceed ± ·15 day. The probable errors of the standard deviations will in all cases be lower. In calculating probable errors the actual number of deaths in the given group and period was taken as n.

A number of points of general interest regarding infantile mortality are brought out by this table. Before considering the main question for which the work was undertaken some of these may be discussed.

(*a*) The mean duration of life is uniformly greater in the case of the legitimates than in the case of the illegitimates. The legitimate males have an average excess of 14·59 days, and the legitimate females an average excess of 13·96 days. The excess is almost exactly the same for each of the three periods, although the mean changes. This uniformity is remarkable, and indicates to what an extent those differences in "nurture" (both pre- and post-natal) to which the difference in mean duration of life of legitimate and illegitimate infants must be attributed are uniform in long periods of time. Associated with the low mean duration of life in the case of the illegitimate infants there is of course a high death-rate as compared with the legitimate group.

(*b*) The illegitimate infants are markedly less variable with respect to duration of life (as indicated by the standard deviation) than the legitimate. The average difference in the case of the males is 9·88 days, and in the case of the females it is 11·53 days. To adopt the illustration which has been used by Pearson, it may be said that the marksman, Death, shoots faster and with deadlier aim at illegitimate than at legitimate infants. This lower variability in the case of the illegitimates may conceivably be the result of a more sharply selective mortality than in the case of the legitimate infants.

(*c*) The mean duration of life of those dying within a year after birth is greater in the two later periods considered than in the first. Apparently, between 1877 and 1896 there has been a gain of about a day. Does this represent a real evolutionary tendency, or is it merely a chance fluctuation? In order to get light on this question I dealt with the mortality of each of the following years separately; 1882, '83, '84, '85, '86, '89, 1890, '91, '94, '95 and '96. These were all the years for which I could get separate data. The groups of male and female, legitimate and illegitimate were treated separately as in the other cases. The results obtained were very interesting in many particulars, but as they fall outside the scope of this note I shall not consider them in detail here. The general tendency from about 1884 on is for the mean duration of life to decrease, with considerable fluctuations from year to year. Thus in 1886 and in 1890 there was an unusually high duration of life. These two years explain why the 1882–1890 group in Table II. shows such high means. In general, a study of these individual year data makes it very clear that there is no steady tendency towards lengthening of the mean duration of life of infants under 1, *within* the period under consideration.

Another interesting point brought out by the single year records is that in the case of both legitimate and illegitimate infants, there is a definitely marked tendency for an increase in the mean duration of life in any year to be associated with an increase in the death-rate for that year. This appears to indicate that in general there is a tendency for any increase in the infantile death-rate to be the result of an increased number of deaths of older rather than younger infants (under 1). It will be noted, however, that this positive relation between death-rate and mean duration of life which appears *within* both legitimate and illegitimate groups is exactly reversed when the two groups are themselves compared. Thus in the legitimate group as a whole we have a condition of low death-rate and high mean duration of life, while in the illegitimate group as a whole the opposite condition obtains.

The single year means are given in Table III.

TABLE III.

Mean Age in Days of Individuals dying under 1 *in the Designated Years.*
(The death-rate for the same group is given in brackets below each mean
age value.)

Year	♂		♀	
	Legitimate	Illegitimate	Legitimate	Illegitimate
1882	111·71 (202·57)	96·39 (341·90)	117·89 (173·02)	104·17 (314·21)
1883	112·91 (204·94)	100·04 (356·52)	117·72 (175·92)	103·80 (322·44)
1884	114·07 (205·96)	98·86 (360·83)	119·70 (178·13)	103·09 (322·02)
1885	113·91 (197·55)	98·15 (341·24)	119·34 (169·97)	106·26 (311·06)
1886	117·15 (217·29)	101·79 (377·47)	121·46 (188·98)	108·91 (343·06)
1889	112·97 (200·00)	98·00 (363·40)	118·28 (172·90)	104·72 (330·00)
1890	114·62 (204·92)	100·53 (360·65)	122·99 (177·27)	109·05 (324·83)
1891	110·96 (196·55)	96·37 (356·66)	116·26 (166·29)	102·54 (321·75)
1894	113·61 (190·80)	98·15 (339·52)	119·23 (163·65)	104·16 (304·46)
1895	111·89 (207·18)	98·58 (376·59)	116·89 (175·36)	104·07 (337·16)
1896	111·67 (187·62)	95·85 (328·69)	116·83 (159·29)	101·79 (296·99)

(*d*) The mean duration of life is uniformly greater in the females than in the males. The real basis for the difference is not clear. It can hardly be due to differing conditions of "nurture" because there is apparently no reason to suppose that in the long run the environment of a male infant differs in any marked and constant way from that of a female infant during the first year after birth. Along with the lowering of the mean age at death in the male there is a smaller amount of variation in this character.

We may turn now to the practical question as to where the first year mortality shall be centred. The values of the mean for the different groups range between 3 and 4 months and each may be taken as the centering point of its own group. It is desirable, however, to have a single value for the total first year mortality, when males and females, and legitimates and illegitimates are grouped together. In order to obtain such a value resort was had to the plan of a weighted average of all the individual values. Several schemes of weighting were tried, but as all gave very closely accordant results, and the one to be described seemed the most logical, it alone need be detailed here. Examination of all the available data indicated that the absolute number of deaths of males under 1 stood in about the ratio of 1·25 : 1 to the absolute number of

deaths of females under 1. So then the mean age at death of males and females together for each period was determined, the two contributory means being weighted in this proportion 1·25 to 1. Legitimates and illegitimates were dealt with separately. Then the mean age for the whole period 1877–95 for both legitimates and illegitimates was determined. In getting these values the three periods 1877–81, 1882–90, 1891–95 were weighted in the proportions 1 : 2 : 1 respectively. In this way was obtained the mean duration of life of legitimates and illegitimates irrespective of sex. It remained to combine into a single average the legitimates and illegitimates. I found that during the period under discussion there died, on the average, 6·556 legitimate infants under 1, to one illegitimate infant. Averaging, then, the legitimate and illegitimate means previously obtained, with weights of 6·556 and 1, respectively, I found for the mean duration of life of all infants dying under age 1, during the period 1877–95 in Prussia, a value of 113·14 days. With a standard month of 30 days this equals 3·771 months, or ·3142 year. Reckoning 365 days to the year we have from the days directly the mean equal to ·30997 year.

I conclude that the deaths occurring in the first year of life may be centred at ·3 year with sufficient accuracy for ordinary purposes*.

We may turn now to two other practical problems which arise in connection with infantile mortality. These are

1. At what age shall the deaths occurring in the first month of life be centred?

2. At what age shall the deaths recorded in census returns as falling in the age period 0–5 years be centred?

For a solution of the first problem I have taken the Prussian statistics for deaths in the first month of life, during the period 1882–90 (Table I.); I find the following values for the means:

Age at Death of those Dying before completing the First Month of Life.

Legitimate		Illegitimate	
♂	♀	♂	♀
10·82 days	11·25 days	12·41 days	12·75 days

From these results it seems reasonable to conclude that the deaths during the first month of life may be centred, with sufficient accuracy for practical purposes, at ·3 month.

In order to test this result I have calculated the mean age at death of English infants dying under age 1, from the following statistics given in Newsholme's " Vital Statistics †."

Annual Death-rate per 1000 Living at each Month of Age.
Healthy Districts.

Age in Months	Death-rate	Age in Months	Death-rate
0	447·51	6	70·54
1	145·49	7	65·97
2	102·05	8	61·85
3	87·16	9	58·32
4	81·09	10	55·28
5	75·54	11	52·86

* The error made by centering each element of the frequency distributions of Table I. at the mid-point would tend to raise slightly the mean age at death for the year. Hence it is probable that ·3 year comes closer to the true value than ·31 year.

† Second Edition, p. 105.

Centering the mortality of the first month at ·3 month and that of each succeeding month at the mid-point I find for the mean 113·09 days, a result in very close agreement with that obtained from the German statistics.

For determining the mean duration of life of those dying within the age class "0–5 years" I have taken as a basis for computation the age distribution of the deaths falling in the first five years of life from the Report of the Twelfth Census of the United States, for the Registration Area. The distribution is as follows :

Age	Under 1	1	2	3	4	Total
Deaths	102220	25986	12020	7825	5520	153571

Now, evidently, the mean age at death for these five groups will fall between

$$\frac{102220 \times ·3 + 25986 \times 1·3 + 12020 \times 2·3 + 7825 \times 3·3 + 5520 \times 4·3}{153571} = ·9224,$$

and

$$\frac{102220 \times ·3 + 25986 \times 1·5 + 12020 \times 2·5 + 7825 \times 3·5 + 5520 \times 4·5}{153571} = ·9893,$$

if we assume essentially the same distribution of deaths under 1 year in the American population as is shown in the Prussian*. The mean of these two values is ·956 year. The true value probably lies somewhere between ·95 year and ·99 year. With a sufficient degree of accuracy for most cases the deaths recorded in the class "Under 5" may be centred at 1 year.

Briefly stated the results are, then, as follows:

(*a*) The deaths recorded in the age class "Under 1 month" may in practical work be centred at ·3 month.

(*b*) Those recorded in the age class "Under 1 year" may be centred at ·3 year.

(*c*) Those recorded as "Under 5 years" may be centred at 1 year.

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* Since the deaths in the second, third, fourth and fifth years of life clearly will not centre lower than those of the first year, nor higher than the mid-point of each year.

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